2 CHAPTER 2 Polynomials

WHAT THIS CHAPTER BUILDS

You will learn to find where a polynomial is zero, read those zeros off its graph, and tie them to its coefficients through the sum and the product.

ALGEBRA UNIT, BOARD CORE

IN THIS CHAPTER

Ananya and Aarav look up at a stone arch that rises over the river and meets the ground on each bank.

Getting started

Where a polynomial is zero

A STORY

Where the arch meets the ground

Ananya and Aarav stand by the river, looking up at the stone arch. The arch leaves the ground on one bank. It rises high overhead. It comes back down to the ground on the far bank.

"It meets the ground twice," Ananya says. "Once here, once on the far bank."

Aarav points up at the arch, then across to the far bank. "That is 40 m from here," he says. "I paced it out on the way here."

"The engineer had a rule for the arch before it was built," Ananya says. "A rule that gives both distances, without walking either one."

"Just from an equation?" Aarav asks.

"Just from an equation," Ananya says. "Two numbers come straight out of it. They are exactly where the arch meets the ground."

What equation would give you both of those numbers, before you walk to either bank?

Take $p(x) = x - 2$. Put $x = 2$ into it. $p(2) = 2 - 2 = 0$. A number that sends a polynomial's value to 0 is called a ZERO.

Say the same rule with a letter. k is a zero of $p(x)$ when $p(k) = 0$.

A zero is also a point on a graph. Draw $y = p(x)$ and you will see it. The zero is the point where the curve meets the x-axis. For $p(x) = x - 2$, that point is $(2, 0)$. Look at the picture and the zero sits right there. No working needed.

We use one fact for two jobs in this chapter.

- **Job one:** read a zero off the graph, at the x-axis crossing.
- **Job two:** find a zero from the equation, by working it out.

Both jobs land on the same numbers. For a quadratic $ax^2 + bx + c$, splitting the middle term finds both zeros. Their sum is $-b/a$. Their product is c/a. We reach both by the end of the chapter.

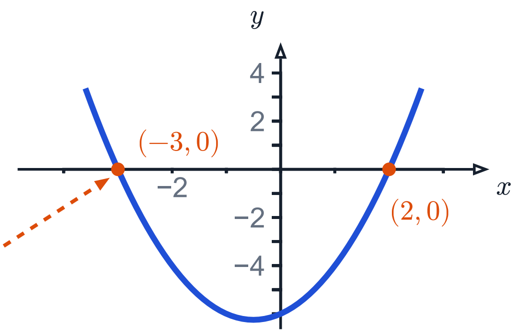
Your turn: is $x = 3$ a zero of $p(x) = x - 2$? (Answer: no. $p(3) = 3 - 2 = 1$, not 0.)

RULE
Before you trust a zero, check the graph. The number of crossings should match the number of roots you found.

KEY
A zero of a polynomial is not only computed. It is a point you can point to, right on the graph.

A zero, in the arithmetic and on the graph

$$\begin{aligned} p(x) &= x^2 + x - 6 \\ p(-3) &= (-3)^2 + (-3) - 6 \\ &= 9 - 3 - 6 \\ &= 0 \end{aligned}$$



Only the left crossing is worked out on the card, and the right one is marked and left for you to check.

A SCHOLAR INDIA REMEMBERS

Brahmagupta worked at Bhillamala, in today's Rajasthan, around 628 CE. He was the first to write down rules for zero as a number in its own right. You can add zero, take it away, and multiply by it. For example, $5 + 0 = 5$ *and* $5 \times 0 = 0$*. A zero of a polynomial is a value of* x *that makes the polynomial equal to that number,* 0*.*

Brahmagupta · the brahminy kite

IN THE WILD

A bridge's arch, seen from the side, is a curve. It meets the road at exactly two points, one at each foot. Write the arch's height as a polynomial in the distance along the road, and those two feet are its zeros. You can put a finger on them, not only solve for them.

CHECK YOURSELF

1. A zero of a polynomial $p(x)$ is a value k such that

- **A.** $k = 0$
- **B.** $p(x)$ equals k for every x
- **C.** $p(k) = 0$
- **D.** k is the degree of $p(x)$

2. For $p(x) = x^2 - 5x + 6$, which value of x is a zero?

- **A.** $x = 2$, since $p(2) = 4 - 10 + 6 = 0$
- **B.** $x = 6$, since 6 is the constant term
- **C.** $x = 5$, since 5 is the coefficient of x
- **D.** $x = 1$, since $p(1) = 1 - 5 + 6 = 2$ is close to zero

1 more in this set online.

IN EVERY CHAPTER

Before you start

DO THIS FIRST

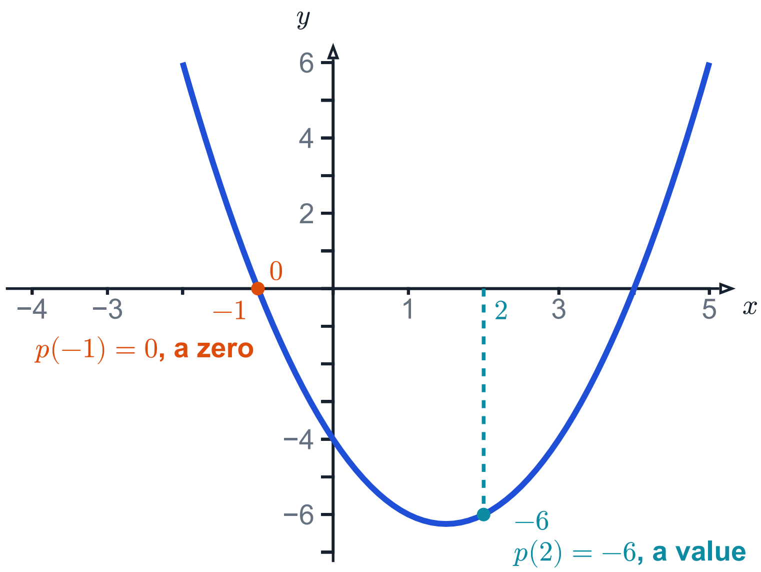
You have used letters for numbers before. Now those letters make a new kind of expression. Try each check below. It takes a minute.

- **Degree of a polynomial** (Class 9, Ganita Manjari, page 18).
  Here: a quadratic, like $x^2 + 5x + 1$, is named for reaching degree 2.
  Check: $x^2 + 5x + 1$ has degree 2, the highest power of x.
- **Substituting into a polynomial** (Class 9, Ganita Manjari, page 20).
  Here: finding $p(k)$ means substituting k for x, exactly as here.
  Check: $2 \cdot 4 + 3 = 11$.
- **A point on a line** (Class 9, Ganita Manjari, page 28).
  Here: a zero of $p(x)$ is a point where its graph crosses the x-axis.
  Check: $(7, 15)$ satisfies $y = 2x + 1$, since $2 \cdot 7 + 1 = 15$.
- **Difference of two squares** (Class 9, Ganita Manjari, page 77).
  Here: this identity turns $x^2 - 3$ into its two irrational zeros.
  Check: $5^2 - 2^2 = (5 - 2)(5 + 2) = 21$.
- **Factorising a quadratic** (Class 9, Ganita Manjari, page 81).
  Here: every split starts from two numbers with this sum and product.
  Check: $a + b = 7$ and $a \cdot b = 12$ give $a = 3$, $b = 4$.
- **Expanding two brackets** (Class 8, Ganita Prakash Part 1, page 140).
  Here: splitting the middle term ends in this bracket multiplication.
  Check: $(5 - 2)(3 + 4) = 5 \cdot 3 - 2 \cdot 3 + 5 \cdot 4 - 2 \cdot 4 = 21$.

If any of these felt new, read the page named before going on.

The value of a polynomial at a point

Two values of $p(x)$, one of them a zero



$p(2) = 4 - 6 - 4 = -6$ is a value. $p(-1) = 1 + 3 - 4 = 0$ is a value too — the special one we call a zero.

Every input gives a value, but only the inputs whose value is 0 earn the name zero.

Take $p(x) = x^2 - 3x - 4$. Put $x = 2$ in place of x. $p(2) = 2^2 - 3 \cdot 2 - 4$. Work it out one piece at a time: $2^2 = 4$, $3 \cdot 2 = 6$, so $p(2) = 4 - 6 - 4 = -6$. That number, -6, is the VALUE of $p(x)$ at $x = 2$.

Say it again with a letter. The value of $p(x)$ at $x = k$ is written $p(k)$. Find it the same way we just did. Put k in place of x everywhere in $p(x)$, then work through the arithmetic in order.

Your turn: find $p(-1)$ for the same polynomial. (Answer: $p(-1) = 1 + 3 - 4 = 0$.)

A zero is the special case where this value comes out to 0: $p(k) = 0$ means k is a zero of $p(x)$.

TRY
For $p(x) = x^2 - 3x - 4$, what is $p(2)$?

Answer: $p(2) = 4 - 6 - 4 = -6$.

BACK
Where a polynomial is zero · p. 37

KEY
$p(k)$ means put k in place of x in $p(x)$. Nothing more.

CHECK YOURSELF

3. For a polynomial $p(x)$, the value $p(3)$ means

- **A.** multiply $p(x)$ by 3
- **B.** substitute 3 for x
- **C.** add 3 to $p(x)$
- **D.** solve $p(x) = 3$ for x

4. If $p(x) = x^2 + 2x - 3$, then $p(-1)$ equals

- **A.** 0
- **B.** -4
- **C.** -6
- **D.** -1

1 more in this set online.

Degree names the polynomial

Look at $3x^2 - x + 5$. Find its highest power of x. It is 2. So its DEGREE is 2.

Let us say the same rule with words. The degree of a polynomial is the highest power of the variable in it.

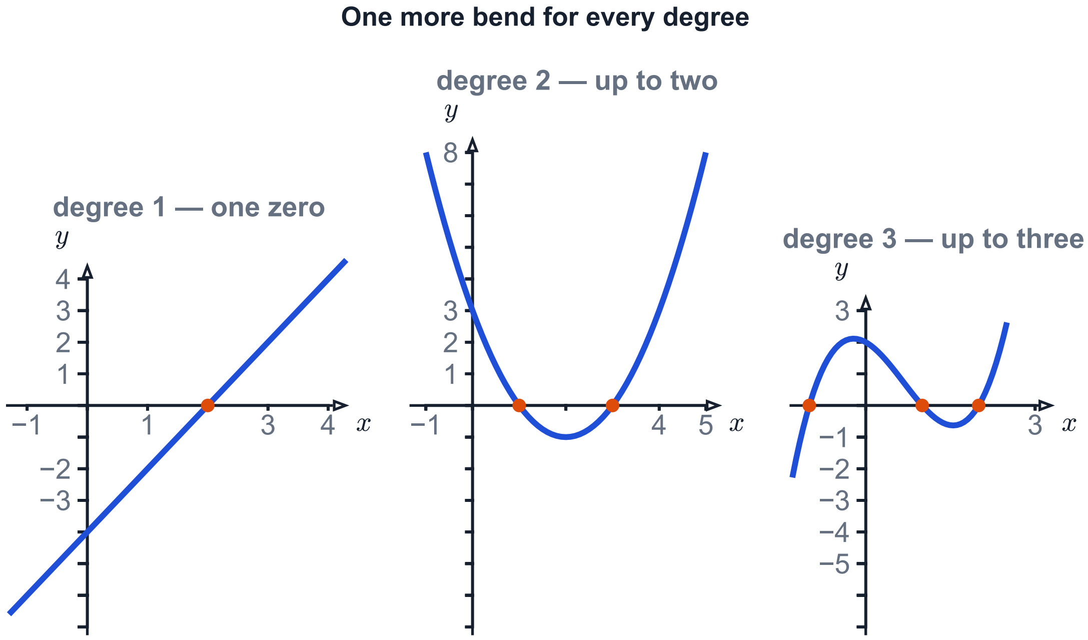
Your turn: what is the degree of $7x - 1$? (Answer: 1, the highest power of x.)

Degree gives each polynomial a name. Degree 1 is LINEAR, written $ax + b$. Degree 2 is QUADRATIC, written $ax^2 + bx + c$. Degree 3 is CUBIC. In every one of these, the leading coefficient is not 0. For the quadratic form, $a \neq 0$ is part of the name, not a separate check we do afterward.

A quadratic always has $a \neq 0$: drop that, and $ax^2 + bx + c$ is not a quadratic any more, it is the linear polynomial $bx + c$.

RULE
A quadratic always has $a \neq 0$. If $a = 0$, it drops to a linear polynomial.

KEY
Degree names the shape: 1 is a line, 2 is a parabola, 3 has one more bend.



A line crosses the x-axis once, a parabola crosses it twice, and a cubic crosses it three times.

CHECK YOURSELF

5. The degree of $p(x) = 5x^3 - 2x^2 + 7$ is

- **A.** 5
- **B.** 3
- **C.** 2
- **D.** 7

6. A quadratic polynomial is written $ax^2 + bx + c$ with the condition

- **A.** $b \neq 0$
- **B.** $c \neq 0$
- **C.** $a = 1$
- **D.** $a \neq 0$

1 more in this set online.

The ideas

A zero is where the graph meets the x-axis

Take the line $p(x) = 2x - 6$. Solve $2x - 6 = 0$. Add 6 to both sides: $2x = 6$. Divide by 2: $x = 3$. That is the zero.

Now picture the graph of $y = p(x)$, and you find the same zero. At $x = 3$, the curve sits exactly on the x-axis, at the point $(3, 0)$. Every point on the x-axis has $y = 0$. So a zero of $p(x)$ is the x-coordinate of a point where its graph meets the x-axis.

For any linear polynomial $ax + b$ with $a \neq 0$, the graph is a straight, tilted line. A tilted line meets the x-axis at exactly one point. Solve $ax + b = 0$ and we find that point directly: $x = -b/a$.

Your turn: check it on $p(x) = 2x - 6$. (Answer: $a = 2$, $b = -6$, and $-b/a = 6/2 = 3$, the same zero found above.)

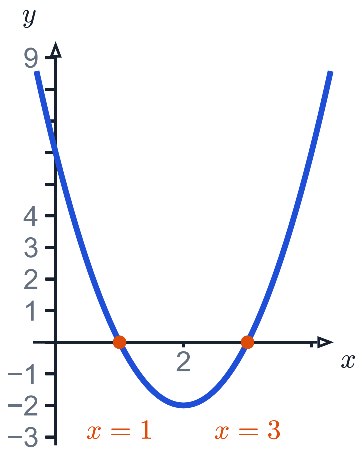
The graph and the algebra find the same zero: read it where the curve meets the x-axis, or solve $p(x) = 0$. Two ways, one answer.

AHEAD
A root satisfies the equation · p. 114

RULE
Read the zero straight off the graph. It is the x-coordinate where the curve touches or crosses the x-axis.

KEY
A zero is an x-axis crossing, not a y-axis crossing.

The crossings ARE the factors, read off the graph



The parabola crosses the x-axis at its two zeros, 1 and 3, exactly where the factored form says.

A SCHOLAR INDIA REMEMBERS

When we write $p(x) = 0$, what is the 0 telling you? It is the height of the graph. Take $p(x) = x^2 - 4$. At $x = 2$, $p(2) = 4 - 4 = 0$. At $x = -2$, $p(-2) = 4 - 4 = 0$. At both points the graph has height 0. So both points sit on the x-axis.

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CHECK YOURSELF

7. For the linear polynomial $p(x) = 2x - 6$, its zero is the x-coordinate where its graph meets the x-axis, namely

- **A.** $x = 3$
- **B.** $x = -3$
- **C.** $x = 6$
- **D.** $x = 2$

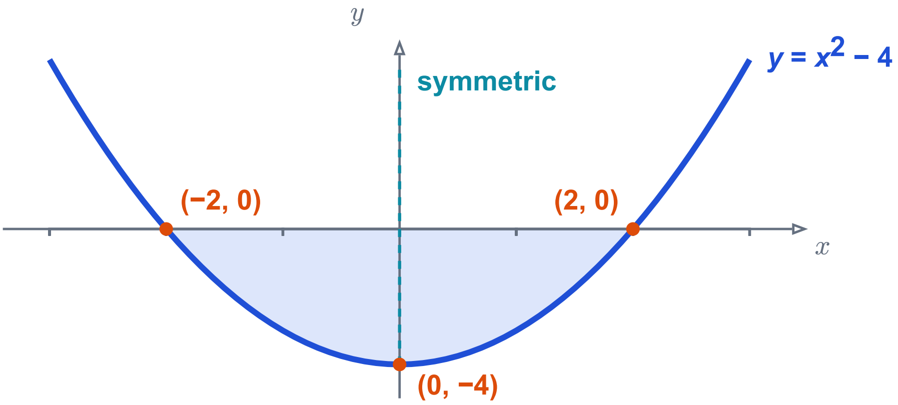
8. The graph of $y = p(x)$ passes through $(4, 0)$. What does this say about $p(x)$?

- **A.** $p(x) = 4$ for all x
- **B.** the y-intercept of $p(x)$ is 4
- **C.** the degree of $p(x)$ is 4
- **D.** 4 is a zero of $p(x)$

2 more in this set online.

The graph of a quadratic is a parabola

Three points make the parabola



three points, one smooth cup, lowest point at the bottom

Plot (−2, 0), (0, −4) and (2, 0), join them smoothly, and the cup appears: one turning point at the bottom, the two arms rising either side of the line $x = 0$.

Three points are enough to see the shape every quadratic graph shares: a cup with one turning point and a line of symmetry through it.

Take $y = x^2 - 4$. At $x = 0$, $y = -4$. At $x = 2$, $y = 0$. At $x = -2$, $y = 0$. Plot the three points and join them yourself. You get a smooth, symmetric curve with one turning point, a PARABOLA. Every quadratic $y = ax^2 + bx + c$, with $a \neq 0$, graphs the same way.

Let us say it again with the sign of a. When $a > 0$, the parabola opens upward, like a cup, and its turning point is the lowest point on the curve. When $a < 0$, it opens downward, like a dome, and the turning point is the highest point.

Your turn: which way does $y = x^2 - 4$ open? (Answer: upward, because $a = 1 > 0$.)

Where the curve actually crosses the x-axis is a separate question, taken up next.

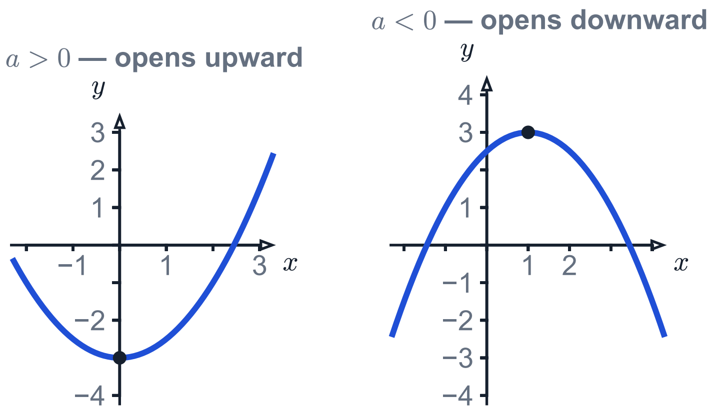
BACK
Degree names the polynomial · p. 40

TRY
Does $y = -2x^2 + x + 1$ open upward or downward?

Answer: Downward: $a = -2 < 0$.

KEY
Sign of a decides the direction: positive opens up, negative opens down.

The sign of a decides which way it opens



Two parabolas share the same size of leading coefficient. One opens upward like a cup; the other opens downward like a dome.

CHECK YOURSELF

9. The graph of $y = -3x^2 + x - 1$

A. opens upward, since -1 is the constant term

B. opens downward, since x has a positive coefficient

C. opens upward, since the highest power is even

D. opens downward, since $a = -3 < 0$

10. A parabola $y = ax^2 + bx + c$ **opens upward. What can be said about** a**?**

A. $a > 0$

B. $a < 0$

C. $a = 0$

D. $c > 0$

1 more in this set online.

Two zeros, one zero, or none

Let us look at three graphs.

- **Two zeros:** $y = x^2 - 4$ crosses the x-axis at $x = -2$ and $x = 2$.
- **One zero:** $y = (x-1)^2$ only touches the x-axis, at $x = 1$.
- **No zero:** $y = x^2 + 1$ never reaches the x-axis at all.

A parabola meets the x-axis in exactly one of these three ways, and no others.

A touch is one repeated zero. It is not zero zeros, because the curve does meet the axis, at $x = 1$. It is not two zeros either, because the curve never crosses to the other side.

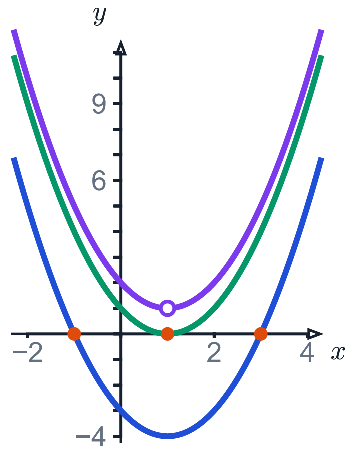
Your turn: check it yourself on $y = (x-1)^2$. Try an x value just below 1 and one just above 1. (Answer: both give a positive y, so the curve stays on one side of the axis both times.)

BACK
A zero is where the graph meets the x-axis · p. 41

RULE
A touch point is one repeated zero, not zero zeros and not two.

KEY
Two crossings, one touch, or no meeting at all: the only three shapes.

Two crossings, one, or none — the same family, sliding



blue, $c = -3$: two crossings
green, $c = 1$: one touch
purple, $c = 2$: no crossing

One frame with three parabolas sharing the same middle term. One crosses the x-axis twice, one touches it once, one never meets it.

A SCHOLAR INDIA REMEMBERS

Your graph never meets the x-axis. Did you find no zeros, or did you not draw far enough? We do not know yet, so let us check. Take $x^2 + 1$. x^2 is never below 0. So $x^2 + 1$ is at least 1, for every x. It never reaches 0. There are no zeros to find.

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CHECK YOURSELF

11. A parabola can meet the x-axis in

- **A.** two, one, or zero points
- **B.** exactly two points, always
- **C.** one point, always
- **D.** any number of points

12. A parabola touches the x-axis at exactly one point without crossing it. This means $p(x)$ has

- **A.** two distinct zeros
- **B.** one repeated zero
- **C.** no real zero, since the curve does not cross the axis
- **D.** an undefined number of zeros

2 more in this set online.

Degree caps the number of zeros

Take $y = x^2 - 5x + 6$. Its graph crosses the x-axis at $x = 2$ and $x = 3$. That is two zeros. Its degree is 2.

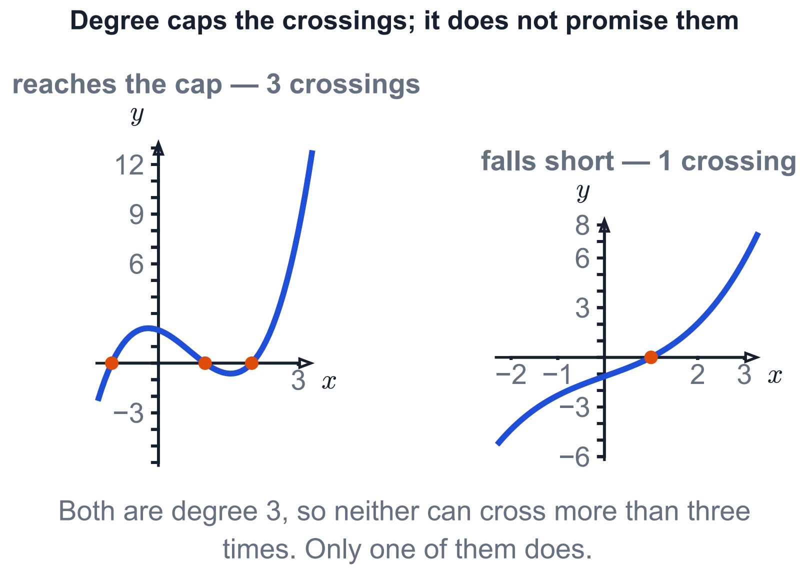
Try to add a third crossing to that same graph. We cannot: a quadratic's degree caps it at 2 zeros. A cubic, degree 3, can have at most 3 zeros. Say it with a letter: a degree-n polynomial has at most n zeros.

This turns counting into something you read straight off a graph, with no algebra at all. Count the points where the curve meets the x-axis. That count is the number of zeros. A single touch point counts once, not twice.

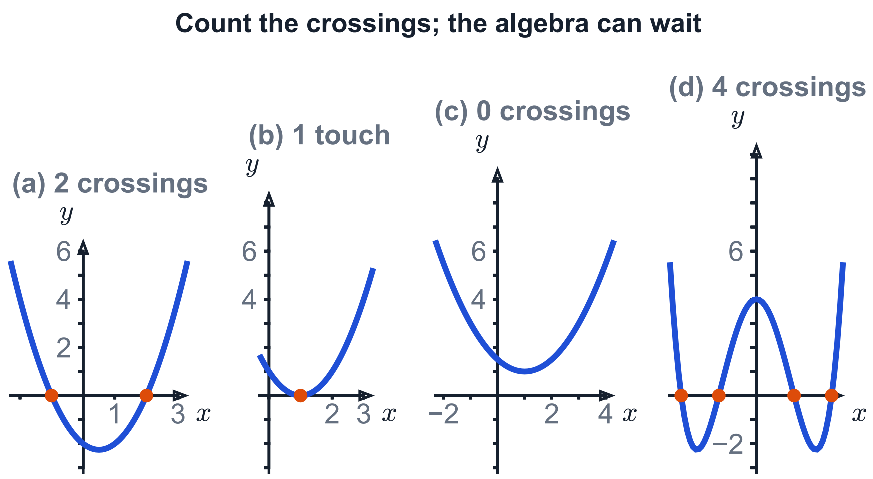
Your turn: a graph crosses the x-axis at four points. What is the smallest degree the polynomial can have? (Answer: 4, one for each crossing.)

CAUTION
A graph can meet the x-axis fewer times than the degree. It can never meet it more.

KEY
Degree caps the count: at most n zeros for a degree-n polynomial.



Read the degree as a ceiling on the number of zeros, not a promise that the graph reaches it.



Four small graphs cross the x-axis two, one, zero, and four times, read straight off each picture.

CHECK YOURSELF

13. A polynomial of degree 5 has, at most,

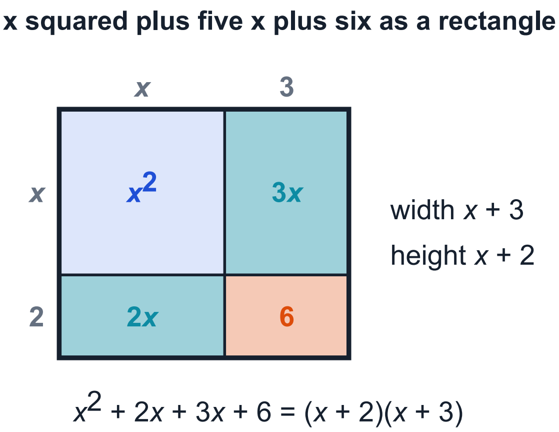
- **A.** 5 real zeros
- **B.** 4 real zeros
- **C.** 10 real zeros
- **D.** an unlimited number of real zeros

14. The graph of $y = p(x)$ meets the x-axis at 4 distinct points. What is the smallest possible degree of $p(x)$?

- **A.** 2
- **B.** 4
- **C.** 8
- **D.** 3

1 more in this set online.

Splitting the middle term finds the zeros



Split the middle term into 2x and 3x and the four pieces tile one rectangle whose sides are x + 2 and x + 3. The factors are the sides.

A rectangle of area x squared plus 2x plus 3x plus 6 splits into sides x plus 2 and x plus 3.

Take $x^2 + 5x + 6$. We need two numbers that multiply to 6 and add to 5. Those numbers are 2 and 3. This is splitting the middle term. It turns a quadratic into two factors, and each factor hands over one zero for free.

Say it again for any quadratic. For $ax^2 + bx + c$, find two numbers that multiply to ac and add to b. Use those two numbers to split the middle term bx into two pieces. We now have four terms instead of three.

Group the four terms in pairs, and pull the common factor out of each pair. The same bracket should appear behind both pairs. That shared bracket is one factor, and what is left outside it is the other.

Your turn: split $x^2 + 5x + 6$ this way. Group $x^2 + 2x + 3x + 6$ into two pairs. (Answer: $x(x + 2) + 3(x + 2)$, and $(x + 2)$ appears both times.)

Each zero of the quadratic is the value of x that makes one of the two factors equal to 0. Find the right pair of numbers first. Guessing at the grouping stage wastes the work already done.

AHEAD
Solving by factorisation · p. 117

RULE
Find the two numbers first, before you group. Guessing the split wastes the work.

KEY
Split bx into two terms. They multiply to ac and add to b. Then group.

CHECK YOURSELF

15. To split the middle term of $ax^2 + bx + c$, the two new coefficients must

- **A.** multiply to b and add to ac
- **B.** multiply to ac and add to b
- **C.** add to ac and multiply to b
- **D.** multiply to c and add to a

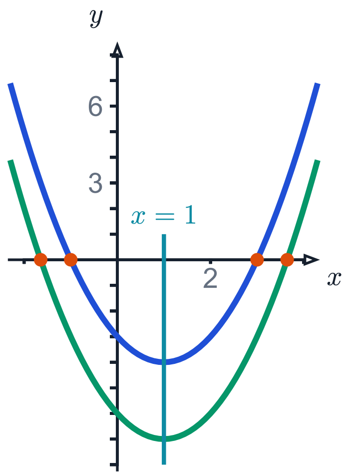
16. To split the middle term of $x^2 + 7x + 10$, the two numbers needed are

- **A.** 1 and 10
- **B.** 2 and 5
- **C.** -2 and -5
- **D.** 7 and 10

2 more in this set online.

Sum and product of a quadratic's zeros

Fix a and b: the sum stays pinned, the pair moves



blue: $c = -3$ **green:** $c = -6$

Two parabolas share the same axis of symmetry, so both pairs of zeros add to the same sum, 2, even though their products differ.

Take $x^2 - 5x + 6$, with zeros 2 and 3. Add them: $2 + 3 = 5$. Multiply them: $2 \cdot 3 = 6$.

Now check both against the coefficients. $-b/a = -(-5)/1 = 5$, and that matches the sum. $c/a = 6/1 = 6$, and that matches the product. Once we know a quadratic's coefficients, we can read off its zeros' sum and product directly. No factorising is needed first.

Say it again for any quadratic. For $ax^2 + bx + c$ with $a \neq 0$ and zeros α, β: the sum $\alpha + \beta$ equals $-b/a$, and the product $\alpha\beta$ equals c/a.

Your turn: for $x^2 + x - 12 = (x + 4)(x - 3)$, the zeros are -4 and 3. (Answer: sum $-4 + 3 = -1$, matching $-b/a = -1/1 = -1$; product $-4 \cdot 3 = -12$, matching $c/a = -12/1 = -12$.)

The minus sign belongs to the sum only, never to the product.

RULE
Sum uses $-b/a$. Product uses c/a. The minus sign belongs to the sum only.

KEY
You do not need to solve for the zeros first. Their sum and product come straight from a, b, c.

CHECK YOURSELF

17. For $ax^2 + bx + c$ with zeros α, β, the sum $\alpha + \beta$ equals

- **A.** $-b/a$
- **B.** b/a
- **C.** $-c/a$
- **D.** a/b

18. For $ax^2 + bx + c$ with zeros α, β, the product $\alpha\beta$ equals

- **A.** $-c/a$
- **B.** c/a
- **C.** b/a
- **D.** a/c

3 more in this set online.

Building a quadratic from sum and product

Suppose we want a quadratic whose zeros add to -3 and multiply to 2. Try $x^2 - (-3)x + 2 = x^2 + 3x + 2$. Factorise it: $(x+1)(x+2)$, with zeros -1 and -2. You can check: they add to -3 and multiply to 2, exactly as wanted.

Say it again with letters. Given any target sum s and product p, the polynomial $x^2 - sx + p$ has exactly those zeros.

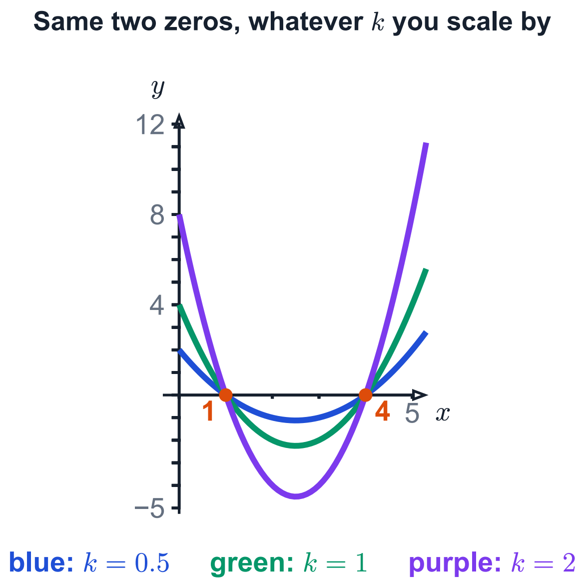
Multiply that polynomial by any nonzero constant k. The result, $kx^2 - ksx + kp$, has the same pair of zeros. The constant scales every term equally, so it cancels out of both the sum and the product.

Your turn: build a quadratic whose zeros add to 5 and multiply to 6. (Answer: $x^2 - 5x + 6$.)

TRY
Sum $= 4$, product $= 1$. Which quadratic?

Answer: $x^2 - 4x + 1$.

KEY
Any target sum s and product p give $x^2 - sx + p$ as one matching quadratic.



Three parabolas, scaled by 0.5, 1 and 2, all cross the x-axis at the same two zeros, 1 and 4.

CHECK YOURSELF

19. Given a target sum s and product p for the zeros, a matching quadratic is

- **A.** $x^2 + sx + p$
- **B.** $x^2 - sx + p$
- **C.** $x^2 - sx - p$
- **D.** $x^2 - px + s$

20. A quadratic whose zeros have sum 4 and product -3 is

- **A.** $x^2 + 4x - 3$
- **B.** $x^2 - 4x + 3$
- **C.** $x^2 - 4x - 3$
- **D.** $x^2 - 3x + 4$

2 more in this set online.

Common mistakes

A zero is not the y-intercept

Take $p(x) = x^2 - 3x - 4$. Its zeros are -1 and 4: $p(-1) = 0$ and $p(4) = 0$. Both points sit on the x-axis.

Now we look at $x = 0$. $p(0) = -4$. That value, -4, is where the curve crosses the y-axis, the Y-INTERCEPT of $p(x)$.

A zero and a y-intercept are two different questions about the same graph. A zero asks where the curve meets the x-axis. The y-intercept asks only what $p(0)$ is. It is easy to reach for the wrong one, because both are single numbers read straight off a graph.

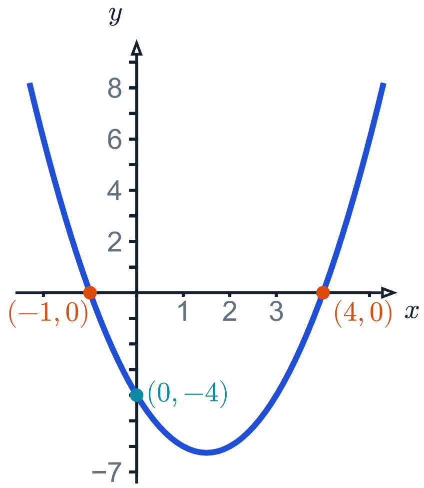
Your turn: is -4, the y-intercept above, also a zero of $p(x)$? (Answer: no. $p(-4) = 16 + 12 - 4 = 24$, not 0.)

CAUTION
A zero and a y-intercept are different numbers unless 0 itself is a zero.

BACK
A zero is where the graph meets the x-axis · p. 41

KEY
Always check: does the graph cross the x-axis there, or the y-axis?

The zero and the y-intercept are different points



The graph shows three points: the two zeros, at minus 1 and 4, and the y-intercept, at minus 4.

IS THE CONSTANT TERM ALWAYS A ZERO YOU CAN READ STRAIGHT OFF A QUADRATIC?

WEAK
A student is asked for a zero of $p(x) = x^2 - x - 6$. They look at the constant term. “The zero is -6,” they say. “That is where the graph crosses the axis.”

They do not check it. Put -6 into $p(x)$: $p(-6) = 36 + 6 - 6 = 36$, not 0. -6 is not a zero at all.

STRONG
Factorise first. $x^2 - x - 6 = (x - 3)(x + 2)$, so the zeros are 3 and -2: $p(3) = 0$ and $p(-2) = 0$.

The constant term, -6, is the y-intercept, $p(0) = -6$. It answers a different question: where the curve crosses the y-axis, not the x-axis.

A SCHOLAR INDIA REMEMBERS
Two different zeros are at work here. Keep them apart. Take $p(x) = x^2 - 5x + 6$. Put $x = 0$ and you get 6. That is where the graph crosses the y-axis. Now put $p(x) = 0$. You get $x = 2$ and $x = 3$. Those are the zeros. One zero goes in, the other comes out.
Brahmagupta · the brahminy kite

WATCH OUT
The trap: The student computes $p(0) = -4$ for the quadratic and marks -4 as one of its zeros.

The correction: A zero is where $p(x)$ crosses the x-axis, not the y-axis; the y-intercept, $p(0)$, is a different number unless 0 happens to be a zero too.

CHECK YOURSELF

21. A student claims: “the zero of a polynomial $p(x)$ is the point where its graph crosses the y-axis.” Is this correct?

- **A.** yes, since both describe a point where the graph meets an axis
- **B.** no, because a zero is always the same number as the y-intercept
- **C.** no — it crosses the x-axis, not the y-axis
- **D.** no, because a polynomial’s graph never touches the y-axis

22. For $p(x) = x^2 - 3x - 4$, the zeros are -1 and 4. What is the y-intercept?

A. $p(0) = -4$

B. -1, since that is one of the zeros

C. 4, since that is the other zero

D. 0, since the y-intercept is where $x = 0$ crosses the curve

2 more in this set online.

The sum and the product are divided by a

Take $p(x) = 2x^2 - 8x + 6$, with zeros 1 and 3. Add them: $1 + 3 = 4$. That uses a: $-b/a = -(-8)/2 = 4$. Multiply them: $1 \cdot 3 = 3$. That uses a too: $c/a = 6/2 = 3$.

We now turn to b and c alone, without a. $-b = 8$ and $c = 6$. Those numbers are not the sum and the product here. Dividing by a is not optional. It is part of both relations.

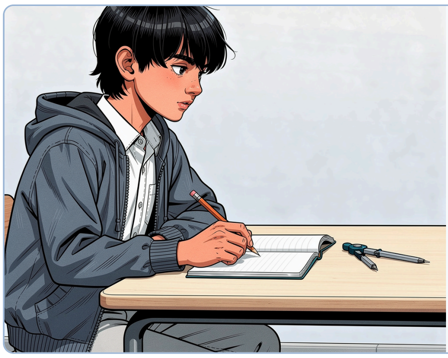
Your turn: for $p(x) = 3x^2 - 9x + 6$, what is a? (Answer: $a = 3$, so both the sum and the product must be divided by 3.)

Sum $= -b/a$. Product $= c/a$. Never $-b$ and c on their own, unless $a = 1$.

BACK
Sum and product of a quadratic's zeros · p. 48

CAUTION
Before writing the sum or the product, look at a. If $a \neq 1$, divide by it.

KEY
Sum $= -b/a$, product $= c/a$; never $-b$ and c on their own.



FIRST TRY
~~For $2x^2 - 8x + 6$ I took the sum of the zeros as 8 and the product as 6.~~

SECOND LOOK
Those numbers are $-b$ and c only when $a = 1$. Here $a = 2$, so the sum is $-(-8)/2 = 4$ and the product is $6/2 = 3$. Both match the zeros 1 and 3.

The sum of the zeros is $-b/a$ and the product is c/a. Never drop the leading coefficient a.

DO THE SUM AND PRODUCT EVER SKIP DIVIDING BY THE LEADING COEFFICIENT?

WEAK
A student is asked for the sum and product of the zeros of $p(x) = 3x^2 - 9x + 6$. They write sum $= -b = 9$ and product $= c = 6$. "That is what the rule says," they say, pointing at $-b$ and c straight from the coefficients.

This feels right, because it worked on every earlier example, where a happened to be 1. It is wrong here, because $a = 3$, not 1.

STRONG
Look at a first. Here $a = 3$. Sum $= -b/a = -(-9)/3 = 3$. Product $= c/a = 6/3 = 2$.

Check it against the zeros themselves: $3x^2 - 9x + 6 = 3(x-1)(x-2)$, whose zeros are 1 and 2. They add to 3 and multiply to 2, exactly as the coefficients say.

Worked examples

WORKED EXAMPLE · **Zeros of $2x^2 - 8x + 6$ by splitting the middle term**

1.	$(-6) + (-2) = -8$ and $(-6) \cdot (-2) = 12 = 2 \cdot 6$	*We need two numbers that multiply to $ac = 2 \cdot 6 = 12$ and add to $b = -8$. Those numbers are -6 and -2.*
2.	$2x^2 - 8x + 6 = 2x^2 - 6x - 2x + 6$	*Split the middle term into the two pieces we just found.*
3.	$= 2x(x-3) - 2(x-3)$	*Group the four terms in pairs. Pull the common factor out of each pair.*
4.	$= (2x-2)(x-3) = 2(x-1)(x-3)$	*The shared bracket $(x-3)$ factors out. What is left behind is the second factor.*
5.	$x = 1$ or $x = 3$	*Each zero makes one factor equal to 0.*
6.	$p(1) = 2 - 8 + 6 = 0, \quad p(3) = 18 - 24 + 6 = 0$	CHECK ***Check both zeros yourself. Put them back into the original polynomial.*** *Both give 0, so 1 and 3 really are the zeros.*

RULE
Find the split first. Two numbers multiply to ac and add to b. Then group in pairs.

BACK
Splitting the middle term finds the zeros · p. 47

TRY
Find the zeros of $p(x) = 2x^2 - 6x + 4$ by splitting the middle term.

Answer: Zeros 1 and 2: split -6 as -2 and -4 ($(-2) \cdot (-4) = 8 = 2 \cdot 4$).

FIND THE ZEROS OF ANY QUADRATIC BY SPLITTING THE MIDDLE TERM.

1. **Find the two numbers** For $2x^2 - 8x + 6$, find two numbers that multiply to $ac = 2 \cdot 6 = 12$ and add to $b = -8$. Those numbers are -6 and -2.
2. **Split the middle term** Rewrite $-8x$ as $-6x - 2x$. $2x^2 - 8x + 6 = 2x^2 - 6x - 2x + 6$.
3. **Group in pairs** Group the first two terms and the last two. Pull the common factor out of each pair. $2x(x-3) - 2(x-3)$.
4. **Factor out the shared bracket** The bracket $(x-3)$ appears in both pairs. $(2x-2)(x-3) = 2(x-1)(x-3)$.
5. **Read off the zeros** Each factor equal to 0 gives a zero. $x = 1$ or $x = 3$. Check: $p(1) = 2 - 8 + 6 = 0$, $p(3) = 18 - 24 + 6 = 0$.

The ball rises from Meera's kick and comes back down to the ground, where its height is zero.

CHECK YOURSELF

23. In factorising $2x^2 - 8x + 6$, the middle term $-8x$ is split into

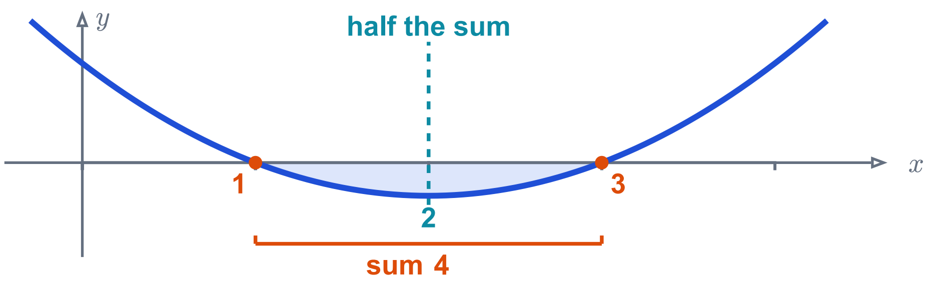
- **A.** $-3x$ and $-5x$
- **B.** $-6x$ and $-2x$
- **C.** $-1x$ and $-7x$
- **D.** $-4x$ and $-4x$

24. After splitting, $2x^2 - 8x + 6$ becomes $2x^2 - 6x - 2x + 6$, which groups into

- **A.** $2x(x-3) - 2(x-3)$
- **B.** $2x(x-3) + 2(x-3)$
- **C.** $2x(x+3) - 2(x+3)$
- **D.** $(2x-6)(x-2)$

1 more in this set online.

The zeros sit either side of the axis



the zeros 1 and 3 sit either side of $x = 2$; their sum 4 is −*b*/*a* = 8/2

The sum of the zeros is −*b*/*a*, and half of it, 2, is the line the parabola is symmetric about. The two zeros of $2x^2$ − 8x + 6 sit one unit either side of it.

The coefficients leave a mark on the picture: minus b over a fixes where the two zeros balance, and the curve folds over that line.

WORKED EXAMPLE · Where the sum-product relation comes from

1.	$ax^2 + bx + c = k(x-\alpha)(x-\beta)$	*Any quadratic with zeros α, β is some nonzero constant k times these two factors.*
2.	$k(x-\alpha)(x-\beta) = kx^2 - k(\alpha+\beta)x + k\alpha\beta$	*Expand the right-hand side.*
3.	$a = k$	*Compare the coefficients of x^2 on both sides.*
4.	$b = -k(\alpha+\beta)$	*Compare the coefficients of x.*
5.	$c = k\alpha\beta$	*Compare the constant terms.*
6.	$\alpha+\beta = -b/a$, $\alpha\beta = c/a$	***Put $k = a$ into the last two equations, and solve for the sum and product yourself.*** *This is where $-b/a$ and c/a come from.*
7.	$1+3 = 4 = -(-8)/2$ and $1 \cdot 3 = 3 = 6/2$	CHECK ***Check the new relation against the zeros already found for $2x^2 - 8x + 6$.*** *Sum 4 and product 3 match $-b/a$ and c/a exactly.*

RULE
Compare the coefficients of x^2, x and the constant separately. Three equations pin down k, the sum and the product.

CHECK YOURSELF

25. In deriving the sum-product relation, $ax^2 + bx + c$ is first written as

- **A.** $k(x+\alpha)(x+\beta)$
- **B.** $k(x-\alpha)(x-\beta)$
- **C.** $(x-\alpha)(x-\beta)$ with no constant k
- **D.** $k(x-\alpha)+(x-\beta)$

26. Expanding $k(x-\alpha)(x-\beta)$ gives $kx^2 - k(\alpha+\beta)x + k\alpha\beta$. Comparing this to ax^2+bx+c term by term gives

- **A.** $a = k$, $b = k(\alpha+\beta)$, $c = k\alpha\beta$
- **B.** $a = -k$, $b = k(\alpha+\beta)$, $c = -k\alpha\beta$
- **C.** $a = k\alpha\beta$, $b = -k(\alpha+\beta)$, $c = k$
- **D.** $a = k$, $b = -k(\alpha+\beta)$, $c = k\alpha\beta$

1 more in this set online.

WORKED EXAMPLE · Zeros of x^2-3, verified

1.	$x^2-3 = \left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)$	*x^2-3 is a difference of squares, $x^2-\left(\sqrt{3}\right)^2$. Factor it using $a^2-b^2=(a-b)(a+b)$.*
2.	$x=\sqrt{3}$ or $x=-\sqrt{3}$	*Each zero makes one factor equal to 0.*
3.	$\text{sum} = \sqrt{3}+\left(-\sqrt{3}\right)=0$	*Add the two zeros.*
4.	$-b/a = -0/1 = 0$	***This polynomial has no x term, so $b=0$.** The sum matches.*
5.	$\text{product} = \sqrt{3}\cdot\left(-\sqrt{3}\right) = -3$	*Multiply the two zeros.*
6.	$c/a = -3/1 = -3$	CHECK ***The product matches too. Check it yourself.** Both zeros are irrational, but the sum and the product still come out as whole numbers.*

TRY
Find the zeros of $p(x)=x^2-5$ using $a^2-b^2=(a-b)(a+b)$.

Answer: Zeros $\sqrt{5}$ and $-\sqrt{5}$; product $\sqrt{5}\cdot\left(-\sqrt{5}\right)=-5$.

KEY
A sum of 0 is still a sum. It means the two zeros are equal and opposite.

Two irrational zeros, mirrored exactly about 0

A number line marks the two zeros of the quadratic, the square root of 3 and the same distance on the other side of 0.

FIND AND CHECK THE ZEROS OF A QUADRATIC WITH NO MIDDLE TERM.

1. **Write it as a difference of squares** x^2-3 is $x^2-\left(\sqrt{3}\right)^2$. Factor it as $\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)$.
2. **Read off the zeros** Each factor equal to 0 gives a zero. $x=\sqrt{3}$ or $x=-\sqrt{3}$.
3. **Add the zeros and check** $\sqrt{3}+\left(-\sqrt{3}\right)=0$. Since $b=0$ here, $-b/a=-0/1=0$. It matches.
4. **Multiply the zeros and check** $\sqrt{3}\cdot\left(-\sqrt{3}\right)=-3$. And $c/a=-3/1=-3$. It matches too.

CHECK YOURSELF

27. $p(x) = x^2 - 3$ **factorises, using** $a^2 - b^2 = (a-b)(a+b)$**, as**

- **A.** $(x-3)(x+1)$
- **B.** $\left(x-\sqrt{3}\right)\left(x-\sqrt{3}\right)$
- **C.** $\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)$
- **D.** $(x-9)(x+9)$

28. **For** $p(x) = x^2 - 3$**, there is no** x **term, so in** $ax^2 + bx + c$ **this means**

- **A.** $b = 0$
- **B.** $a = 0$
- **C.** $c = 0$
- **D.** $b = 3$

1 more in this set online.

WORKED EXAMPLE · **Building a quadratic from sum -3, product 2**

1.	$s = -3, p = 2$	*Read off the target sum and product.*
2.	$x^2 - sx + p = x^2 - (-3)x + 2 = x^2 + 3x + 2$	*Put s and p into $x^2 - sx + p$.*
3.	$x^2 + 3x + 2 = (x+1)(x+2)$	***Check the answer by factorising it.*** *Splitting the middle term should recover the target zeros.*
4.	zeros -1 and -2, sum -3, product 2	**CHECK** ***Both match the original target.*** *Factorise your own answer the same way, and check that it comes out too.*

BACK
Building a quadratic from sum and product · p. 49

TRY
Sum $= 5$, product $= 6$. Which quadratic has these zeros?

Answer: $x^2 - 5x + 6$.

RULE
Build the quadratic. Then factor it back. Check both the sum and the product.

BUILD A QUADRATIC WHEN YOU ONLY KNOW THE SUM AND PRODUCT OF ITS ZEROS.

1. **Read off the target sum and product** For a target sum -3 and product 2, set $s = -3$ and $p = 2$.
2. **Build the quadratic** Put s and p into $x^2 - sx + p$. $x^2 - (-3)x + 2 = x^2 + 3x + 2$.
3. **Check by factorising** $x^2 + 3x + 2 = (x+1)(x+2)$. The zeros are -1 and -2. They add to -3 and multiply to 2, matching the target.

CHECK YOURSELF

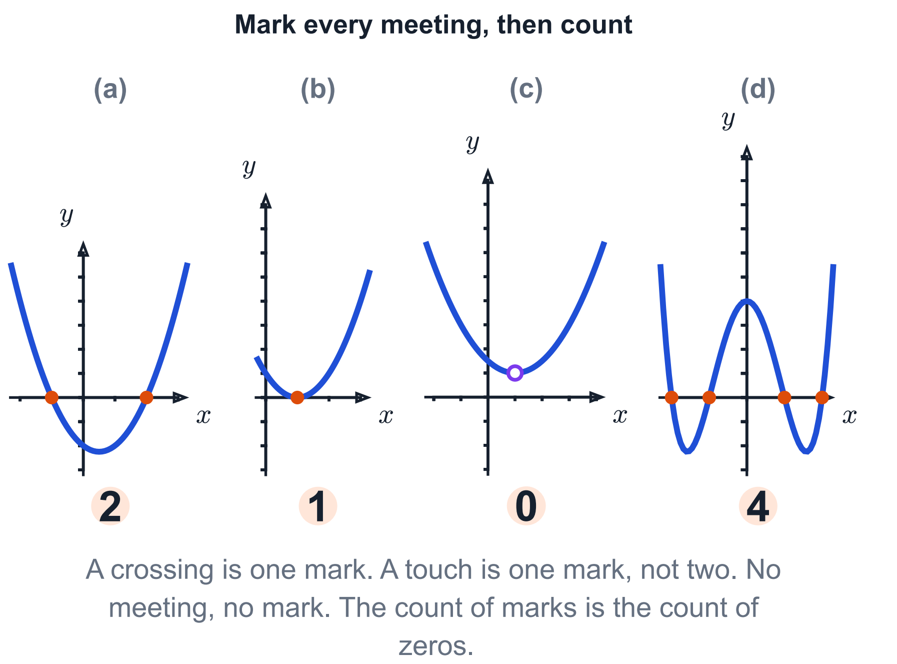
29. A quadratic whose zeros have sum -3 and product 2 is built as $x^2 - sx + p$ with $s = -3, p = 2$, giving

- **A.** $x^2 - 3x + 2$
- **B.** $x^2 + 3x + 2$
- **C.** $x^2 + 3x - 2$
- **D.** $x^2 - 3x - 2$

30. Checking $x^2 + 3x + 2 = (x+1)(x+2)$ by factorising, the zeros are

- **A.** 1 and 2
- **B.** -1 and 2
- **C.** 3 and 2
- **D.** $-1, -2$

1 more in this set online.



Four graphs meet the x-axis 2, 1, 0 and 4 times, giving each one's count of zeros.

WORKED EXAMPLE · Counting zeros from four graphs

1. graph (a) meets the x-axis at exactly two points — *Two distinct crossings give two distinct zeros.*
2. graph (b) touches the x-axis at exactly one point, without crossing it — ***A touch counts as one zero, not two and not zero.*** *It is a single repeated zero.*
3. graph (c) never meets the x-axis — *No crossing and no touch means no real zero at all.*
4. graph (d) meets the x-axis at exactly four points — *The polynomial behind it must have degree at least 4. A graph can never meet the x-axis more times than its degree.*

WORKED EXAMPLE · **Counting zeros from four graphs**

5. marks on (a) 2, on (b) 1, on (c) 0, on (d) 4

CHECK ***Go back over the four graphs yourself. Put a mark wherever the curve meets the axis, then count the marks.*** *The four numbers must match the counts just read off.*

BACK
Two zeros, one zero, or none · p. 44

TRY
A graph of $y = p(x)$ meets the x-axis at 3 points. What is the smallest degree $p(x)$ can have?

Answer: Degree 3. Each of the 3 crossings is a zero, and a polynomial has no more zeros than its degree.

KEY
Touching counts as one zero, not two and not zero.

COUNT THE ZEROS OF A POLYNOMIAL STRAIGHT OFF ITS GRAPH, WITH NO ALGEBRA.

1. **Count the crossings** Count every point where the curve meets the x-axis. Each crossing is one zero.
2. **Count a touch as one** A curve that only touches the axis, without crossing to the other side, still counts as one zero, not two and not zero.
3. **Match the count to the degree** The crossing count is the smallest degree the polynomial can have. A graph never meets the x-axis more times than its degree.

CHECK YOURSELF

31. A graph of $y = p(x)$ meets the x-axis at exactly two points. How many zeros does $p(x)$ have?

A. 1

B. 4

C. an unknown number, since the graph alone cannot say

D. 2

32. A graph touches the x-axis at one point without crossing it. This is described as

A. 1 repeated zero

B. 0 zeros, since the curve does not cross

C. 2 zeros at the same point

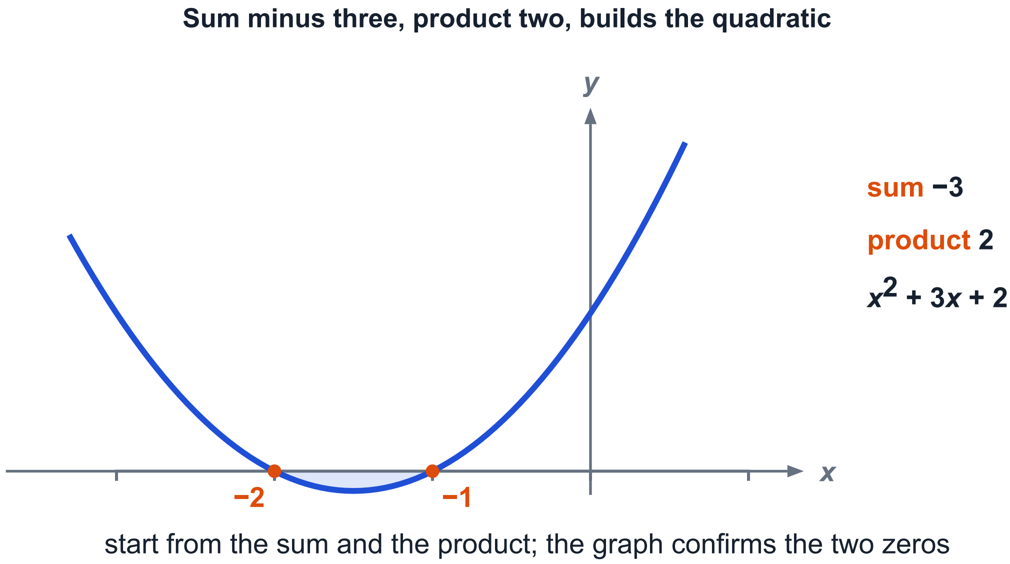
D. an undefined number of zeros

1 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap



Choose the sum and the product, write x^2 − (sum)x + product, and the graph meets the axis exactly where the algebra said: at −1 and at −2.

The curve y equals x squared plus 3x plus 2 crosses the x-axis at minus 2 and minus 1, sum minus 3, product 2.

This one polynomial shows a zero, a graph point, a degree cap, and a sum and product, all together. Take $p(x) = 2x^2 - 8x + 6$.

- **Zero:** $p(k) = 0$. Here $p(1) = 0$.
- **Graph:** a zero is a point where $y = p(x)$ meets the x-axis. Here, $(1, 0)$ and $(3, 0)$.
- **Degree cap:** a quadratic has at most 2 zeros. This one has exactly 2, matching its degree.
- **Sum and product:** the zeros add to $-b/a$ and multiply to c/a. Here a is 2, b is -8 and c is 6. So they add to $8/2$, which is 4, and multiply to $6/2$, which is 3. The zeros 1 and 3 fit both.
- **Reverse:** a target sum -3 and product 2 build $x^2 + 3x + 2$, whose zeros are -1 and -2.

We looked at one number, $k = 1$, three ways. It is an algebra fact and a graph point. It is also part of a sum and a product, both read off a, b, c.

Your turn: check the second zero, 3, against the sum and product above. (Answer: 3 is the other zero used in the sum $1 + 3 = 4$ and the product $1 \cdot 3 = 3$ already shown.)

BACK
A zero is where the graph meets the x-axis · p. 41

KEY
A zero is where $p(k) = 0$ and where the graph meets the x-axis. For $ax^2 + bx + c$, the zeros add to $-b/a$ and multiply to c/a.

CHECK YOURSELF

33. This chapter's zero-finding method (splitting the middle term) and its reverse (building a quadratic from a sum and product) are related because

A. they run the same relation in opposite directions

B. they use entirely different formulas with no shared relation

C. the reverse method only works when the zeros are irrational

D. splitting the middle term and building from sum-product always give the same final quadratic

34. A quadratic's zero, seen algebraically as $p(k) = 0$ and seen graphically as a point on the x-axis, are

A. two separate properties that happen to coincide sometimes

B. the same fact about $p(x)$, described in two different ways

C. only equal when the polynomial is linear

D. different concepts, since one is a number and the other is a point

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

35. A zero of a polynomial $p(x)$ is a value k with $p(k) = 0$. On the graph of $y = p(x)$, what does a zero correspond to?

A. the value of $p(x)$ when $x = 0$

B. the highest point on the curve

C. the point where the curve crosses the y-axis

D. a point where the curve meets the x-axis

36. The graph of the linear polynomial $p(x) = 2x - 6$ is a straight line. Where does it meet the x-axis?

A. at $(0, -6)$

B. at $(3, 0)$

C. at $(0, 0)$

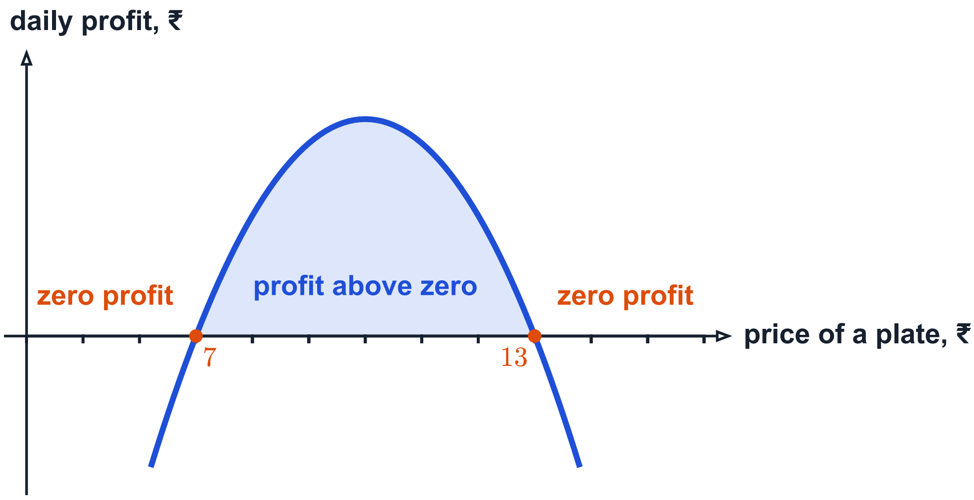
D. at $(-3, 0)$

3 more in this set online.

IN EVERY CHAPTER

Where you will meet this

The profit curve crosses the price axis twice



Profit is $-(x-7)(x-13)$. It is 0 at $x=7$ and $x=13$: two zeros, two break-even prices.

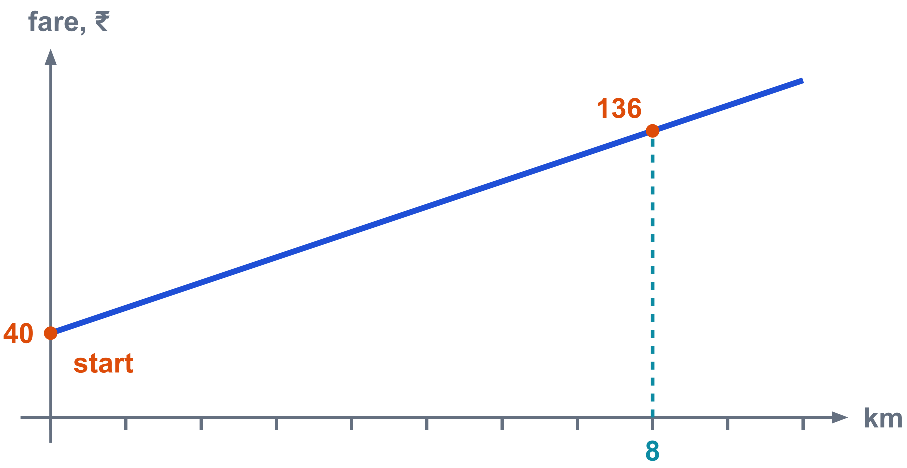
Between the two zeros the stall earns; outside them it loses, so the zeros are the prices to stay between.

You may never solve a polynomial for fun after Class 10. But shops price things using formulas just like these. Here are seven of those places.

- **Paying for a cab ride.** Your cab charges a fixed ₹40 to start. Then it charges ₹12 for every km you ride.
  The maths: the fare for x km is the value of that polynomial at x.
  For $x = 8$ km: $40 + 12 \cdot 8 = 136$ rupees.
- **Pricing a custom rug.** A shop prices a square rug at ₹150 for every square foot of its area.
  The maths: the price is degree 2 in the side length, since area is the side squared. That makes it a quadratic.
  For a side of 6 feet: $150 \cdot 6^2 = 5400$ rupees.
- **Reading a stall's profit chart.** A stall's daily profit is plotted against the price of a plate. The curve crosses the price axis at two points.
  The maths: each point where the curve crosses the axis is a zero of the profit polynomial. Two zeros here means two prices give zero profit.
  Profit is $-(x-7)(x-13)$ rupees, for a plate priced at x rupees. It is 0 at $x = 7$ and $x = 13$.

BACK
The value of a polynomial at a point · p. 39

The fare line starts at forty



fare at 8 km is 40 + 12 · 8 = 136

A fixed charge is where the line meets the fare axis; the per-kilometre rate is its slope. Reading the fare for 8 km is evaluating the polynomial at $x = 8$.

A taxi meter runs a degree-one polynomial, and every fare it shows is one value of that polynomial.

- **Finding a lemonade stand's break-even prices.** Your lemonade stand's daily profit is $-x^2 + 34x - 240$ rupees, for a glass priced at x rupees.
  The maths: splitting the middle term finds the exact prices where profit is 0.
  Split 34 into 10 and 24: $10 \cdot 24 = 240$ and $10 + 24 = 34$. So profit is 0 at $x = 10$ and $x = 24$.
- **Checking a stall's break-even claim.** A friend says their samosa stall breaks even at ₹9 and ₹15 a plate. Their profit formula is $-x^2 + 24x - 135$.
  The maths: the sum and product of the zeros come straight from the coefficients, with no solving.
  Sum $= 24$ and product $= 135$. And $9 + 15 = 24$, $9 \cdot 15 = 135$. It matches.
- **Writing a vendor's profit formula.** A vendor breaks even selling 3 kg or 12 kg of guavas a day. That is all you know about the profit formula.
  The maths: knowing just the sum and product of the zeros builds the whole quadratic shape.
  Sum $= 3 + 12 = 15$ and product $= 3 \cdot 12 = 36$. So the profit is a multiple of $x^2 - 15x + 36$, which is 0 at $x = 3$ and $x = 12$.
- **Planning a rectangular garden.** You have 40 m of fence for a rectangular garden. If one side is x m, the other is $20 - x$ m.
  The maths: the area $20x - x^2$ has degree 2, so it has at most two zeros.
  The zeros are $x = 0$ and $x = 20$, the two widths with no garden at all. At $x = 10$ the area is 100 square m.

Your turn. A juice stall's profit is $-x^2 + 16x - 39$ rupees, for a glass priced at x rupees. At what two prices does it break even? *Answer:* Split 16 into 3 and 13: $3 \cdot 13 = 39$ and $3 + 13 = 16$. So it breaks even at $x = 3$ and $x = 13$ rupees.

Practice: Exercise 2.1

PRACTICE SET · **Count the zeros from the graph**

1. PRACTICE A graph of $y = p(x)$ meets the x-axis at $x = -2$ and at $x = 3$, and nowhere else. How many zeros does $p(x)$ have, and what are they? (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

	Step	Reason
1.	the graph meets the x-axis at $x = -2$ and at $x = 3$	*a zero of $p(x)$ is an x-value where the graph meets the x-axis*
2.	$p(-2) = 0$ and $p(3) = 0$	*meeting the x-axis means the height of the graph there is 0*
3.	2 zeros, namely -2 and 3	*two meeting points, each crossed once, so two zeros*

2. PRACTICE A graph of $y = p(x)$ meets the x-axis at $x = -1$, at $x = 0$ and at $x = 5$, and nowhere else. How many zeros does $p(x)$ have? (Same first step — list the x-values where the graph meets the axis, then count them.)

3. PRACTICE A graph of $y = p(x)$ touches the x-axis at $x = 2$ without crossing it there, and crosses the axis at $x = -4$. How many zeros does $p(x)$ have, counted with repetition? (A touch point counts twice.)

4. PRACTICE A graph of $y = p(x)$ crosses the x-axis at exactly two points and does not touch it anywhere else. How many zeros does $p(x)$ have?

5. PRACTICE A graph of $y = p(x)$ touches the x-axis at exactly one point, without crossing it, and meets the axis nowhere else. How many zeros does $p(x)$ have, counted with repetition?

6. PRACTICE A graph of $y = p(x)$ does not meet the x-axis at all. How many real zeros does $p(x)$ have?

7. PRACTICE A graph of $y = p(x)$ meets the x-axis at exactly four points. What is the least possible degree of $p(x)$?

8. PRACTICE Can the graph of a quadratic polynomial meet the x-axis at three distinct points? Give a reason.

ANSWERS **1.** 2 zeros — -2 and 3. **2.** 3 zeros — -1, 0 and 5.
3. 3 zeros counted with repetition — the touch at $x = 2$ counts twice, and the crossing at $x = -4$ counts once.
4. 2 zeros.
5. 2 zeros counted with repetition — the touch point is a double root, so there is one distinct zero, counted twice.
6. No real zero. **7.** Degree at least 4.
8. No — a quadratic's graph meets the x-axis in at most 2 points, since its degree caps the zero count at 2.
Full solutions: **manthika.com/learn/math10-ch02**

Further practice

PRACTICE SET · **Exercise 2.1 — further practice, counting zeros**

9. PRACTICE The graph of $y = p(x)$ comes down from the upper left, reaches its lowest point below the x-axis, and then rises to the upper right, crossing the x-axis at exactly two points along the way. It meets the axis nowhere else. How many zeros does $p(x)$ have?

10. PRACTICE The graph of $y = p(x)$ crosses the x-axis at exactly three distinct points and meets the axis nowhere else. How many zeros does $p(x)$ have?

11. PRACTICE The graph of $y = p(x)$ touches the x-axis at two different points, without crossing the axis at either one, and meets the axis nowhere else. How many zeros does $p(x)$ have?

12. PRACTICE The graph of $y = p(x)$ touches the x-axis at exactly one point without crossing it there, and crosses the axis at two further points. It meets the axis nowhere else. How many zeros does $p(x)$ have?

A. 1
B. 2
C. 3
D. 4

13. PRACTICE The graph of $y = p(x)$ meets the x-axis at exactly six distinct points and meets it nowhere else. What is the least possible degree of $p(x)$?

14. PRACTICE A quadratic polynomial's graph opens upward, and its lowest point lies exactly on the x-axis. How many zeros does the polynomial have, and are the two zeros equal or different?

ANSWERS **9.** 2 zeros. **10.** 3 zeros. **11.** 2 zeros (each a repeated zero). **12.** C — 3 zeros (the touch point is a repeated zero). **13.** Degree at least 6. **14.** 2 equal zeros — the lowest point sitting on the x-axis makes it a double root, so the two zeros coincide at that one point.
Full solutions: **manthika.com/learn/math10-ch02**

Practice: Exercise 2.2

PRACTICE SET · **Zeros and coefficients**

1. PRACTICE Find the zeros of $2x^2 - 10x + 12$, and verify the relation between the zeros and the coefficients. (Worked in full below — it is the worked example with new numbers.)

SOLUTION — Q1 · **Worked in full**

1.	$2x^2 - 10x + 12 = 2(x^2 - 5x + 6)$	*take the common factor 2 out first, so the middle term is easier to split*
2.	$x^2 - 5x + 6 = (x - 2)(x - 3)$	*split the middle term: -2 and -3 multiply to 6 and add to -5*
3.	$x = 2$ or $x = 3$	*each zero makes one factor equal to 0*

SOLUTION — Q1 · **Worked in full**

4.	$2 + 3 = 5$, and $-b/a = -(-10)/2 = 5$	*the sum of the zeros; here $a = 2$, so divide by a instead of reading $-b$ off*
5.	$2 \cdot 3 = 6$, and $c/a = 12/2 = 6$	*the product of the zeros; divide by a here too*

2. PRACTICE Find the zeros of $3x^2 - 12x + 9$, and verify the relation between the zeros and the coefficients. (Same first step — take the common factor 3 out, then split the middle term. Keep $a = 3$ in the check.)

3. PRACTICE Find a quadratic polynomial whose zeros have sum -4 and product -5. (Use $x^2 - sx + p$, with s the sum and p the product.)

4. PRACTICE Find the zeros of $x^2 - 2x - 8$, and verify the relation between the zeros and the coefficients.

5. PRACTICE Find the zeros of $6x^2 - 7x - 3$, and verify the relation between the zeros and the coefficients.

6. PRACTICE Find the zeros of $4x^2 - 4x + 1$, and verify the relation between the zeros and the coefficients.

7. PRACTICE Find a quadratic polynomial whose zeros are $1/4$ and -1.

8. PRACTICE Find a quadratic polynomial whose sum and product of zeros are 5 and 6 respectively.

ANSWERS **1.** Zeros 2 and 3; sum $5 = -(-10)/2$, product $6 = 12/2$.
2. Zeros 1 and 3; sum $4 = -(-12)/3$, product $3 = 9/3$.
3. $x^2 + 4x - 5$ (or any nonzero multiple).
4. Zeros -2 and 4; sum $2 = -(-2)/1$, product $-8 = -8/1$.
5. Zeros $-1/3$ and $3/2$; sum $7/6 = -(-7)/6$, product $-1/2 = -3/6$.
6. Zeros $1/2$ and $1/2$ (repeated); sum $1 = -(-4)/4$, product $1/4 = 1/4$.
7. $4x^2 + 3x - 1$ (or any nonzero multiple). **8.** $x^2 - 5x + 6$.
Full solutions: **manthika.com/learn/math10-ch02**

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Further practice

PRACTICE SET · **Exercise 2.2 — further practice, zeros and coefficients**

9. PRACTICE Find the zeros of $x^2 - 7x + 12$, and verify the relation between the zeros and the coefficients.

SOLUTION — Q9 · **Worked in full**

1.	$x^2 - 7x + 12 = (x - 4)(x - 3)$	*split the middle term: -3 and -4 multiply to 12 and add to -7*
2.	$x = 4$ or $x = 3$	*each zero makes one factor equal to 0*
3.	$4 + 3 = 7$	*the sum of the zeros; matches $-b/a = -(-7)/1 = 7$*
4.	$4 \cdot 3 = 12$	*the product of the zeros; matches $c/a = 12/1 = 12$*

10. PRACTICE Find the zeros of $x^2 + x - 12$, and verify the relation between the zeros and the coefficients.

11. PRACTICE Find the zeros of $5u^2 + 10u$, and verify the relation between the zeros and the coefficients.

12. PRACTICE Find a quadratic polynomial whose zeros have sum -2 and product -15.

13. PRACTICE For the quadratic $2x^2 - 3x - 5$ with zeros α and β, what is $\alpha + \beta$?

A. $3/2$
B. $-3/2$
C. $5/2$
D. $-5/2$

14. PRACTICE Find the zeros of $2x^2 - 5x + 2$, and verify the relation between the zeros and the coefficients.

15. PRACTICE Find a quadratic polynomial whose zeros are $2/3$ and -3.

16. PRACTICE Which of these is a quadratic polynomial whose zeros have sum 0 and product -9?

A. $x^2 - 9$
B. $x^2 + 9$
C. $x^2 - 9x$
D. $x^2 + 9x$

17. PRACTICE Find the zeros of $x^2 - 20$, and verify the relation between the zeros and the coefficients.

18. PRACTICE Find the zeros of $-x^2 + x + 6$, and verify the relation between the zeros and the coefficients.

ANSWERS **9.** Zeros 3 and 4; sum $7 = -(-7)/1$, product $12 = 12/1$.
10. Zeros 3 and -4; sum $-1 = -1/1$, product $-12 = -12/1$.
11. Zeros 0 and -2; sum $-2 = -10/5$, product $0 = 0/5$.
12. $x^2 + 2x - 15$ (or any nonzero multiple). **13.** A — $\alpha + \beta = 3/2$.
14. Zeros $1/2$ and 2; sum $5/2 = -(-5)/2$, product $1 = 2/2$.
15. $3x^2 + 7x - 6$ (or any nonzero multiple). **16.** A — $x^2 - 9$.
17. Zeros $2\sqrt{5}$ and $-2\sqrt{5}$; sum $0 = -0/1$, product $-20 = -20/1$.
18. Zeros 3 and -2; sum $1 = -1/(-1)$, product $-6 = 6/(-1)$.
Full solutions: **manthika.com/learn/math10-ch02**