4 CHAPTER 4 Quadratic Equations

WHAT THIS CHAPTER BUILDS

You will learn to solve an equation with a squared unknown by factorising and by the quadratic formula, and to read from the discriminant how many roots it has.

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IN THIS CHAPTER

Meera fits a new paving square into place while Bhavisha lifts the next one from the stack beside her.

Getting started

When the unknown is squared

A STORY

Forty-eight squares

Meera fits a grey paving square into place. Bhavisha lifts the next one from the stack.

“We have 48 squares,” Bhavisha says. “Enough for a rectangle, if the long side has 2 more squares than the short side.”

“How many along the short side, then?” Meera asks.

“Call it x,” Bhavisha says. “The long side is $x + 2$, so $x(x + 2) = 48$.”

Meera tries numbers under her breath. “6 works. 6 times 8 is 48.”

“So does -8,” Bhavisha says. “-8 times -6 is 48 too.”

Meera laughs. “Nobody lays -8 squares.”

“No,” Bhavisha says. “But the equation has two answers all the same. When the unknown is squared, there can be two. There can never be more than two.”

“And we keep the one that fits the ground,” Meera says.

You will learn to find both answers every time, and to tell before you start how many there will be.

Solve $x + 5 = 12$ and you get one answer, in one step. Now square the unknown instead. Try solving $x^2 = 9$ yourself, before you read on. (Answer: $x = 3$ and $x = -3$.) No single algebra move finds both at once.

Let us write that down properly. An equation of the form $ax^2 + bx + c = 0$, with $a \neq 0$, is called a QUADRATIC EQUATION. It never has more than two real roots.

This chapter shows you two ways to solve one. One way factorises it into two linear pieces. The other applies a formula, and that one works even when factorising is hard to spot. **One number, computed straight from a, b and c, tells you which case you are in before you solve anything: it is called the discriminant, $b^2 - 4ac$.** We will compute it first, every time, and know what to expect before we start.

RULE
Compute the discriminant first. Let its sign choose the method, and the number of roots to expect.

KEY
A quadratic can have two answers, one answer, or none. The discriminant tells you which, before you solve it.

A SCHOLAR INDIA REMEMBERS

Brahmagupta wrote his book, the Brahmasphutasiddhanta, around 628 CE. In it he gave rules for negative numbers, as numbers in their own right. That matters here, because a root can be negative. The equation $x^2 = 9$ has two roots, 3 and -3. Check: $(-3)^2 = 9$. The negative root is as good an answer as the positive one.

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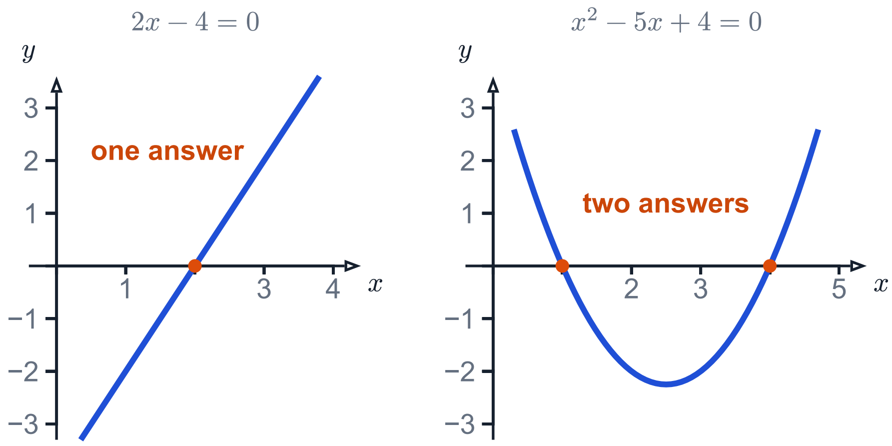
One answer, or as many as two



Each curve is the equation's left-hand side, so an answer is where the curve meets the axis. A straight line leaves the axis and never returns; squaring the unknown makes the curve turn back to it.

The line meets the axis once and the parabola twice, so the power on the unknown, not the arithmetic, sets the number of answers.

IN THE WILD

A cricket ball is hit straight up. It passes every height below its peak twice, once going up and once coming down. It passes its peak height exactly once. It never reaches any height above the peak. Ask when the ball is at 15 metres, and you have written a quadratic equation in time. The discriminant tells you which of these three cases you are in, before you solve it.

CHECK YOURSELF

1. Which condition must always hold for $ax^2 + bx + c = 0$ to be a quadratic equation in x?

- **A.** $a \neq 0$
- **B.** $b \neq 0$
- **C.** $c \neq 0$
- **D.** a, b, c all positive

2. The discriminant $b^2 - 4ac$ can be found before the equation is solved. What does this let a student do?

- **A.** find the exact value of both roots directly
- **B.** skip checking whether $a \neq 0$
- **C.** decide the root count first
- **D.** avoid writing the equation in standard form

1 more in this set online.

IN EVERY CHAPTER

Before you start

DO THIS FIRST

You have solved equations with x before. Now x gets squared. Try each check below. It takes a minute.

- **Square root of a number** (Class 8, Ganita Prakash Part 1, page 8).
  Here: a root is a number that fits an equation, just as 7 fits $x^2 = 49$.
  Check: $7^2 = 49$, so 7 is a square root of 49.
- **A situation as an equation** (Class 9, Ganita Manjari, page 17).
  Here: a situation becomes an equation once the unknown is named x.
  Check: if $l = 3$, the wire fencing costs $200 \cdot 3 = 600$.
- **Splitting the middle term** (Class 9, Ganita Manjari, page 81).
  Here: factorising starts from exactly this pair, $a + b = 7$, $a \cdot b = 12$.
  Check: $(x + 3)(x + 4) = x^2 + 7x + 12$, since $3 + 4 = 7$ and $3 \cdot 4 = 12$.
- **Expanding a bracket** (Class 8, Ganita Prakash Part 1, page 140).
  Here: expanding turns $x(x + 2)$ into $x^2 + 2x$ before its degree is checked.
  Check: At $x = 3$, $x(x + 2) = 3 \cdot 5 = 15$ and $x^2 + 2x = 9 + 6 = 15$.
- **Naming an equation by degree** (Class 9, Ganita Manjari, page 21).
  Here: an equation with highest power 2 is named quadratic, like this.
  Check: $10x - x^2$ has degree 2, so it is quadratic.
- **Area of a rectangle** (Class 9, Ganita Manjari, page 139).
  Here: a garden's area is length times breadth, just like this rectangle's.
  Check: a rectangle 8 by 12 has area 96.

If any of these felt new, read the page named before going on.

Standard form of a quadratic equation

Let us take any quadratic equation. Write its terms in descending powers of x. Put them all on one side, equal to 0. Do that, and you get $ax^2 + bx + c = 0$. This is the equation's STANDARD FORM. a, b, c are the three real numbers that name the whole equation.

The equation stays quadratic only when $a \neq 0$. That single condition keeps the x^2 term alive. Drop it, and $ax^2 + bx + c = 0$ collapses to $bx + c = 0$: a linear equation, with a different number of roots. Check this yourself. Set $a = 0$ in $2x^2 + 3x - 5 = 0$. (Answer: it leaves $3x - 5 = 0$, a linear equation.)

Reading an equation against this pattern is the first move every method makes. Before you factorise, or reach for a formula, write the equation in standard form. Then read a, b and c straight off it.

BACK
When the unknown is squared · p. 111

RULE
a can never be 0. That is what keeps the equation quadratic, not linear.

KEY
$ax^2 + bx + c = 0$, $a \neq 0$: three real numbers name the whole equation.

CHECK YOURSELF

3. Which of these is the standard form of a quadratic equation in x?

A. $ax^2 + bx + c = 0$, with $b \neq 0$

B. $ax + bx^2 + c = 0$, with $a \neq 0$

C. $ax^2 + bx + c = 0$, with $a \neq 0$

D. $ax^2 + bx + c$

4. Why does the standard form of a quadratic equation require $a \neq 0$?

- **A.** if $a = 0$ the equation has no solution at all
- **B.** if $a = 0$ the discriminant cannot be computed
- **C.** the x^2 term disappears
- **D.** if $a = 0$ the coefficient c must also be 0

1 more in this set online.

A root satisfies the equation

A real number k is a ROOT, or a SOLUTION, of $ax^2 + bx + c = 0$, if putting $x = k$ into it makes the equation true. That means $ak^2 + bk + c = 0$. Checking a claimed root costs you nothing more than one substitution.

Let us try one. Take $x^2 - 5x + 6 = 0$. Try $x = 2$: $4 - 10 + 6 = 0$. It works, so 2 is a root. Now try $x = 1$: $1 - 5 + 6 = 2 \neq 0$, so 1 is not a root. Try $x = -1$ yourself. (Answer: $1 + 5 + 6 = 12 \neq 0$, so -1 is not a root either.)

A root of the equation is the same number as a zero of the quadratic polynomial $ax^2 + bx + c$. That idea is from an earlier chapter, carried forward under a new name. A quadratic equation has at most two real roots, never more.

TRY
Is $x = 2$ a root of $x^2 - 5x + 6 = 0$?

Answer: Yes: $4 - 10 + 6 = 0$.

KEY
Root of the equation and zero of the polynomial: the same number, two names.

A SCHOLAR INDIA REMEMBERS

Is -3 a root of $x^2 + x - 6 = 0$? Do not guess. Put it in and look. $(-3)^2 + (-3) - 6 = 9 - 3 - 6 = 0$. The left side is 0, so yes, -3 is a root. A negative root counts as fully as a positive one. The other root is 2. Check it the same way.

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CHECK YOURSELF

5. A real number k is called a root of $ax^2 + bx + c = 0$ when

- **A.** $ak^2 + bk + c = 0$
- **B.** $ak^2 + bk + c > 0$
- **C.** k is a factor of c
- **D.** $k^2 = ax^2 + bx + c$

6. How is a root of the quadratic equation $ax^2 + bx + c = 0$ related to the quadratic polynomial $ax^2 + bx + c$?

- **A.** a root of the equation is the same number as a zero of the polynomial
- **B.** a root of the equation is always larger than every zero of the polynomial
- **C.** a root exists only if the polynomial has no zero
- **D.** a root is the value of the polynomial at $x = 0$

1 more in this set online.

The ideas

Simplify first, then check the form

An equation may not arrive looking like $ax^2 + bx + c = 0$. You may need to expand brackets first. Collect every term onto one side. Only then does the equation show what it really is.

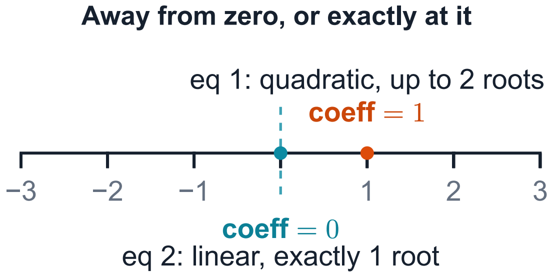
Expand every bracket, every time. Read the coefficient of x^2 only after simplifying, never before.

Let us watch for the trap. An x^2 term can look present at the start, then cancel once you collect both sides. What is left is a linear equation, not a quadratic one. Check the two equations in the worked example that follows. Watch this happen to one of them yourself. The x^2 you see before simplifying promises nothing about the one left after.

CAUTION
An x^2 term that cancels out during simplification leaves a linear equation, not a quadratic one.

BACK
Standard form of a quadratic equation · p. 113

RULE
Expand and collect everything onto one side before deciding whether an equation is quadratic.



A number line marks the coefficient of x squared for two equations: one stays quadratic, the other collapses to linear.

CHECK YOURSELF

7. To check whether an equation is quadratic once every bracket is expanded, what must be examined?

- **A.** whether the x^2 coefficient survives simplifying
- **B.** whether an x^2 term appears anywhere before simplification
- **C.** whether the equation has exactly three terms
- **D.** whether the highest power written is 2

8. A student expands $(x-1)^2 = x^2 - 1$ and says: 'There is an x^2 on both sides, so this must be quadratic.' Is this correct?

- **A.** Yes — any equation with x^2 written down is quadratic
- **B.** No — because the equation has no constant term
- **C.** Yes — because both sides have degree 2 before simplifying
- **D.** No — the x^2 terms cancel, leaving a linear equation

2 more in this set online.

Turning a situation into a quadratic

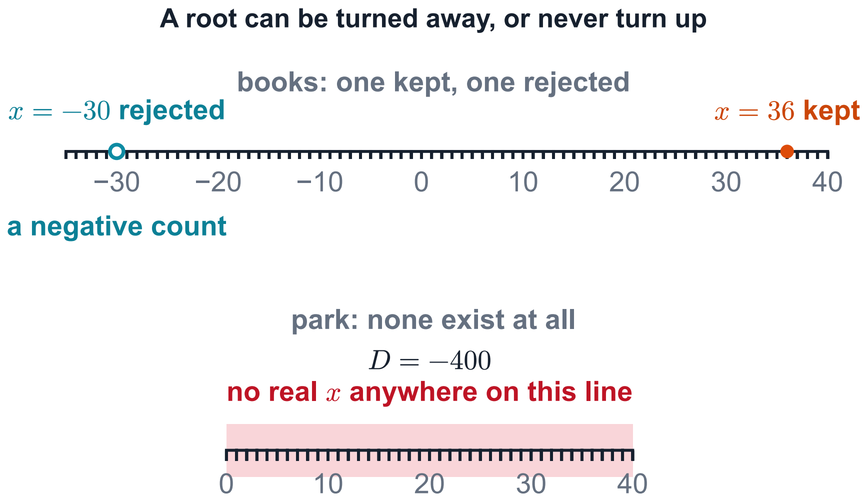
A day-to-day situation can be an area, a product of two ages, or a train's speed against its journey time. Every one of them turns into a quadratic equation the same way. Let us name the unknown first: call it x, in words, before writing any equation. Then write every other quantity in terms of that one x.

The situation gives you one condition. An area must work out to a fixed number, or a product must equal one. Turn that condition into a single equation in x. Simplify it into the standard form $ax^2 + bx + c = 0$. That is the quadratic equation the situation satisfies.

A root drawn from a real situation may still need rejecting, once you solve it. A negative length, or a negative count of objects, is not a value the situation allows, even when it satisfies the equation perfectly. In Science, a projectile's height at time t often has this same shape, $h(t) = ut - ct^2$. Asking when it lands is the same problem, in a different dress. Try writing that landing condition as an equation yourself.

CAUTION
A root drawn from a real situation may still need rejecting. A negative length, or a negative count, is not a valid answer.

RULE
Name the unknown first, in words, before writing a single equation.



Two number lines: one shows a kept root beside a rejected negative root, the other shows no real root at all.

IN THE WILD
In Science, a projectile's height at time t often has the form $h(t) = ut - ct^2$, for constants u and c. Asking when it lands means solving $h(t) = 0$, the same problem in a different dress. Area problems turn into a quadratic equation the same way. A plot, a frame, a sheet of fixed area, with one side named x, and the area condition written in x.

CHECK YOURSELF

9. To turn a day-to-day situation into a quadratic equation, what is the first step?

- **A.** guess a value of x that seems to fit the situation
- **B.** assume the situation is already quadratic before checking it
- **C.** name the unknown quantity as x
- **D.** write the discriminant of the situation directly

10. A rectangular plot has area 75 square metres. Its length is 2 metres more than its breadth. Taking the breadth as x metres, which equation must x satisfy?

- **A.** $x^2 - 2x - 75 = 0$
- **B.** $x(x+2) = 75x$
- **C.** $2x^2 = 75$
- **D.** $x^2 + 2x - 75 = 0$

2 more in this set online.

Solving by factorisation

When $ax^2 + bx + c$ factorises into two linear pieces, you can solve $ax^2 + bx + c = 0$ with nothing more than arithmetic. A product of two things is 0 exactly when one of them is 0. So once you have the two factors, you already have the two roots.

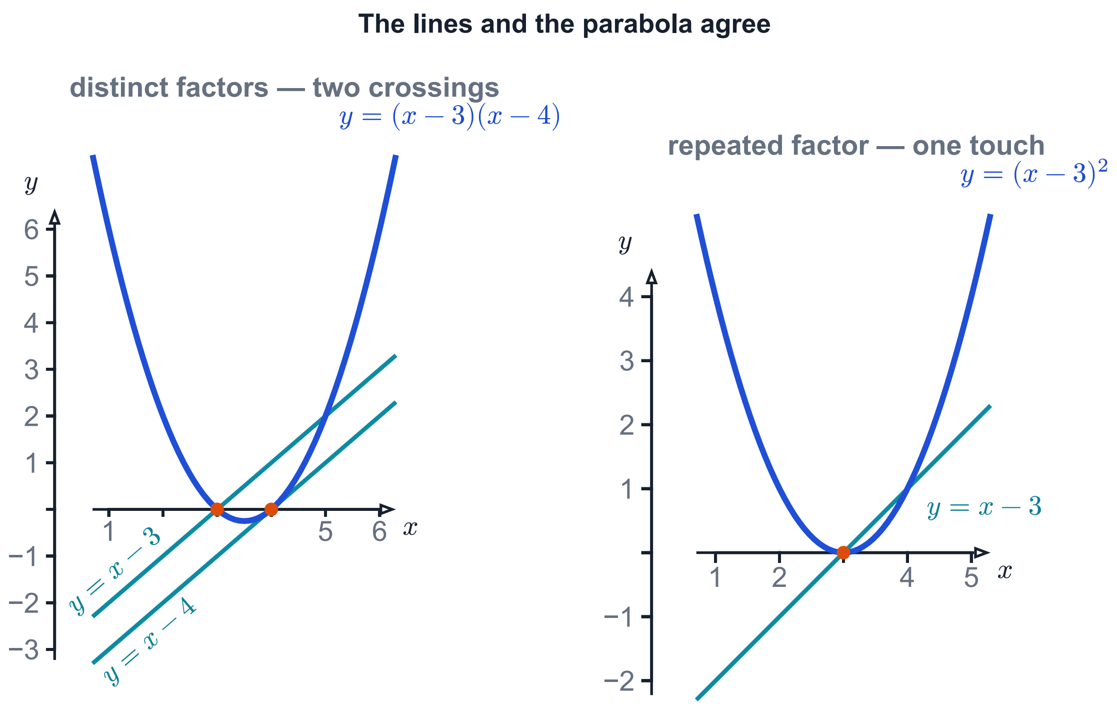
The standard route there splits the middle term bx into two pieces. Their coefficients multiply to ac and add to b. Let us group the four terms in pairs, then pull the common factor out of each pair.

Finding the right split is the one step worth doing carefully. Once you have it, the grouping and the factoring that follow are mechanical. Try the worked example next, and watch each step fall into place.

BACK
A root satisfies the equation · p. 114

RULE
Find the split first: two numbers multiplying to ac, adding to b.

KEY
Factorise, then set EACH factor to 0 separately: two factors, two roots.



Two crossing lines multiply into a parabola that crosses the axis twice. A line squared multiplies into a parabola that only touches it.

CHECK YOURSELF

11. To factorise $ax^2 + bx + c$ by splitting the middle term, the two split coefficients must

- **A.** multiply to b and add to ac
- **B.** multiply to c and add to a
- **C.** add to ac and multiply to b
- **D.** multiply to ac and add to b

12. To factorise $x^2 + 7x + 10$, a student splits $7x$ as $3x + 4x$, since $3 + 4 = 7$. Is this split usable?

- **A.** Yes — matching the sum to 7 is the only requirement
- **B.** No — the two numbers must both be even
- **C.** No — the product must be 10, not 12
- **D.** Yes — because 10 is close enough to 12

2 more in this set online.

When both factors are the same

Factorisation does not always hand back two different factors. Sometimes the same linear factor appears twice. Then $ax^2 + bx + c = 0$ becomes $a(x - r)(x - r) = 0$, for one number r.

Setting either copy of the factor to 0 gives the same equation, $x = r$. This root is called a REPEATED, or EQUAL, root.

Let us picture the parabola this equation describes. Sketch it yourself on paper if you like. At a repeated root, the curve only touches the x-axis at that one point. It does not cross from one side to the other, the way it does at two distinct roots.

CAUTION
A repeated root is ONE number, counted twice, not two different roots.

KEY
Same factor twice: the parabola touches the x-axis, but does not cross it.

CHECK YOURSELF

13. When factorisation gives the same linear factor twice, such as $(x - 3)(x - 3) = 0$, the equation is said to have

- **A.** one repeated root, counted twice
- **B.** two different roots, both equal to 3
- **C.** no real root, since the factor did not vary
- **D.** an undefined root, since factorisation failed

14. $(x - 5)^2 = 0$ gives the repeated root $x = 5$. Why is this root counted twice even though only one value of x satisfies it?

- **A.** because $x = 5$ must be substituted into the equation twice to confirm it
- **B.** because the same factor $(x - 5)$ appears twice in the factorisation
- **C.** because the equation has two terms
- **D.** because 5 is an odd number of factors away from 0

1 more in this set online.

The quadratic formula

Factorisation is not always easy to spot. For some equations, no simple integer split exists at all. The QUADRATIC FORMULA reaches every real root of $ax^2 + bx + c = 0$ directly from a, b and c, with no factorising step needed. This is the one formula worth learning by heart:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

One equation, two roots: one from the $+$, one from the $-$, in front of the square root.

The formula needs no separate proof. Substitute either root back into $ax^2 + bx + c$, and you will see it makes the expression 0. That holds when the quantity under the square root is not negative. Let us try that check ourselves, on the very first worked example that uses this formula. Then try it yourself on every equation you solve with the formula next.

RULE
The formula works even when factorisation is hard or impossible to spot.

KEY
Read a, b, c off the standard form first. The formula only works on that form.

CHECK YOURSELF

15. The quadratic formula for the roots of $ax^2 + bx + c = 0$ is

- **A.** $x = (b \pm \sqrt{b^2 - 4ac})/(2a)$
- **B.** $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$
- **C.** $x = (-b \pm \sqrt{b^2 - 4ac})/a$
- **D.** $x = (-b \pm \sqrt{b^2 + 4ac})/(2a)$

16. Why does the quadratic formula need no factorising step to find a root?

- **A.** it gives any real root directly from a, b, c
- **B.** it only works after the equation has already been factorised
- **C.** it works only when the roots happen to be whole numbers
- **D.** it replaces the need to write the equation in standard form

2 more in this set online.

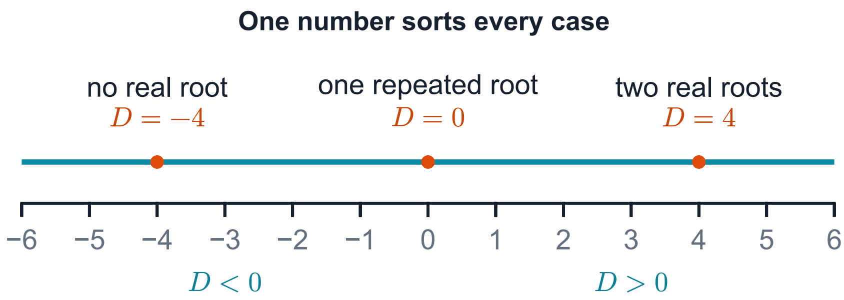
The discriminant

Look again at the quantity under the square root in the quadratic formula: $b^2 - 4ac$. It matters enough to carry its own name, the DISCRIMINANT. You can find its value from a, b, c alone, before attempting to solve the equation.

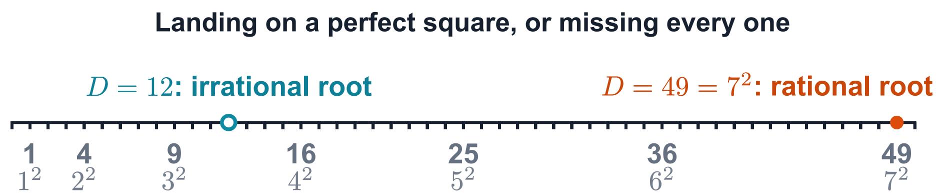
For $2x^2 + 3x - 5 = 0$, that means $3^2 - 4 \cdot 2 \cdot (-5) = 9 + 40 = 49$: one number, found in one line. Let us try another. Compute it yourself for $x^2 - 2x - 2 = 0$, before you read further.

The discriminant is not a step inside solving the equation. It is a number you compute in advance. As the next section shows, that number already tells you what kind of answer to expect.

KEY
Discriminant $= b^2 - 4ac$: compute it first, from a, b, c alone.



A number line marks three discriminants: one negative with no real root, one zero with a repeated root, one positive with two real roots.



A number line marks two discriminants. One lands on a perfect square and gives a rational root, the other does not, giving an irrational root.

A SCHOLAR INDIA REMEMBERS

What if the discriminant comes out negative? Do not rub it out. Read it. For $x^2 + 2x + 5 = 0$, $b^2 - 4ac = 4 - 20 = -16$. A negative discriminant is a result, and it tells you something. This equation has no real roots.

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CHECK YOURSELF

17. For $ax^2 + bx + c = 0$, the discriminant is

- **A.** $b^2 - 4ac$
- **B.** $b^2 + 4ac$
- **C.** $\sqrt{b^2 - 4ac}$
- **D.** $4ac - b^2$

18. The discriminant of $ax^2 + bx + c = 0$ can be computed before the equation is solved because

- **A.** it needs the roots to already be known
- **B.** it needs only a, b, c
- **C.** it needs the equation to be factorised first
- **D.** it needs the equation checked against a graph first

2 more in this set online.

What the discriminant's sign tells you

The sign of the discriminant $b^2 - 4ac$ decides the NATURE of the roots of $ax^2 + bx + c = 0$. It does this before you find either root. There are three cases, and only three.

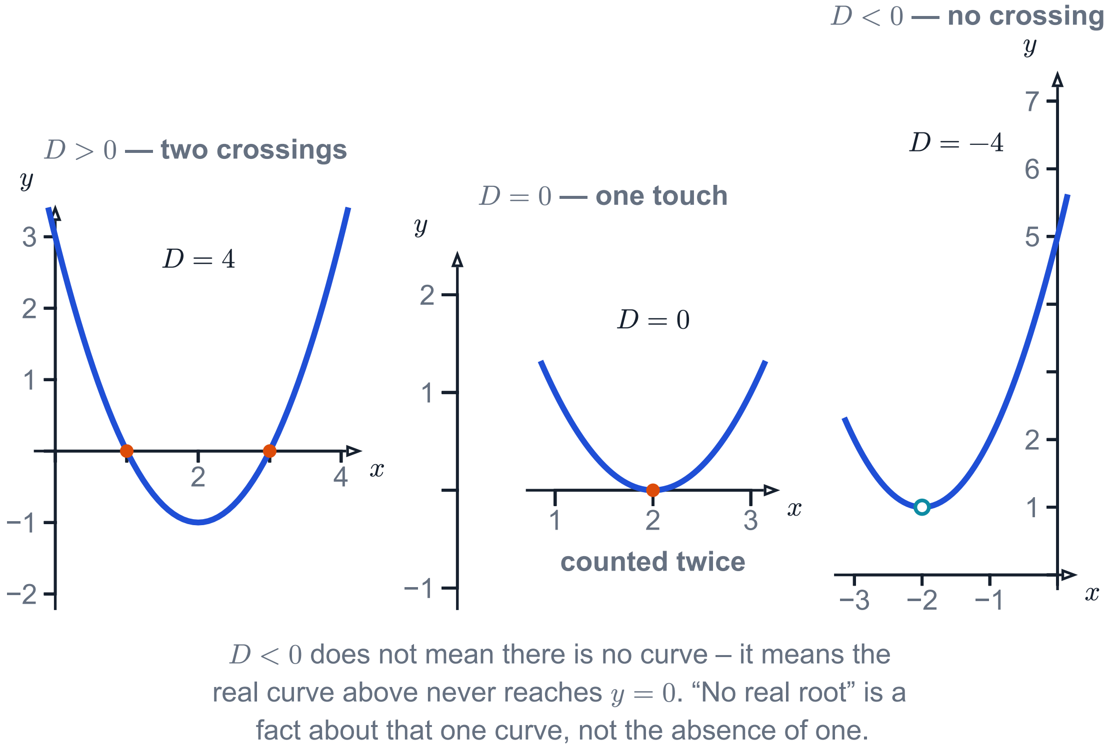
$b^2 - 4ac > 0$ gives two distinct real roots. $b^2 - 4ac = 0$ gives one repeated real root. $b^2 - 4ac < 0$ gives no real root at all. We work with real roots only here, so a negative discriminant is where your search simply stops.

Knowing the case in advance turns solving from a search into a checked calculation. Look back at the equation you tried last section. Say out loud which of the three cases it falls into, before you check.

RULE
Check the discriminant's sign before reaching for either solving method.

KEY
Positive: two distinct roots. Zero: one repeated root. Negative: no real root.

One test, three outcomes



Three parabolas, one for each discriminant sign. The first crosses the x-axis twice, the second touches once, the third never meets it.

A SCHOLAR INDIA REMEMBERS

Now let the discriminant be 0. Does 0 mean nothing happened? No. It tells you something exact. For $x^2 - 6x + 9 = 0$, $b^2 - 4ac = 36 - 36 = 0$. The two roots are equal. Both are 3, because $x^2 - 6x + 9 = (x-3)^2$.

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CHECK YOURSELF

19. If $b^2 - 4ac > 0$ for $ax^2 + bx + c = 0$, the equation has

A. two equal real roots

B. two distinct real roots

C. no real root

D. exactly one real root and one complex root

20. For $ax^2 + bx + c = 0$, $b^2 - 4ac = 0$. What does this say about the roots, without solving the equation?

A. the equation has two distinct real roots

B. the equation has no real root

C. the equation cannot be solved

D. the equation has one repeated real root

3 more in this set online.

Common mistakes

A negative constant term makes the discriminant larger

The minus sign inside $b^2 - 4ac$ is easy to lose. Once you write c down with its own sign, you must carry that sign through the multiplication, not drop it.

Take $2x^2 + 3x - 5 = 0$: here $a = 2$, $b = 3$, $c = -5$. Compute $-4ac$ as $-4 \cdot 2 \cdot (-5)$, and the two minus signs give a plus: $-4 \cdot 2 \cdot (-5) = +40$.

Drop that sign, and $D = 9 - 40 = -31$ looks like no real root. Keep it, and $D = 9 + 40 = 49$, giving two real roots, $x = 1$ and $x = -5/2$.

Let us check which is right. $2x^2 + 3x - 5$ factorises as $(2x + 5)(x - 1)$, so real roots plainly exist. Try the same computation yourself, slowly, and watch where the sign decides the answer.

CAUTION
Write a, b, c down with their signs before substituting. Put a negative c in brackets.

BACK
The discriminant · p. 119

KEY
If your discriminant looks wrong, try factorising directly instead. A factorisation that works means you lost a sign somewhere.

FIRST TRY
~~Here $a = 2$, $b = 3$ and $c = -5$. I wrote~~
~~$D = 9 - 4 \cdot 2 \cdot 5 = 9 - 40 = -31$, so no real roots.~~

SECOND LOOK
The minus sign is already in the formula, so c keeps its own sign too. $D = 9 - 4 \cdot 2 \cdot (-5) = 9 + 40 = 49$, giving two roots, $x = 1$ and $x = -5/2$.

Let c carry its own sign into the discriminant, always.

IF C IS NEGATIVE, DOES ITS SIGN ALREADY SHOW UP CORRECTLY IN THE DISCRIMINANT?

WEAK
A student solves $3x^2 + 2x - 8 = 0$. They write $D = 2^2 - 4 \cdot 3 \cdot 8 = 4 - 96 = -92$. They read off $c = -8$, but drop the sign when they multiply. They conclude there is no real root.

Check with a quick factorisation. $3x^2 + 2x - 8$ factorises as $(3x - 4)(x + 2)$, and that gives two real roots straight away, so the discriminant above must be wrong.

STRONG
Keep the sign of c through the whole computation. $D = 2^2 - 4 \cdot 3 \cdot (-8) = 4 + 96 = 100$.

$D = 100$ is a perfect square, so $\sqrt{100} = 10$, and $x = (-2 \pm 10)/6$ gives $x = 4/3$ and $x = -2$, matching the factorisation exactly.

Check the discriminant before choosing a method

You might be tempted to guess a factorisation for any quadratic equation. You might treat the formula as a fallback, only for when guessing takes too long.

That shortcut skips a check that would have saved the wasted search. $b^2 - 4ac < 0$ means no real root exists, so no factorisation, guessed or otherwise, will ever be found. A discriminant that is positive but not a perfect square means the roots are irrational, so no integer or simple rational split turns up either. The formula is then your only route.

Let us check D first, always. It costs one line of arithmetic. It tells you, before a single guess, whether guessing was ever going to work. Try it yourself on $x^2 + x + 3 = 0$: is a real root even worth searching for? (Answer: $D = 1 - 12 = -11 < 0$, so no real root exists.)

SHOULD YOU ALWAYS TRY GUESSING A FACTORISATION FIRST, AND USE THE FORMULA ONLY WHEN GUESSING FAILS?

WEAK

A student meets $x^2 + 2x + 5 = 0$. They start guessing pairs of numbers that multiply to 5 and add to 2. They try 1 and 5. They try -1 and -5. Nothing works, so they try a few more pairs, then give up and reach for the formula.

Compute D first, and the guessing was never going to succeed. $D = 2^2 - 4 \cdot 1 \cdot 5 = 4 - 20 = -16$, and $D < 0$ means no real root exists at all. No pair of numbers, guessed or not, was ever going to factorise this equation.

STRONG

Compute D before guessing anything. For $x^2 - 4x + 1 = 0$: $D = (-4)^2 - 4 \cdot 1 \cdot 1 = 16 - 4 = 12$.

$D = 12$ is positive, so two real roots exist, but 12 is not a perfect square. No integer or simple rational split will ever be found. Go straight to the formula: $x = (4 \pm \sqrt{12})/2 = 2 \pm \sqrt{3}$.

CAUTION

A quadratic with no real root has NO factorisation to find. Checking D first saves the wasted search.

KEY

D not a perfect square means irrational roots: reach for the formula, not for integer guesses.

WATCH OUT

The trap: The student spends several minutes guessing integer factors before ever computing $b^2 - 4ac$ to check they exist.

The correction: Check D first: $b^2 - 4ac < 0$ means no factorisation exists; a non-perfect-square D means irrational roots, reachable only by the formula.

CHECK YOURSELF

21. A student says: 'Any quadratic equation can be solved by guessing a factorisation — the quadratic formula is only a fallback for when guessing takes too long.' Is this student correct?

- **A.** Yes — guessing eventually works for every quadratic equation, given enough time
- **B.** No — some equations cannot be factorised at all
- **C.** No — but only because factorisation is always the slower method
- **D.** Yes — the quadratic formula is only needed as a shortcut

22. For $x^2 + 2x + 5 = 0$, the discriminant is $4 - 20 = -16$. Why will guessing a factorisation never succeed here?

- **A.** guessing would work eventually, with more patience
- **B.** the equation must first be checked for a repeated root
- **C.** no real root exists, so no factorisation is possible
- **D.** factorisation fails only because the coefficients are too large

1 more in this set online.

Worked examples

WORKED EXAMPLE · Is x(x + 2) = 3(x − 4) quadratic?

1.	$x(x+2) = 3(x-4)$	*Let us expand both sides first, then collect everything onto one side.*
2.	$x^2 + 2x = 3x - 12$, so $x^2 - x + 12 = 0$	*Expand both sides, then move every term to one side.*
3.	coefficient of x^2 is $1 \neq 0$	*So this equation is quadratic.*
4.	$(x-1)^2 = x^2 - 1$	*Now a second equation, checked the same way.*
5.	$x^2 - 2x + 1 = x^2 - 1$, so $-2x + 2 = 0$	*Expand and collect. The x^2 terms cancel on both sides.*
6.	this is a linear equation, not a quadratic one	*The coefficient of x^2 is 0, once everything is collected, even though x^2 appeared before simplifying.*
7.	at $x = 2$: $x(x+2) = 8$ and $3(x-4) = -6$, a difference of 14, matching $x^2 - x + 12 = 4 - 2 + 12 = 14$	CHECK ***Check the first simplification: test one value in both the original equation and the simplified form. They agree.*** *Try the second equation the same way yourself. At $x = 2$: $(x-1)^2 = 1$ and $x^2 - 1 = 3$, matching $-2x + 2 = -2$.*

TRY
Is $x(x-3) = 2(x+1)$ a quadratic equation once simplified?

Answer: Yes: it simplifies to $x^2 - 5x - 2 = 0$, coefficient of x^2 is $1 \neq 0$.

BACK
Simplify first, then check the form · p. 115

RULE
Expand fully and collect onto one side before reading off the coefficient of x^2.

HOW DO YOU KNOW IF A MESSY EQUATION IS REALLY QUADRATIC?

1. **Expand every bracket** Multiply out both sides completely. Do not skip a single bracket.
2. **Collect onto one side** Move every term to one side, equal to zero. Now the equation is in standard form.
3. **Read the coefficient** Look only at the number in front of x squared, after collecting. If it is not zero, the equation is quadratic.

4. **Watch for cancelling** Sometimes the x squared terms cancel once collected. Then the equation is linear, not quadratic, even though x squared appeared before.

CHECK YOURSELF

23. Following the same check used for $x(x+2)=3(x-4)$, is $2x(x+1)=x^2+2x-5$ quadratic once simplified?

- **A.** No — the x terms cancel, so nothing remains
- **B.** Yes — $x^2+5=0$ remains after simplifying
- **C.** Yes — because both sides already show an x^2 term before simplifying
- **D.** No — because the constant term is negative

24. Using the same method, is $(x+3)^2=x^2+6x+9$ quadratic once simplified?

- **A.** Yes — an x^2 term is written on both sides
- **B.** No — every term cancels to $0=0$
- **C.** No — because the coefficient of x is too large
- **D.** Yes — because expanding a square always gives a quadratic

1 more in this set online.

WORKED EXAMPLE · A garden's area as a quadratic

1.	breadth $= x$ metres, length $= (x+4)$ metres	*Let us name the unknown first. The length is stated in terms of the breadth, so call the breadth x.*
2.	$x(x+4)=96$	*Area is length times breadth, and here the area is 96 square metres.*
3.	$x^2+4x-96=0$	*Expand, then move every term to one side. This is the quadratic equation the breadth must satisfy.*
4.	check $x=8$: $8(8+4)=8\cdot12=96$	CHECK ***Check it: does a breadth of 8 metres satisfy the equation?*** *Then the length is 12 metres, and $8\cdot12=96$ matches the garden's area. The equation checks out.*

BACK
Turning a situation into a quadratic · p. 116

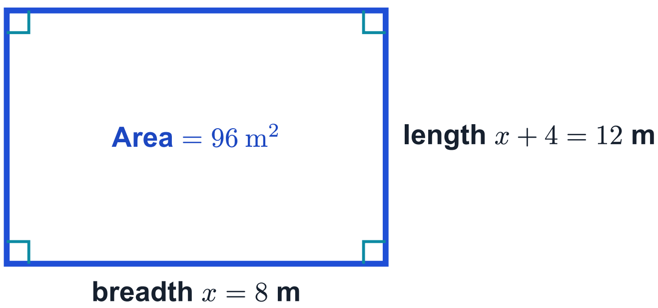
TRY
A rectangle has area 40 square metres; its length is 3 metres more than its breadth. Form the equation.

Answer: $x^2+3x-40=0$, taking the breadth as x metres.

KEY
Name the breadth x. Every other length in the problem is written in terms of that one x.

Aarav marks out a rectangular garden bed with four pegs, its length longer than its breadth.

The garden’s own shape writes the equation



A rectangle, eight metres by twelve, shows the garden’s breadth and length. Together they give ninety-six square metres.

HOW DO YOU TURN AN AREA PROBLEM INTO A QUADRATIC EQUATION?

1. **Name the unknown** Call the smaller length x. Write every other length in terms of that same x.
2. **Write the area condition** Multiply length by breadth. Set the product equal to the area you are given.
3. **Simplify to standard form** Expand the brackets and move every term to one side. This is the quadratic equation x must satisfy.
4. **Check your answer** Substitute your value of x back in. Confirm the area comes out right, and reject a negative length.

CHECK YOURSELF

25. Following the garden method, a rectangular hall has area 150 square metres and its length is 5 metres more than its breadth. Taking the breadth as x metres, which equation results?

- **A.** $x^2 - 5x - 150 = 0$
- **B.** $x^2 + 5x + 150 = 0$
- **C.** $x^2 + 5x - 150 = 0$
- **D.** $x(x+5) = 150x$

26. Solving $x^2 + 5x - 150 = 0$ from the hall problem gives $x = 10$ and $x = -15$. Which value is the breadth?

- **A.** $x = -15$ metres, taken as a magnitude of 15 metres
- **B.** both values, since the equation gives two roots
- **C.** neither value, since a breadth this large is unrealistic
- **D.** $x = 10$ metres, since a breadth cannot be negative

1 more in this set online.

WORKED EXAMPLE · Factorising to find the roots

1.	solve $x^2 - 7x + 12 = 0$	*Let us split the middle term first.*
2.	$(-3) + (-4) = -7$ and $(-3) \cdot (-4) = 12$	*Find two numbers that multiply to 12 and add to -7.*
3.	$x^2 - 3x - 4x + 12 = x(x-3) - 4(x-3) = (x-3)(x-4)$	*Split the middle term using those numbers. Then group in pairs, and factor each pair.*
4.	$x = 3$ or $x = 4$	*Set each factor to 0, in turn.*
5.	now solve $x^2 - 6x + 9 = 0$ the same way	*Now a second equation, solved the same way.*
6.	$(-3) + (-3) = -6$ and $(-3) \cdot (-3) = 9$	*The split this time lands on the same number, twice.*
7.	$(x-3)(x-3) = (x-3)^2 = 0$	*The same factor appears twice: not two different factors.*
8.	$x = 3$, a repeated root	*One factor set to 0 gives one root, counted twice.*
9.	$3^2 - 7 \cdot 3 + 12 = 9 - 21 + 12 = 0$ and $4^2 - 7 \cdot 4 + 12 = 16 - 28 + 12 = 0$	CHECK ***Check both roots: substitute them back into the original equation. Both give 0, so 3 and 4 really are the roots.*** *Try the same check yourself, on the repeated root $x = 3$ of the second equation.*

BACK
Solving by factorisation · p. 117

TRY
Solve $x^2 - 5x + 6 = 0$ by factorisation.

Answer: $(-2) + (-3) = -5$, $(-2) \cdot (-3) = 6$: $(x-2)(x-3) = 0$, roots $x = 2$ and $x = 3$.

RULE
Verify a factorisation by expanding it back out before trusting the roots it gives.

HOW DO YOU SOLVE A QUADRATIC BY FACTORISING?

1. **Write in standard form** Get the equation as $ax^2 + bx + c = 0$ first.
2. **Find the split** Find two numbers that multiply to ac and add to b.
3. **Split and group** Split the middle term using those two numbers. Group the four terms in pairs.
4. **Factor each pair** Pull the common factor out of each pair. The same bracket should appear in both.
5. **Set each factor to zero** Each factor gives one root. Two factors give two roots, unless they are the same factor twice.

CHECK YOURSELF

27. Using the same splitting method as $x^2 - 7x + 12 = (x-3)(x-4)$, how does $x^2 - 9x + 20$ factorise?

A. $(x-2)(x-10)$

B. $(x+4)(x+5)$

C. $(x-4)(x+5)$

D. $(x-4)(x-5)$

28. Using the same method, does $x^2 - 10x + 25$ factorise to a repeated root, and if so what is it?

A. $(x-5)^2 = 0$, giving the repeated root $x = 5$

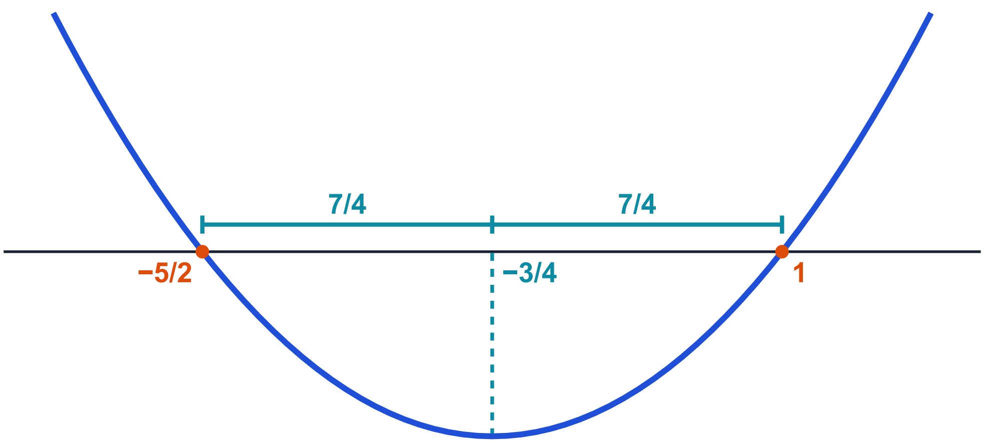
B. $(x-5)(x+5) = 0$, giving roots 5 and -5

C. no repeated root exists, since 25 is not a perfect square of 10

D. $(x-25)(x-1)$, giving roots 25 and 1

2 more in this set online.

The ± steps both ways from the middle



The formula's −3/4 is the curve's middle line; ± 7/4 steps the same distance each way to the two roots, −5/2 and 1.

Whatever the equation, its two roots are mirror images of each other in the axis of the parabola.

WORKED EXAMPLE · Solving with the formula

1.	solve $2x^2 + 3x - 5 = 0$: $a = 2$, $b = 3$, $c = -5$	*Let us read the coefficients off the standard form, before touching the formula.*
2.	$D = b^2 - 4ac = 3^2 - 4 \cdot 2 \cdot (-5) = 9 + 40 = 49$	*Compute the discriminant first, as one number.*
3.	$\sqrt{49} = 7$	*The discriminant is a perfect square here, so its square root is a whole number.*
4.	$x = (-3 \pm 7)/4$	*Substitute into the quadratic formula.*
5.	$x = 1$ or $x = -5/2$	*Evaluate both signs of $\pm$, separately.*
6.	$2(1)^2 + 3(1) - 5 = 0$ and $2(-5/2)^2 + 3(-5/2) - 5 = 0$	CHECK ***Substitute each root back into the original equation, and confirm it works.*** *This check is what the formula's correctness rests on, not a separate proof. Try it yourself, on any equation you solve with the formula from here on.*

BACK
The quadratic formula · p. 119

TRY
Solve $2x^2 - 5x + 2 = 0$ using the quadratic formula.

Answer: Discriminant $(-5)^2 - 4 \cdot 2 \cdot 2 = 9$, $\sqrt{9} = 3$: $x = (5 \pm 3)/4$, so $x = 2$ and $x = 1/2$.

RULE
Compute the discriminant first, as one number, before touching the rest of the formula.

HOW DO YOU SOLVE A QUADRATIC WITH THE FORMULA?

1. **Read off a, b, c** Write the equation in standard form first. Then read a, b, c straight off it.
2. **Compute the discriminant** Work out $b^2 - 4ac$ as one number, before anything else.
3. **Find its square root** Take the square root of the discriminant. Simplify it if you can.
4. **Substitute into the formula** Put a, b and the square root into $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$.
5. **Evaluate both signs** Work out the plus case and the minus case separately. Each gives you one root.

CHECK YOURSELF

29. Using the same formula method as $2x^2 + 3x - 5 = 0$, what is the discriminant of $3x^2 - 2x - 8 = 0$?

- **A.** 4
- **B.** -92
- **C.** 192
- **D.** 100

30. Continuing with $3x^2 - 2x - 8 = 0$, whose discriminant is 100, what are the roots?

- **A.** $x = -2$ and $x = 4/3$
- **B.** $x = 2$ and $x = -4/3$
- **C.** $x = 12$ and $x = -8$
- **D.** $x = 2$ and $x = 4/3$

1 more in this set online.

WORKED EXAMPLE · **Reading the nature of roots**

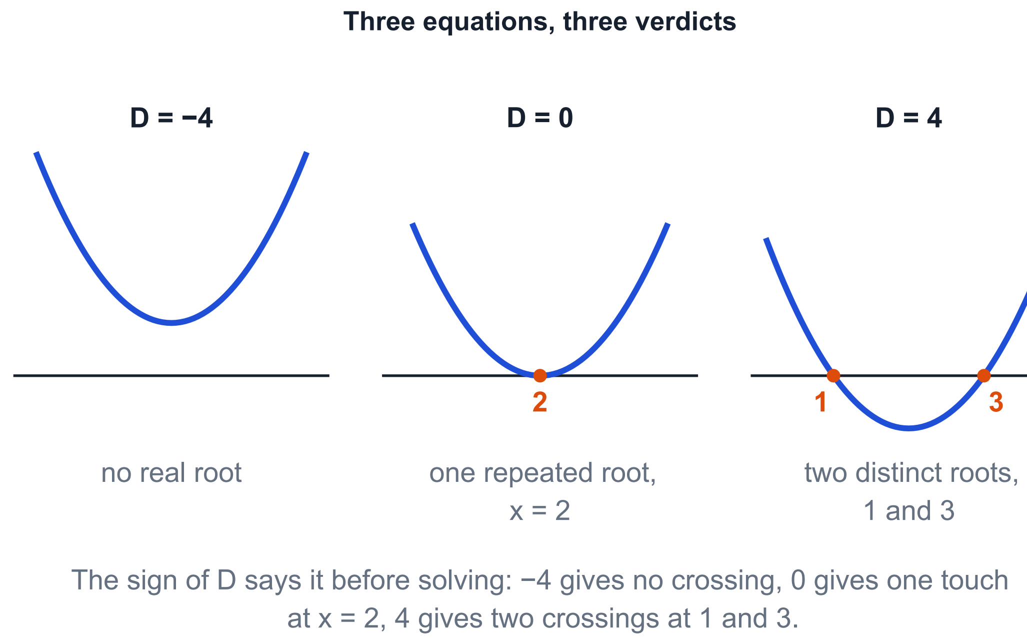
1.	$x^2 + 4x + 5 = 0$: $b^2 - 4ac = 16 - 20 = -4$	*Let us compute the discriminant first.*
2.	$D < 0$, so there is no real root	*A negative discriminant rules out any real root.*
3.	$x^2 - 4x + 4 = 0$: $b^2 - 4ac = 16 - 16 = 0$	*Now a second equation, discriminant computed the same way.*
4.	$D = 0$, so there is one repeated real root, $x = 2$	*A zero discriminant means the equation factors as one factor, squared.*
5.	$x^2 - 4x + 3 = 0$: $b^2 - 4ac = 16 - 12 = 4$	*A third equation.*
6.	$D > 0$, so there are two distinct real roots, $x = 1$ and $x = 3$	*A positive discriminant means two different real roots.*
7.	$2^2 - 4 \cdot 2 + 4 = 4 - 8 + 4 = 0$, and $1^2 - 4 \cdot 1 + 3 = 0$, $3^2 - 4 \cdot 3 + 3 = 9 - 12 + 3 = 0$	CHECK ***Check the roots from the second and third equations: substitute them back.*** *Every one gives 0, confirming the discriminant predicted the right number of roots, each time.*

BACK
What the discriminant's sign tells you · p. 120

TRY
Classify the roots of $x^2 + 6x + 9 = 0$ by its discriminant.

Answer: $b^2 - 4ac = 36 - 36 = 0$: one repeated real root, $x = -3$.

KEY
Same shape of check, three different discriminant signs, three different outcomes.



Three parabolas, three discriminants, three verdicts on how many times each meets the axis, all known without finding a single root.

HOW DO YOU TELL HOW MANY ROOTS AN EQUATION HAS, WITHOUT SOLVING IT?

1. **Read off a, b, c** Write the equation in standard form, then read a, b, c off it.
2. **Compute the discriminant** Work out $b^2 - 4ac$ as one number.
3. **Read its sign** A positive discriminant means two distinct roots. A zero discriminant means one repeated root. A negative discriminant means no real root.
4. **Check your answer** If you go on to solve the equation, the number of roots you find should match what the sign predicted.

CHECK YOURSELF

31. Following the same three-case method, what is the nature of the roots of $x^2 + 6x + 9 = 0$?

- **A.** one repeated real root
- **B.** two distinct real roots
- **C.** no real root
- **D.** two complex roots, since the equation cannot be factorised

32. What is the nature of the roots of $x^2 + x + 1 = 0$?

- **A.** two distinct real roots
- **B.** one repeated real root
- **C.** two real roots, both negative
- **D.** no real root

1 more in this set online.

WORKED EXAMPLE · **Checking the discriminant before factoring**

1. solve $x^2 - 2x - 2 = 0$ *Let us check D before attempting to factorise.*

WORKED EXAMPLE · **Checking the discriminant before factoring**

2. $D = b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot (-2) = 4 + 8 = 12$

Compute the discriminant from $a = 1$, $b = -2$, $c = -2$.

3. 12 is positive but not a perfect square

A real root exists, since $D > 0$. But no integer or simple rational split of the middle term will be found: the roots are irrational.

4. $x = (2 \pm \sqrt{12})/2 = 1 \pm \sqrt{3}$

The quadratic formula reaches the roots directly. No factorisation attempt is needed.

5. $\left(1+\sqrt{3}\right)^2 - 2\left(1+\sqrt{3}\right) - 2 = 4 + 2\sqrt{3} - 2 - 2\sqrt{3} - 2 = 0$

CHECK ***Check the root: substitute it back into the original equation.*** *The square-root terms cancel, leaving 0, so the irrational root really does work. Check the other root, $1 - \sqrt{3}$, the same way yourself.*

BACK
Check the discriminant before choosing a method · p. 123

KEY
$D = 12$: positive, so real roots exist. But not a perfect square, so they are irrational.

CHECK YOURSELF

33. Following the same check-before-factoring method, what does the discriminant of $x^2 - 6x + 2 = 0$ tell you before attempting to factorise?

- **A.** $b^2 - 4ac = 28$, so the equation has no real root
- **B.** positive, not a perfect square — formula needed
- **C.** $b^2 - 4ac = 28$, so factorisation with integers must eventually be found by trying more pairs
- **D.** $b^2 - 4ac = 28$, so the equation has a repeated root

34. Having confirmed $D = 28$ for $x^2 - 6x + 2 = 0$ is not a perfect square, what are the roots?

- **A.** $x = 3 \pm \sqrt{7}$
- **B.** $x = 6 \pm \sqrt{7}$
- **C.** $x = 3 \pm \sqrt{28}$
- **D.** $x = 3 \pm 7$

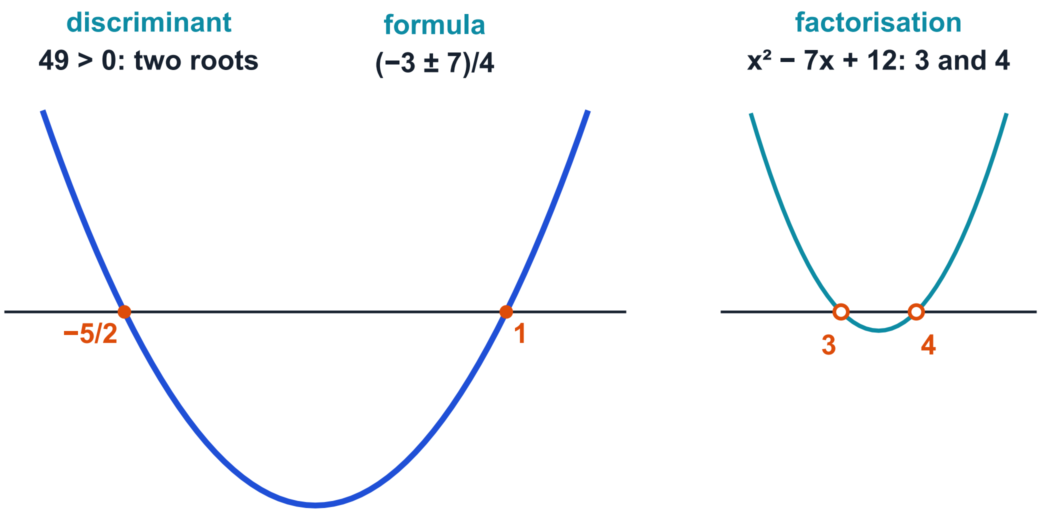
1 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap

Three lenses on the same equation



Three ways to read the same equation: the discriminant 49 says two roots exist, the formula names them, 1 and −5/2, and where a split exists, as −3 and −4 do for $x^2 - 7x + 12$, the factors name them too, 3 and 4.

Whichever lens you pick, the same roots come out; use the discriminant first to know how many to look for.

- STANDARD FORM: $ax^2 + bx + c = 0$, $a \neq 0$. For $2x^2 + 3x - 5 = 0$: $a = 2$, $b = 3$, $c = -5$.
- DISCRIMINANT: $D = b^2 - 4ac$. Here $D = 3^2 - 4 \cdot 2 \cdot (-5) = 49$.
- NATURE OF ROOTS: $D > 0$ gives two distinct roots, $D = 0$ gives one repeated root, $D < 0$ gives no real root. Here $D = 49 > 0$, so two distinct real roots.
- QUADRATIC FORMULA: $x = (-b \pm \sqrt{D})/(2a)$. Here $x = (-3 \pm 7)/4$, giving $x = 1$ or $x = -5/2$.
- FACTORISATION: split bx into parts multiplying to ac and adding to b. For $x^2 - 7x + 12 = 0$, the split is -3 and -4, giving roots 3 and 4.

Let us look back over what we have covered. Every quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, has at most two real roots, never more. Two routes reach them. One factorises the expression into two linear pieces. The other applies the formula, and that works whether or not a factorisation is easy to spot.

One number decides which case you are in, before either method starts. It does this before you find a single root. Skip that check, and you can spend a long time hunting for a factorisation that does not exist.

A situational problem is no different in kind. You solve it by naming the unknown, writing the one stated condition as an equation, and solving it by the same two methods as any other quadratic. The only extra step a real situation adds is checking your answer against the situation itself. A length or a count can never come out negative.

BACK
A root satisfies the equation · p. 114

KEY
Two roots at most, two ways to find them. One number, the discriminant, predicts which kind.

CHECK YOURSELF

35. Which single quantity, computed from a, b, c alone, decides which solving method fits a given quadratic equation before either method is tried?

A. the sum of the coefficients, $a + b + c$

B. the value of c alone

C. the standard form of the equation

D. the discriminant, $b^2 - 4ac$

36. A situational problem is turned into a quadratic equation and solved. What must be checked at the very end, that the discriminant and the formula alone cannot confirm?

A. whether each root fits the real situation

B. whether the discriminant was computed correctly

C. whether the equation was quadratic in the first place

D. whether a repeated root exists

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

37. A quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) has at most two real roots. What tells you, before solving, whether it has two distinct roots, one repeated root, or none?

A. the sign of the constant term c alone

B. the sign of the discriminant $b^2 - 4ac$

C. whether a is positive or negative

D. the number of terms written in the equation

38. $x^2 - x - 12$ factorises as $(x - 4)(x + 3)$. What are the roots of $x^2 - x - 12 = 0$?

A. $x = 4$ and $x = 3$

B. $x = -4$ and $x = -3$

C. $x = 4$ and $x = -3$

D. $x = -4$ and $x = 3$

3 more in this set online.

IN EVERY CHAPTER

Where you will meet this

You will meet quadratics whenever one fact fixes two unknowns together, like a length and a width. Here are seven of those places.

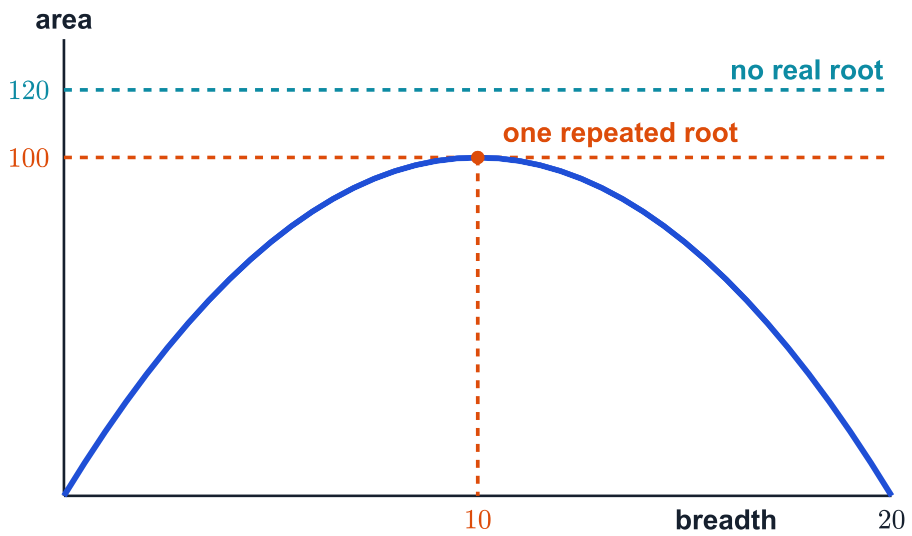
- **Sizing a photo print.** A photo print is 3 cm longer than it is wide. Its area is 108 square cm.
  The maths: name the width as x, then translate the one condition, area $= 108$, into a quadratic equation.
  Width x, length $x + 3$: $x(x + 3) = 108$, so $x^2 + 3x - 108 = 0$. Width 9 cm, length 12 cm.

- **Tiling a courtyard grid.** A courtyard is tiled in rows and columns of square tiles. It has 5 fewer rows than columns, and 150 tiles in all.
  The maths: splitting the middle term factors the quadratic. Each factor then gives one root.
  Columns x: $x(x-5)=150$, so $x^2-5x-150=0$. Split -5 into -15 and 10: $(x-15)(x+10)=0$, so $x=15$ columns.

- **Checking a friend's answer about a trip.** A trip is 240 km. Going 20 km/h faster would save 1 hour. A friend says the usual speed was 60 km/h.
  The maths: a root must make the equation true when you put it back in. Here the equation is $x^2+20x-4800=0$.
  $60^2+20\cdot 60-4800=3600+1200-4800=0$. And $240/60-240/80=4-3=1$ hour. It checks out.

- **Testing if a plot is possible.** A rectangular plot has a fixed perimeter of 40 m. You want its area to be exactly 120 square m.
  The maths: a negative discriminant means the equation has no real root, so no real plot can give that area.
  Breadth x, length $20-x$: $x^2-20x+120=0$. The discriminant is $(-20)^2-4\cdot 1\cdot 120=-80$.

One fence, two areas asked for



40 m of fence gives area $x(20-x)$. 100 square m is met once, at the top, 10 by 10 (one repeated root); 120 square m is above the whole curve (no real root, discriminant -80).

A quadratic has a top, and a target above that top gives an equation with no real root at all.

- **Getting the most from a fixed fence.** You have the same 40 m of fence. This time you want an area of exactly 100 square m.
  The maths: the equation factorises into the same factor twice. So it has one repeated root, and only one shape works.
  Breadth x, length $20-x$: $x^2-20x+100=0$, which is $(x-10)^2=0$. Only the 10 m by 10 m square works.

- **Sizing a rectangular poster.** A rectangular poster has a diagonal of 13 cm and an area of 60 square cm.

The maths: the quadratic formula finds both sides directly, without guessing a factorisation first.
Sides add to $\sqrt{13^2 + 2 \cdot 60} = \sqrt{289} = 17$ cm. The formula gives $t = (17 \pm \sqrt{289 - 240})/2$: 12 and 5 cm.

- **Finding a deposit's yearly rate.** You put ₹10000 in a deposit that adds interest once a year. After 2 years it has grown to ₹12100.
  The maths: name the yearly growth factor x. Two years of growth make the quadratic equation $10000x^2 = 12100$.
  $x^2 = 1.21$, so $x = 1.1$. The other root, -1.1, makes no sense here. So the rate is 10% a year.

Your turn. A rectangular rug is 2 m longer than it is wide. Its area is 24 square m. How wide is it? *Answer:* Width x, length $x + 2$: $x^2 + 2x - 24 = 0$. Since $6 \cdot (-4) = -24$ and $6 + (-4) = 2$, the factors are $(x + 6)(x - 4) = 0$, so $x = 4$ m.

Practice: Exercise 4.1

PRACTICE SET

1. PRACTICE Check whether $x(x + 5) = 2(x - 3)$ is a quadratic equation, after full simplification. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	$x^2 + 5x = 2x - 6$	*expand both sides before judging anything*
2.	$x^2 + 3x + 6 = 0$	*bring every term to one side, so the equation is in standard form*
3.	the coefficient of x^2 is 1, and $1 \neq 0$	*standard form is $ax^2 + bx + c = 0$ with $a \neq 0$, so this is the test*
4.	yes, it is a quadratic equation	*the check — the verdict is read off the simplified form, never the original*

2. PRACTICE Check whether $(x + 2)^2 = x^2 + 9$ is a quadratic equation, after full simplification. (Start the same way — expand both sides and collect everything on one side before looking at the coefficient of x^2.)
3. PRACTICE A rectangular room is 2 metres longer than it is wide, and its area is 48 square metres. Taking the width as x metres, write the quadratic equation that x must satisfy. (Name the width first, then write the length in terms of it.)
4. PRACTICE Check whether $2x^2 - 3x + 1 = (x - 1)(x + 2)$ is a quadratic equation, after full simplification.
6. PRACTICE A rectangular plot's length is 3 metres more than twice its breadth, and its area is 90 square metres. Taking the breadth as x metres, write the quadratic equation the breadth satisfies.
7. PRACTICE The product of two consecutive positive integers is 240. Taking the smaller integer as x, write the quadratic equation it satisfies.
8. PRACTICE A train travels a distance of 300 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 2 hours less for the same journey. Taking the speed as x km/h, write the quadratic equation x satisfies.

5. PRACTICE Check whether $(x+3)^2 = x^2 - 4$ is a quadratic equation, after full simplification.

9. PRACTICE The sum of the ages of two friends is 24 years. Four years ago, the product of their ages in years was 45. Taking the age of one friend as x years, form the quadratic equation this problem gives.

ANSWERS

1. Yes. Expanding gives $x^2 + 5x = 2x - 6$, so $x^2 + 3x + 6 = 0$, and the coefficient of x^2 is $1 \neq 0$.
2. No. Expanding gives $x^2 + 4x + 4 = x^2 + 9$, and the x^2 terms cancel, leaving $4x - 5 = 0$ — a linear equation.
3. $x(x+2) = 48$, that is $x^2 + 2x - 48 = 0$.
4. Yes; expanding and collecting gives $x^2 - 4x + 3 = 0$, coefficient of x^2 is $1 \neq 0$.
5. No — the x^2 terms cancel; expanding and collecting gives the linear equation $6x + 13 = 0$.
6. $2x^2 + 3x - 90 = 0$. **7.** $x^2 + x - 240 = 0$. **8.** $x^2 + 5x - 750 = 0$.
9. $x^2 - 24x + 125 = 0$.
Full solutions: **manthika.com/learn/math10-ch04**

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Simplify first, then check the form · p. 115

Further practice

PRACTICE SET

10. PRACTICE Check whether $(x-4)(x+2) = x^2 - 3x + 6$ is a quadratic equation, after full simplification.

11. PRACTICE Check whether $3x(x-1) = 5(x+2)(x-2) - 4x$ is a quadratic equation, after full simplification.

12. PRACTICE Which of these equations, once fully simplified, is not a quadratic equation?

A. $(x+2)^2 = 3x + 10$
B. $x(x-4) = (x-1)^2 - 2$
C. $2x^2 - 3x = (x+1)^2 - 5$
D. $3x^2 = (x-2)(x+5)$

13. PRACTICE A rectangular hall's carpet area is 126 square metres, and its length is 5 metres more than twice its breadth. Taking the breadth as x metres, write the quadratic equation the breadth satisfies.

14. PRACTICE A cottage industry produces a certain number of pens in a day. The cost of production of each pen, in rupees, is 8 more than the number of pens produced that day. If the total cost of production that day was ₹240, taking the number of pens produced as x, write the quadratic equation x satisfies.

15. PRACTICE A woman is 3 years older than her son. Five years from now, the product of their ages in years will be 150. Taking the son's present age as x years, form the quadratic equation this problem gives.

16. PRACTICE The hypotenuse of a right triangle is 2 metres more than twice its shorter leg, and its longer leg is 1 metre less than twice the shorter leg. Taking the shorter leg as x metres, write the quadratic equation x satisfies, using the Pythagoras theorem.

ANSWERS

10. No — the x^2 terms cancel; expanding and collecting gives the linear equation $x - 14 = 0$.
11. Yes; expanding and collecting gives $2x^2 - x - 20 = 0$, coefficient of x^2 is $2 \neq 0$.
12. B — it simplifies to $-2x + 1 = 0$, a linear equation. 13. $2x^2 + 5x - 126 = 0$.
14. $x^2 + 8x - 240 = 0$. 15. $x^2 + 13x - 110 = 0$. 16. $x^2 - 12x - 3 = 0$.

Full solutions: **manthika.com/learn/math10-ch04**

Practice: Exercise 4.2

PRACTICE SET

1. PRACTICE Solve $x^2 - 9x + 20 = 0$ by factorisation. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	two numbers multiplying to 20 and adding to -9 are -4 and -5	*the middle term is split by the pair that fits both conditions: product c, sum b*
2.	$x^2 - 4x - 5x + 20 = 0$	*rewrite the middle term as that pair*
3.	$x(x - 4) - 5(x - 4) = 0$, so $(x - 4)(x - 5) = 0$	*group in pairs and take out the common bracket*
4.	$x = 4$ or $x = 5$	*a product is zero only when one of its factors is zero*
5.	$16 - 36 + 20 = 0$	*the check — put one root back into the original equation*

2. PRACTICE Solve $x^2 + 2x - 15 = 0$ by factorisation. (Start the same way — find two numbers that multiply to -15 and add to 2. One of them is negative.)
3. PRACTICE Find two numbers whose sum is 15 and whose product is 56. (Call one of them x; the other is then $15 - x$.)
4. PRACTICE Solve $x^2 - 5x - 24 = 0$ by factorisation.
7. PRACTICE The product of two consecutive positive odd integers is 143. Find the integers.
8. PRACTICE The hypotenuse of a right triangle is 13 cm, and one leg is 7 cm longer than the other. Taking the shorter leg as x cm, find the two legs by factorisation.

5. PRACTICE Solve $4x^2 - 12x + 9 = 0$ by factorisation.

6. PRACTICE Find two numbers whose sum is 27 and product is 182.

9. PRACTICE A shopkeeper buys a number of books for ₹720. If the price per book had been ₹4 more, the shopkeeper would have bought 6 fewer books for the same amount. Taking the number of books bought as x, find x by factorisation.

ANSWERS **1.** $x = 4$ and $x = 5$. **2.** $x = -5$ and $x = 3$. **3.** The numbers are 7 and 8. **4.** $x = 8$ or $x = -3$. **5.** A repeated root, $x = 3/2$; $4x^2 - 12x + 9 = (2x-3)^2$. **6.** The two numbers are 14 and 13. **7.** 11 and 13. **8.** The legs are 5 cm and 12 cm ($x = 5$; $x = -12$ is rejected as a negative length). **9.** $x = 36$ books, originally priced at ₹20 each ($x = -30$ is rejected).
Full solutions: **manthika.com/learn/math10-ch04**

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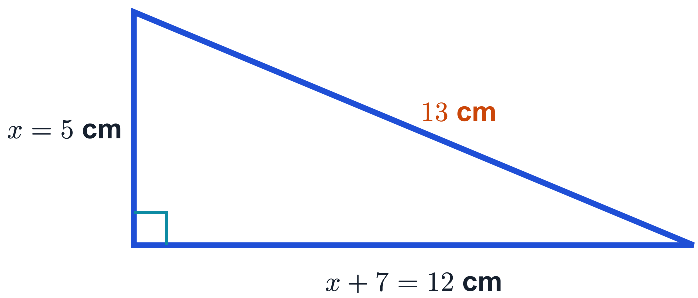
Solving by factorisation · p. 117

Further practice

Where the equation comes from



A right triangle: legs of five and twelve centimetres, hypotenuse thirteen, with the shorter leg named x.

PRACTICE SET

10. PRACTICE Solve $x^2 - 2x - 35 = 0$ by factorisation.

SOLUTION — Q10 · **Worked in full**

1.	$(-7) + 5 = -2$ and $(-7) \cdot 5 = -35$	*find two numbers that multiply to -35 and add to -2*
2.	$x^2 - 2x - 35 = x^2 - 7x + 5x - 35 = x(x-7) + 5(x-7) = (x-7)(x+5)$	*split the middle term using those two numbers, then group in pairs and factor each pair*
3.	$x = 7$ or $x = -5$	*set each factor to 0 in turn*

11. PRACTICE Solve $6x^2 + x - 2 = 0$ by factorisation.

16. PRACTICE A rectangular park's length is 4 metres more than its breadth, and its area is 192 square metres. Taking the breadth as x metres, find the length and breadth by factorisation.

12. PRACTICE Which of these quadratic equations has a repeated root?
A. $x^2 - 5x + 6 = 0$
B. $x^2 - 6x + 9 = 0$
C. $x^2 - x - 6 = 0$
D. $x^2 + x - 6 = 0$

13. PRACTICE Find two positive numbers whose difference is 5 and whose product is 204.

14. PRACTICE Find two consecutive positive even integers whose product is 624.

15. PRACTICE The sum of the squares of two consecutive positive integers is 181. Find the integers.

17. PRACTICE The hypotenuse of a right triangle is 25 m, and one leg is 17 m longer than the other. Taking the shorter leg as x m, find the two legs by factorisation.

18. PRACTICE A cottage industry produces a number of table lamps in a day. The cost of production of each lamp, in rupees, is 7 more than twice the number of lamps produced that day. If the total cost of production that day was ₹114, taking the number of lamps produced as x, find x by factorisation.

ANSWERS **10.** $x = 7$ or $x = -5$. **11.** $x = -2/3$ or $x = 1/2$. **12.** B — $x^2 - 6x + 9 = 0$.
13. The numbers are 12 and 17 ($x = -17$ is rejected).
14. The integers are 24 and 26 ($x = -26$ is rejected).
15. The integers are 9 and 10 ($x = -10$ is rejected).
16. Breadth $= 12$ m, length $= 16$ m ($x = -16$ is rejected).
17. The legs are 7 m and 24 m ($x = -24$ is rejected).
18. 6 lamps were produced, at ₹19 each ($x = -19/2$ is rejected).
Full solutions: **manthika.com/learn/math10-ch04**

Practice: Exercise 4.3

PRACTICE SET

1. PRACTICE Find the discriminant of $3x^2 + 5x - 2 = 0$ and state the nature of its roots. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	$a = 3, b = 5, c = -2$	*write all three down with their signs before substituting anything*
2.	$D = b^2 - 4ac = 5^2 - 4 \cdot 3 \cdot (-2)$	*substitute with the sign attached: c is -2, not 2, so keep it in brackets*
3.	$D = 25 + 24 = 49$	*minus four times a negative c turns into a plus, which is where the sign is usually lost*
4.	$D = 49 > 0$, so there are two distinct real roots	*the check — a positive discriminant means the graph crosses the x-axis twice*

2. PRACTICE Find the discriminant of $x^2 + 6x + 9 = 0$ and state the nature of its roots. (Start the same way — write a, b and c down with their signs first.)

6. PRACTICE Solve $3x^2 - 5x - 2 = 0$ using the quadratic formula.

7. PRACTICE Find the value of k for which the equation $kx^2 + 4x + 1 = 0$ has equal roots.

3. PRACTICE Find the value of k for which $x^2 + kx + 9 = 0$ has equal roots. (Equal roots means the discriminant is zero.)

4. PRACTICE Find the discriminant of $2x^2 - 3x + 5 = 0$ and state the nature of its roots.

5. PRACTICE Find the discriminant of $5x^2 - 6x - 2 = 0$ and state the nature of its roots.

8. PRACTICE Find the value of p for which the equation $2x^2 + px + 8 = 0$ has real and equal roots.

9. PRACTICE A community wants to design a rectangular park with a perimeter of 80 metres and an area of 500 square metres. Taking the length as x metres, check using the discriminant whether such a park is possible, before attempting to solve for x.

ANSWERS **1.** $D = 49$, which is positive, so the equation has two distinct real roots.
2. $D = 0$, so the equation has one repeated real root, $x = -3$. **3.** $k = 6$ or $k = -6$.
4. $D = 9 - 40 = -31 < 0$; no real root.
5. $D = 36 + 40 = 76 > 0$; two distinct real roots (irrational, since 76 is not a perfect square).
6. $x = 2$ or $x = -1/3$. **7.** $k = 4$. **8.** $p = 8$ or $p = -8$.
9. Breadth $= (40 - x)$, so $x^2 - 40x + 500 = 0$ and $D = 1600 - 2000 = -400 < 0$: no real value of x exists, so a park with these dimensions is not possible.
Full solutions: **manthika.com/learn/math10-ch04**

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The discriminant · p. 119

Further practice

PRACTICE SET

10. PRACTICE Find the discriminant of $x^2 - 8x + 16 = 0$ and state the nature of its roots.

SOLUTION — Q10 · **Worked in full**

1.	$D = b^2 - 4ac = (-8)^2 - 4 \cdot 1 \cdot 16 = 64 - 64 = 0$	*compute the discriminant from $a = 1$, $b = -8$, $c = 16$*
2.	$D = 0$, so there is one repeated real root	*a zero discriminant means the equation factors as a single factor squared*

11. PRACTICE Find the discriminant of $4x^2 + 3x + 2 = 0$ and state the nature of its roots.

12. PRACTICE Which of these quadratic equations has two distinct real roots?
A. $x^2 + 2x + 5 = 0$
B. $x^2 - 4x + 4 = 0$
C. $2x^2 - 3x - 1 = 0$
D. $x^2 + x + 1 = 0$

13. PRACTICE Solve $2x^2 - 7x + 3 = 0$ using the quadratic formula.

15. PRACTICE Find the value of k for which the equation $9x^2 - 24x + k = 0$ has equal roots.

16. PRACTICE Find the value of n for which the equation $nx^2 + nx + 4 = 0$ (with $n \neq 0$) has equal roots.

14. PRACTICE Solve $x^2 - 4x - 1 = 0$ using the quadratic formula.

17. PRACTICE A community wants to design a rectangular hall whose perimeter is 60 metres and whose area is 250 square metres. Taking the length as x metres, check using the discriminant whether such a hall is possible, before attempting to solve for x.

ANSWERS **10.** $D = 0$; one repeated real root ($x = 4$). **11.** $D = -23 < 0$; no real root. **12.** C — $2x^2 - 3x - 1 = 0$. **13.** $x = 3$ or $x = 1/2$. **14.** $x = 2 + \sqrt{5}$ or $x = 2 - \sqrt{5}$. **15.** $k = 16$. **16.** $n = 16$ ($n = 0$ is rejected).
17. $D = -100 < 0$: no real value of x exists, so a hall with these dimensions is not possible.
Full solutions: **manthika.com/learn/math10-ch04**