5

CHAPTER 5

Arithmetic Progressions

WHAT THIS CHAPTER BUILDS

You will learn to recognise a list that grows by a constant step, find any term of it, and add up as many terms as you like with one formula.

ALGEBRA UNIT, PATTERN QUESTIONS

IN THIS CHAPTER

Deo climbs the school staircase one equal step at a time while Nila watches from the bottom.

Getting started

A constant step

A STORY

The same step every time

Deo climbs the school staircase, one step at a time. Nila waits at the bottom, one hand on the first step.

“Every step is the same height,” Nila says. “I measured one last week: 15 cm.”

Deo stops. “Then tell me how high my foot is. It is on the fourth step.”

“Four steps of 15 cm,” Nila says. “60 cm above the floor.”

“And the twentieth step, at the top?”

“20 times 15 is 300 cm, three metres up. The same step every time makes it easy.”

“Now a harder one,” Deo says. “Climb one step and come down. Then two steps and come down. Then three, and so on, up to all twenty. How many steps do you climb in all?”

“One, two, three, and on to twenty,” Nila says slowly. “That is a long sum to add by hand.”

“There is a short way,” Deo says.

You will learn the rule for any one step in a pattern like this, and the rule for adding up all of them.

₹200 goes into a savings box in the first month. Every month after that, ₹40 more goes in than the month before. The monthly deposits climb $200, 240, 280, 320, \cdots$. Your turn: check the gap between each deposit and the one before it. (Answer: every gap is exactly ₹40.)

Let us name that fixed step. A sequence built this way is an ARITHMETIC PROGRESSION, or AP. Each term differs from the one before it by the same fixed number. The fixed number is called d, the common difference.

Two questions follow at once. Picture one particular term, far down the list: could you name it without writing out every term before it? Could you add up a whole run of terms just as fast? We answer both questions from the start, not by handing over a ready-made shortcut.

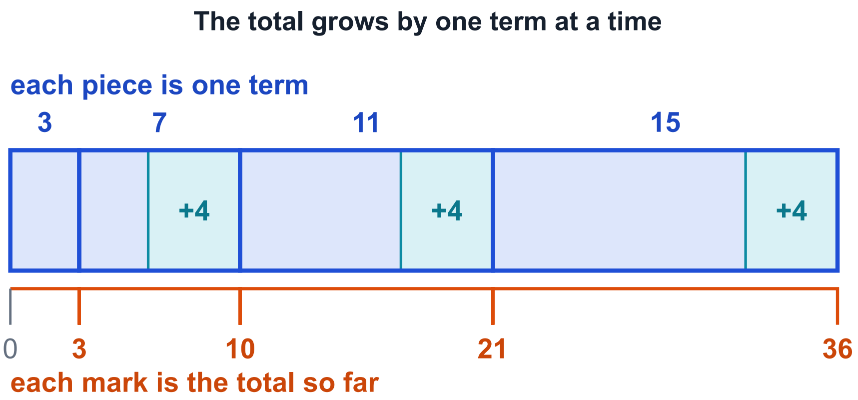
RULE
Before applying a formula, name a, d and n for the sequence at hand, the way $5, 9, 13, \cdots$ names $a = 5$ and $d = 4$. The rest is substitution.

KEY
Once the first term and the common step are known, every later term and every running total are already fixed.

A SCHOLAR INDIA REMEMBERS

Aryabhata wrote a book called the Aryabhatiya in 499 CE, when he was 23. *He lived at Kusumapura, near today’s Patna. His book gave general methods: one method for a whole class of problems. We want the same here. Take the list* 3, 7, 11, 15. *We will not reach the 100th number by writing all* 100. *We will find one rule that gives any number in the list.*

Aryabhata · the hoopoe



Each piece is one term, each piece after the first is four longer, and the marks below give the running total.

IN THE WILD

Picture a stadium where the rows grow by a fixed number of seats as you go back, 20 seats in the first row, 24 in the second, 28 in the third. Once you know the first row's seat count and that fixed step, you can name the fiftieth row's seat count, or add up the seats in all fifty rows, without counting a single seat.

CHECK YOURSELF

1. Which description correctly defines an arithmetic progression (AP)?

- **A.** the step from one term to the next stays fixed
- **B.** each term is double the one before it
- **C.** each term increases, but by a different amount each time
- **D.** the terms have no fixed pattern between them

2. In an AP, what does the common difference d actually control?

- **A.** how many terms the AP has
- **B.** the value of the first term
- **C.** the added amount, term to term
- **D.** which position a term sits at in the list

1 more in this set online.

IN EVERY CHAPTER

Before you start

DO THIS FIRST

You have met sequences before. Now name the fixed step between two terms. Try each check below. It takes a minute.

- **The common difference** (Class 9, Ganita Manjari, page 181).
  Here: d is the fixed step this progression adds, term after term.
  Check: $1, 5, 9, 13, \ldots$ has common difference 4.
- **An arithmetic progression** (Class 9, Ganita Manjari, page 180).
  Here: a list is tested by checking that every gap has the same value.
  Check: $3, 6, 9, 12$ has equal gaps of 3, so it is an AP.
- **The nth term formula** (Class 9, Ganita Manjari, page 182).
  Here: $a_n = a + (n-1)d$ finds the 30th term without listing 29 others.
  Check: $t_n = 1 + (n-1) \cdot 4$ gives $t_3 = 1 + 2 \cdot 4 = 9$.

- **An explicit rule for a sequence** (Class 9, Ganita Manjari, page 176).
  Here: this one rule generates every term, without writing each one out.
  Check: $u_n = 2n - 1$ gives $u_3 = 2 \cdot 3 - 1 = 5$.
- **A sequence's recursive rule** (Class 9, Ganita Manjari, page 183).
  Here: each term here is built from the one before it, deriving the nth term.
  Check: $t_1 = 1$, $t_2 = t_1 + 4 = 5$.
- **Sum of the first n numbers** (Class 9, Ganita Manjari, page 184).
  Here: adding forwards and backwards this way derives the sum formula.
  Check: $1 + 2 + 3 + 4 = 10$, and $4 \cdot 5/2 = 10$.

If any of these felt new, read the page named before going on.

The common difference of an AP

The COMMON DIFFERENCE of an AP is the fixed number d added to each term to get the next one. Pick two consecutive terms, a_k and a_{k+1}. You can find d yourself: $d = a_{k+1} - a_k$, the later term minus the earlier one, in that order.

d can be positive, negative, or zero: all three are legal common differences. A salary rising by a fixed annual raise has a positive d. A savings amount shrinking by a fixed instalment each month has a negative d. A list where every term repeats the same number has $d = 0$. We still call it an AP: just a flat one, a fixed step of zero.

Your turn: find d for the AP $6, 3, 0, -3, \cdots$. (Answer: $d = 3 - 6 = -3$, so each term is three less than the one before it.) Getting the subtraction order backwards, using $a_k - a_{k+1}$ instead, flips the sign of d and every later prediction.

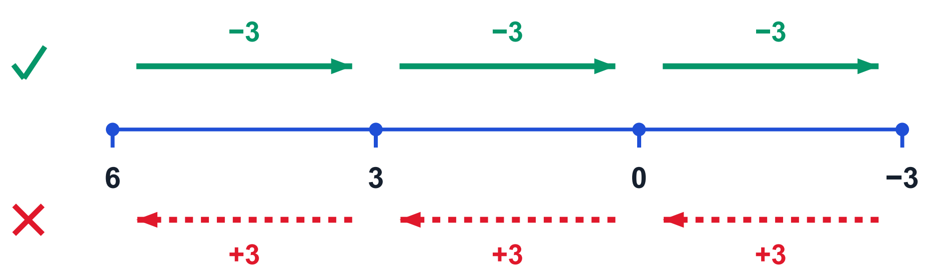
TRY
For the AP $6, 3, 0, -3, \cdots$, what is d?

Answer: $d = 3 - 6 = -3$.

BACK
A constant step · p. 144

KEY
d may be positive, negative, or zero, all three are legal common differences.

Later minus earlier fixes the sign



Every gap read forwards is −3; the same gaps backwards, +3.

The same four terms, subtracted both ways round: only the order the key term names gives d.

A SCHOLAR INDIA REMEMBERS

Is 2, 4, 8, 16 *an AP? Count the steps before you answer.* $4 - 2 = 2$. $8 - 4 = 4$. $16 - 8 = 8$. *The steps are* 2, 4 *and* 8. *They are not equal. So this list is not an AP. An AP takes the same step every time.*

Aryabhata · the hoopoe

CHECK YOURSELF

3. How is the common difference d of an AP correctly found from two consecutive terms?

A. $d = a_k - a_{k+1}$, the earlier term minus the later one

B. $d = a_{k+1} - a_k$, the later term minus the earlier one

C. $d = a_{k+1} + a_k$, the two terms added together

D. $d = a_{k+1}/a_k$, the later term divided by the earlier one

4. An AP's terms get smaller with every step. What must be true of its common difference d?

A. d must be positive

B. d must be zero

C. d must be negative

D. d could be any sign, since the terms are shrinking either way

1 more in this set online.

The general form of an AP

The GENERAL FORM of an AP has first term a and common difference d. Let us write it out: $a, a + d, a + 2d, a + 3d, \cdots$. Every term after the first is built the same way: you take the one before it and add d.

The first term is often written a_1, or simply a, when no confusion is possible. You can see the pattern continue: the second term is a_2, the third a_3, and so on, one subscript higher for every step along the list.

a and d between them fix everything. Nothing else changes what the list looks like. Your turn: write out the first four terms of the AP with $a = 5$ and $d = 3$. (Answer: $5, 8, 11, 14$.)

RULE
a and d between them fix the whole progression, the way $a = 5$ and $d = 3$ fix $5, 8, 11, 14, \cdots$ completely. Nothing else is needed to list it.

KEY
Every term after the first is built the same way, add d to the one before it.

CHECK YOURSELF

5. Which is the correct general form of an AP with first term a and common difference d?

A. $a, 2a, 3a, 4a, \cdots$

B. $a, a + d, a + 4d, a + 9d, \cdots$

C. $a, a + d, a + 2d, a + 3d, \cdots$

D. $a, d, 2d, 3d, \cdots$

6. In the general form, the third term is written $a + 2d$. Why $2d$ and not $3d$?

A. because d is always counted starting from the second term, not the first

B. because the third term is always exactly two-thirds of the way through the AP

C. reaching the third term needs only two additions of d from the first term

D. because a itself already counts as one addition of d

1 more in this set online.

The ideas

Testing whether a list is an AP

A list $a_1, a_2, a_3, \cdots$ is an AP when the difference between every pair of consecutive terms is the same. Compute $a_2 - a_1$, then $a_3 - a_2$, and so on. If every difference you compute comes out the same, the list is an AP, and that shared value is its d.

One matching pair is never enough. You should check at least two or three consecutive differences before deciding. We can be fooled by a single coincidence: two terms may differ by the right amount purely by chance.

Your turn: test $2, 4, 8, 16, \cdots$ the same way. (Answer: $4 - 2 = 2$, but $8 - 4 = 4$, so the differences disagree and this list is not an AP, even though the pattern looked tempting.)

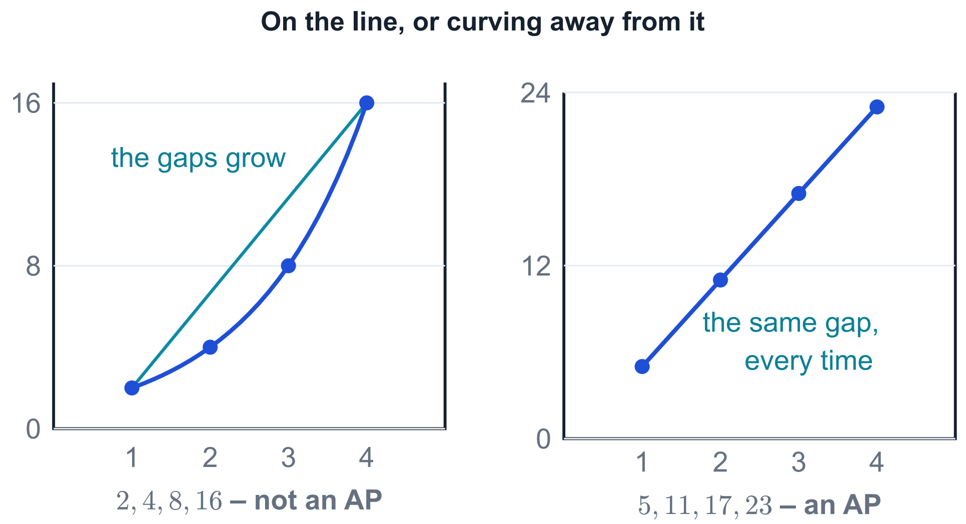
Once a list has actually been confirmed as an AP, the shortcut runs the other way. A single pair of consecutive terms is now enough to read off d: every other pair matches it too.

BACK
The common difference of an AP · p. 146

TRY
Is $2, 4, 8, 16, \cdots$ an AP?

Answer: No: $4 - 2 = 2$, but $8 - 4 = 4$. The differences are not equal.

RULE
Once a list is confirmed to be an AP, one pair of terms is enough to find d.



Points on a straight line have equal gaps between them, and points that curve away do not.

CHECK YOURSELF

7. What must be true for a list of numbers to count as an AP?

- **A.** the difference between the first and last terms is a whole number
- **B.** every term is bigger than the one before it
- **C.** every consecutive pair shares the same difference
- **D.** the terms can be arranged in any order to look evenly spaced

8. To confirm that a list of 5 numbers is an AP, how many consecutive differences must be checked?

- **A.** just the first 1 difference
- **B.** 5 differences, one for every term
- **C.** only the difference between the first and last term
- **D.** all 4 consecutive differences

2 more in this set online.

Finite and infinite APs

An AP can end, or it can run forever. A FINITE AP has a fixed number of terms and stops at a LAST TERM, usually written l. An INFINITE AP never stops: after any term, another one always follows, by the same rule as every term before it.

Let us look at the AP $147, 148, 149, \cdots, 157$. Is it finite or infinite? You can tell from the last term alone: it is finite, because the list is written with an explicit last term, 157, and it stops there. Your turn: is $1, 4, 7, 10, \cdots$ finite or infinite? (Answer: infinite, since it is written with no last term and the pattern continues without limit.)

The number of seats in one row of a hall is a finite AP: the row has an end. The multiples of 3, listed in order, form an infinite AP, since there is no largest multiple of 3.

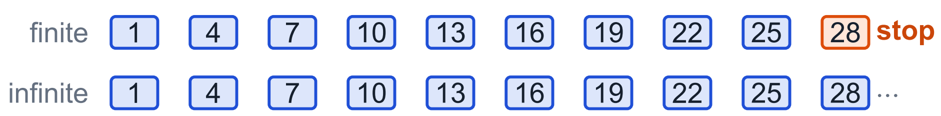
BACK
The general form of an AP · p. 147

TRY
The AP $147, 148, 149, \cdots, 157$: finite or infinite?

Answer: Finite: it ends at the last term 157.

KEY
A last term l is the signature of a finite AP. Its absence marks an infinite one.

The same rule, with or without a last term



The top row stops at a marked last chip, and the bottom row keeps going, trailing off with three dots.

CHECK YOURSELF

9. What distinguishes a finite AP from an infinite one?

- **A.** a finite AP has a common difference d; an infinite AP does not
- **B.** a finite AP stops at a fixed last term; an infinite AP never stops
- **C.** a finite AP always has a negative d; an infinite AP always has a positive d
- **D.** a finite AP has only whole-number terms; an infinite AP can have any terms

10. Which of these is an example of a finite AP?

- **A.** the natural numbers $1, 2, 3, 4, \cdots$, counted without end
- **B.** the odd numbers $1, 3, 5, 7, \cdots$, listed forever
- **C.** any AP where d is a whole number
- **D.** hourly temperatures recorded through one 24-hour day

1 more in this set online.

The nth term formula

The nth term of an AP with first term a and common difference d is $\boldsymbol{a_n = a + (n-1)d}$. Reaching the nth term means starting at a and taking $(n-1)$ steps of size d. That is one step fewer than the term number, because the first term itself needs no step at all. The full build-up of this formula, term by term, follows shortly.

For a finite AP with last term l, the same formula gives $l = a + (n-1)d$, where n is the total number of terms. The last term is simply the nth term, under a different name.

We now have four quantities linked together: a, d, n, and a_n. Know any three of them, and you can find the fourth.

Your turn: first term 5, common difference 2, what is the 6th term? (Answer: $a_6 = 5 + (6 - 1) \cdot 2 = 15$.)

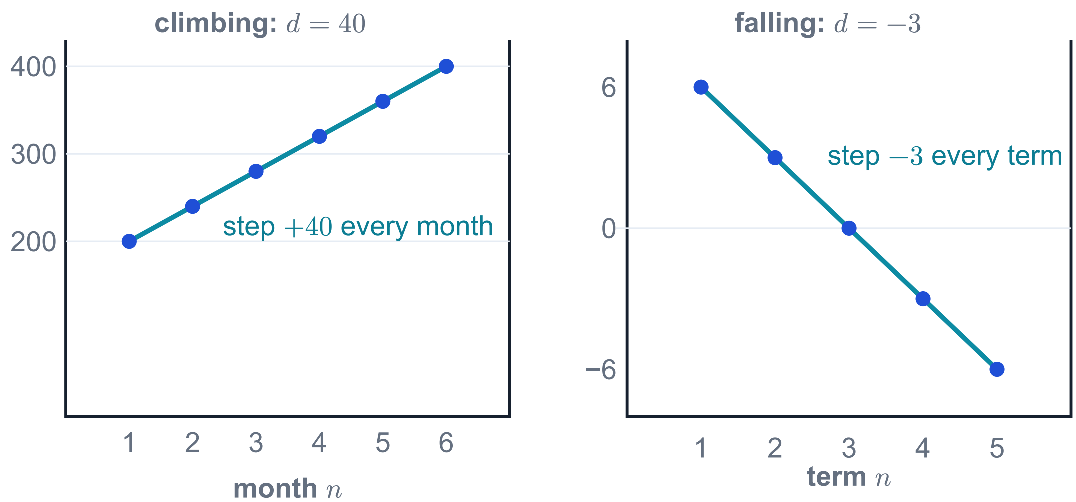
TRY
First term 5, common difference 2: what is the 6th term?

Answer: $a_6 = 5 + (6-1) \cdot 2 = 15$.

RULE
Four quantities are linked by this formula: a, d, n, a_n. Know any three, such as $a = 3$, $d = 4$ and $n = 5$, and the fourth, $a_5 = 19$, follows.

KEY
$(n-1)$, not n, the first term itself needs zero additions of d.

The common difference is the slope



Each dot is one nth-term value, climbing in a straight line when the common difference is positive and falling when it is negative.

A SCHOLAR INDIA REMEMBERS

Will your rule work for the next case too? Test $a_n = a + (n-1)d$ where you can check it. For 3, 7, 11, 15, we have $a = 3$ and $d = 4$. The 4th term is $3 + 3 \times 4 = 15$. That matches the list. Now use it for the 100th term: $3 + 99 \times 4 = 399$.

Aryabhata · the hoopoe

CHECK YOURSELF

11. Which is the correct formula for the nth term of an AP with first term a and common difference d?

A. $a_n = a + (n-1)d$

B. $a_n = a + nd$

C. $a_n = na + d$

D. $a_n = a + (n+1)d$

12. Why does the nth-term formula use $(n-1)$ instead of n?

A. the first term needs no addition at all

B. because d is defined as one less than the actual step size

C. because the formula only works when n is even

D. because the last term is always excluded from the count

2 more in this set online.

The sum of the first n terms

Let us take an AP with first term a and common difference d. The sum of its first n terms is $\boldsymbol{S_n = n/2 \cdot [2a + (n-1)d]}$. When the last term l is known instead of d, the same sum can be written $S_n = n/2 \cdot (a + l)$. That is the average of the first and last term, multiplied by how many terms there are. The full derivation, using a trick from Gauss, follows shortly.

Two equivalent forms exist, for two different situations. Use the d-form when you know the common difference. Use the l-form when you know the last term instead. Finding S_n needs n, and either d or l: never mix the two forms inside one calculation.

Your turn: first term 4, last term 20, 10 terms. Find S_{10}. (Answer: $S_{10} = 10/2 \cdot (4 + 20) = 120$.)

TRY
First term 4, last term 20, 10 terms: what is S_{10}?

Answer: $S_{10} = 10/2 \cdot (4 + 20) = 120$.

RULE
Finding S_n needs n, and either d or l: for 10 terms with $a = 4$ and $l = 20$, that is enough. Never mix the two forms in one calculation.

KEY
Two equivalent forms: use d when it is known, use l when it is not.

A SCHOLAR INDIA REMEMBERS

Before you use the formula, guess the size of the answer. $1 + 2 + 3 + \ldots + 100$ *has* 100 *terms. On average a term is about* 50*. So the sum is about* $100 \times 50 = 5000$*. The formula gives* $100/2 \times (1 + 100) = 5050$*. The guess was close, so the answer makes sense.*

Aryabhata · the hoopoe

CHECK YOURSELF

13. Which is the correct formula for the sum S_n of the first n terms of an AP?

A. $S_n = n \cdot [a + (n-1)d]$

B. $S_n = n/2 \cdot [2a + nd]$

C. $S_n = n/2 \cdot [2a + (n-1)d]$

D. $S_n = a + (n-1)d$

14. In the sum formula, why does the factor $n/2$ appear at all?

- **A.** because an AP always has an even number of terms
- **B.** because a and l are always exactly half of each other
- **C.** because only half the terms in an AP are actually counted
- **D.** forward-plus-backward double-counts every term

2 more in this set online.

How a term and a sum are related

A term and a sum are linked, but never equal. The nth term of an AP equals the difference between two consecutive sums: $\boldsymbol{a_n = S_n - S_{n-1}}$, for $n \geq 2$. A single term is exactly what a sum GAINS by reaching one term further, not the sum itself.

For $n = 1$ there is nothing to subtract: $a_1 = S_1$, since summing just one term gives that term back.

Let us take the AP $3, 7, 11, \cdots$, where $S_3 = 21$ and $S_4 = 36$. Then $a_4 = S_4 - S_3 = 15$. Your turn: check a_4 against the ordinary formula. (Answer: $3 + (4-1) \cdot 4 = 15$, the same answer, confirming the relation both ways.)

This relation LINKS a_n and S_n. It never makes the two equal. The difference between them is exactly what the next section asks you to catch.

CAUTION
This relation LINKS a_n and S_n. It does not make them equal.

BACK
The nth term formula · p. 150

KEY
A single term is what a sum GAINS by going one term further, not the sum itself.

CHECK YOURSELF

15. For $n \geq 2$, how is a single term a_n correctly recovered from sums S_n and S_{n-1}?

- **A.** $a_n = S_n - S_{n-1}$
- **B.** $a_n = S_n + S_{n-1}$
- **C.** $a_n = S_{n-1} - S_n$
- **D.** $a_n = S_n/n$

16. If $S_7 = 84$ and $S_6 = 66$ for an AP, what is a_7?

- **A.** 150
- **B.** 12
- **C.** 18
- **D.** 84

2 more in this set online.

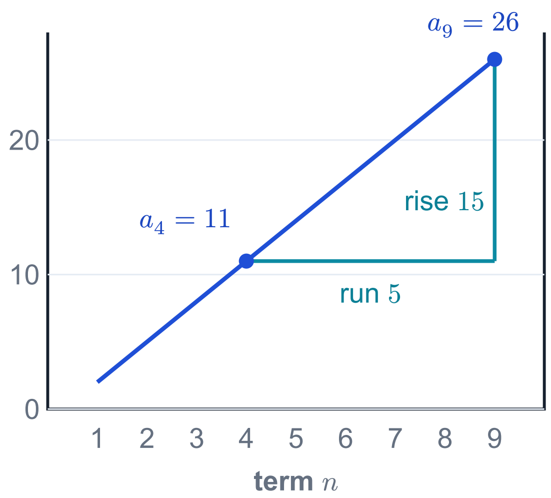
Finding a and d from two given terms

Any two terms of an AP can be written using $a_n = a + (n-1)d$. We now have two linear equations in the two unknowns a and d. Write the two equations, then solve them together, and you will find both unknowns. Every other term of the AP then follows from the same formula.

Subtract the two equations first. Doing so removes a immediately, since a appears in both equations with the same coefficient. That leaves one equation in d alone, which you can solve directly. Your turn: in the equations $a + 3d = 11$ and $a + 8d = 26$, which unknown cancels when you subtract? (Answer: a cancels, leaving $5d = 15$.)

RULE
Subtract the two equations first: for $a + 3d = 11$ and $a + 8d = 26$, that leaves $5d = 15$ at once.

Two points fix the line – and the slope between them



The rise of fifteen over the run of five gives the common difference, three, the slope of this line.

CHECK YOURSELF

17. Knowing any two terms of an AP gives you what, in terms of finding a and d?

- **A.** the value of d directly, with no further work needed
- **B.** one equation with two unknowns, which cannot be solved
- **C.** two linear equations in a and d
- **D.** the sum S_n of the AP directly

18. Given two equations $a + 3d = 11$ and $a + 8d = 26$, what is the most direct way to find d?

- **A.** subtract the first equation from the second, eliminating a
- **B.** add the two equations together
- **C.** guess values of a and d until both equations happen to work
- **D.** divide the second equation by the first

1 more in this set online.

The rth term from the end

The rth term from the end of a finite AP is found by a trick. Let us reverse the AP and read the same list backwards, starting from the last term l. You will find it is itself an AP, with first term l and common difference $-d$: every step that added d moving forward now subtracts d moving backward.

Reversing turns a "from the end" question into an ordinary nth-term question, just starting from l instead of a. **The rth term from the end is $l + (r-1)(-d)$, the ordinary formula applied to the reversed AP.**

Counting from the end restarts the count at l. Do not confuse a term's position counted from the end with its position counted from the start: check which direction a question means before you begin. The two counts run in opposite directions along the same list. Your turn: if an AP has $d = 4$, what is the common difference of its reversed form? (Answer: -4.)

BACK
Finite and infinite APs · p. 149

CAUTION
Counting from the end restarts the count at l. Do not confuse it with the term's position from the start.

KEY
Reversing the AP turns a "from the end" question into an ordinary nth-term question, starting from l.

CHECK YOURSELF

19. What is the correct formula for the rth term from the end of a finite AP with last term l and common difference d?

- **A.** $l + (r-1)d$
- **B.** $l + rd$
- **C.** $a + (r-1)d$
- **D.** $l + (r-1)(-d)$

20. Why can the ordinary nth-term formula be reused, unchanged in form, to find a term counted from the end of a finite AP?

- **A.** because l always equals a in a finite AP
- **B.** because every finite AP secretly has $d = 0$
- **C.** reversing gives another AP: first term l, step $-d$
- **D.** because counting from the end only works when n is odd

1 more in this set online.

Common mistakes

A term is not the same as a sum

Here is the mistake most worth catching: asked for a specific term, it is tempting to compute the sum of the first n terms instead. Or the reverse can happen: asked for a running total, you report a single term instead. The two questions sound similar. They are not the same question, and the two answers are almost never the same number.

Let us take the AP $3, 7, 11, 15, 19$. The fifth term is $a_5 = 19$: one single number, the fifth entry on the list and nothing else. The sum of the first five terms is $S_5 = 55$: every one of those five numbers added together. Both numbers describe the same AP. Neither one stands in for the other.

The two are related, but only through **$a_n = S_n - S_{n-1}$, never through $a_n = S_n$**. A term is what a running total gains by going one step further, not the total itself. Mixing the two is not a small slip. A term and a sum can differ by a wide margin, as 19 and 55 already show.

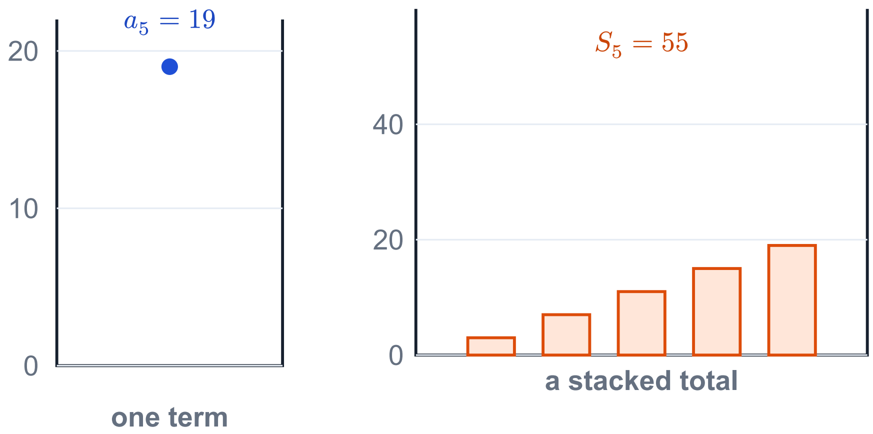
Before you answer a question about an AP, read it again. Ask yourself exactly one thing: does it name a single term, or a total? Your turn: a problem gives the value in the 5th year, and separately asks how much has built up after 5 years. Which formula answers each? (Answer: the value in the 5th year is a term, a_5. The total built up after 5 years is a sum, S_5.) Get this one read wrong, and the whole calculation goes to the wrong formula.

BACK
How a term and a sum are related · p. 152

CAUTION
Read the question again: does it name ONE term, or a TOTAL?

KEY
A term and a sum answer different questions and are almost never the same number.

A term is one dot. A sum is a stack.



The single dot on the left is a term, and all the bars on the right added together are a total.

FIRST TRY
~~Asked for the 5th term of 3, 7, 11, 15, 19, I added all five terms and got $a_5 = 55$.~~

SECOND LOOK
a_n is one term. S_n is the running total. Here $a_5 = 19$, the fifth term alone, while $S_5 = 55$ is all five added.

A term and a sum answer different questions. Never swap them.

A SAVINGS BOX ADDS A FIXED AMOUNT EVERY MONTH. WHAT HAS IT SAVED BY MONTH 9?

WEAK
A reader wants the total saved after 9 months, for a box that holds ₹200 after month 1 and grows by ₹40 every month. They compute $a_9 = 200 + (9 - 1) \cdot 40 = 520$ and give ₹520 as the answer.

But ₹520 is just the amount added in month 9 alone, one single deposit, not everything saved so far. Nine months of deposits, each at least ₹200, must add up to far more than ₹520. The size of that answer alone is the giveaway that the wrong formula was used.

STRONG
The question asks for a total, so it needs S_n, not a_n. $S_9 = 9/2 \cdot [2 \cdot 200 + (9 - 1) \cdot 40] = 9/2 \cdot [400 + 320] = 9/2 \cdot 720 = 3240$.

The total saved after 9 months is ₹3240, not ₹520. Checking the size of an answer against the question asked catches this slip before it is written down.

CHECK YOURSELF

21. A student was asked for the 5th term of the AP $3, 7, 11, 15, 19$ and answered 55. What went wrong?

- **A.** nothing is wrong — 55 is an acceptable way to state the 5th term
- **B.** the student should have said 15 instead, the 4th term
- **C.** 55 is the SUM of all five terms — the 5th term alone is 19

22. A student was asked for S_4 of the AP $5, 8, 11, 14$ and answered 14. What went wrong?

- **A.** nothing is wrong — the last term IS the sum of an AP
- **B.** 14 is a_4 alone — $S_4 = 5 + 8 + 11 + 14 = 38$
- **C.** the student should have said 11 instead, the 3rd term

3 more in this set online.

Worked examples

WORKED EXAMPLE · **Checking whether 5, 11, 17, 23 is an AP**

1.	$11 - 5 = 6$	*Compute the first consecutive difference.*
2.	$17 - 11 = 6$	*Now compute a second consecutive difference.*
3.	$23 - 17 = 6$	*Compute a third difference too, so the pattern is not just a coincidence.*
4.	all three differences equal 6	*Every consecutive pair checked gives $a_{k+1} - a_k = 6$, so the list is an AP.*
5.	the AP has $a = 5$ and $d = 6$	***Three matching differences, not two, is enough to trust the pattern.*** *The first term and the shared difference read off directly.*
6.	$a_4 = 5 + (4 - 1) \cdot 6 = 23$	CHECK *Check the pattern a different way: the formula gives the 4th term as 23 too, the same number the list already showed.*

TRY
Check whether $3, 6, 9, 12, \cdots$ is an AP.

Answer: $6 - 3 = 3$, $9 - 6 = 3$, $12 - 9 = 3$: all equal, so yes, an AP with $a = 3$ and $d = 3$.

BACK
Testing whether a list is an AP · p. 148

RULE
Compute three consecutive differences before concluding, not two: in $5, 11, 17, 23$ all three come to 6. Two alone can be a coincidence.

CHECK WHETHER ANY LIST OF NUMBERS IS AN ARITHMETIC PROGRESSION.

1. **Compute the differences** Subtract each term from the one after it, at least three times in a row.
2. **Check they all match** If every difference comes out the same, the list is an AP, and that shared value is d.
3. **Read off a and d** The first term of the list is a. The common difference is the value you just found.

CHECK YOURSELF

23. Which check correctly confirms that $5, 11, 17, 23, \cdots$ is an AP?

- **A.** $11-5=17-11=23-17=6$
- **B.** $23-5=18$, so the common difference is 18
- **C.** $11/5=2.2$, so the common ratio is 2.2
- **D.** checking that 5 and 23 are both odd numbers

24. For an unfamiliar list of numbers, why must every consecutive pair be checked, rather than just one pair?

- **A.** because the AP definition only applies to lists longer than 4 terms
- **B.** because checking more than one pair changes the value of d
- **C.** a coincidence could make one pair match
- **D.** because the first pair is always the least reliable one to check

1 more in this set online.

WORKED EXAMPLE · Deriving the nth term formula

1.	$a_1 = a$	*The first term, by definition. Nothing has been added yet.*
2.	$a_2 = a_1 + d = a + d$	*Each term is the one before it, plus d.*
3.	$a_3 = a_2 + d = a + 2d$	*Add d once more to the second term.*
4.	$a_4 = a_3 + d = a + 3d$	*The same step, repeated.*
5.	the coefficient of d is one less than the term number	*The pattern from four terms already built: 0, 1, 2, 3 additions of d for terms 1, 2, 3, 4.*
6.	$a_n = a + (n-1)d$	***This generalises the pattern: reaching the n th term takes $(n-1)$ additions of d, one fewer than the term number.***
7.	$a_4 = a + (4-1)d = a + 3d$	**CHECK** *Check the new formula against the pattern already built: putting $n=4$ in gives $a + 3d$, matching the fourth term found above.*

BACK
The general form of an AP · p. 147

RULE
Write out the 2nd, 3rd and 4th terms first: the coefficient of d (one less than the term number) is what generalises.

Term number, steps taken, the term

term $n =$	1	2	3	4	5
steps of $d =$	0	1	2	3	4
$a_n =$	5	7	9	11	13

$a_n = a + (n-1)d$, with $a = 5$, $d = 2$

The steps taken are always one fewer than the term number, which is where the minus one in the formula comes from.

CHECK YOURSELF

25. In deriving $a_n = a + (n-1)d$ term by term ($a_2 = a + d$, $a_3 = a + 2d$, $a_4 = a + 3d$, ...), which observation lets the pattern jump straight to a_n?

- **A.** every term after the second is exactly double the one before it
- **B.** the pattern only holds up to the 4th term shown, and cannot be extended further
- **C.** the count of additions always trails the term number by exactly one
- **D.** d gets larger by one unit at every step of the derivation

26. Why does the count of additions in the derivation always trail the term number by exactly one?

- **A.** a_1 needs zero additions — counting starts there
- **B.** because the derivation always skips the second term
- **C.** because d is only added on even-numbered terms
- **D.** because the last term in a derivation is always one step short

1 more in this set online.

WORKED EXAMPLE · A starting salary rising each year

1.	the salaries form an AP with $a = 15000$ and $d = 800$	*The salary starts at ₹15000 and rises by a fixed ₹800 every year, exactly the definition of a common difference.*
2.	$a_{12} = 15000 + (12-1) \cdot 800$	*Apply $a_n = a + (n-1)d$ with $n = 12$.*
3.	$a_{12} = 15000 + 8800 = 23800$	*Finish the arithmetic.*
4.	the salary in the 12th year is ₹23800	***This is one single term of the AP, a_{12}, not a running total of everything earned so far.***
5.	$a_{11} = 15000 + (11-1) \cdot 800 = 23000$, then $23000 + 800 = 23800$	**CHECK** *Check the answer a second way, from the year before it: the 11th year comes to ₹23000, and one more raise of ₹800 gives ₹23800 again.*

BACK
The nth term formula · p. 150

TRY
A salary starts at ₹10000 with a ₹500 yearly increment. Find the salary in the 6th year.

Answer: $a_6 = 10000 + (6-1) \cdot 500 = 12500$. ₹12500.

KEY
The salary in a given YEAR is one single term, a_n, not a running total.

One marked year, one marked salary

yr	1	2	3	4	5	6
₹	15000	15800	16600	17400	18200	19000
yr	7	8	9	10	11	12
₹	19800	20600	21400	22200	23000	23800

$a_{12} = 15000 + 11 \cdot 800 = 23800$

The chip for year twelve shows one salary, not the total of all twelve years' raises added together.

FIND THE SALARY IN A GIVEN YEAR, WHEN IT STARTS FIXED AND RISES BY A FIXED AMOUNT EVERY YEAR.

1. **Find a and d** The starting salary is a. The fixed yearly raise is d.
2. **Write the term you need** Use $a_n = a + (n-1)d$ for the year number you want.
3. **Do the arithmetic** Multiply $(n-1)$ by d, then add a.
4. **Read it as a term, not a total** The answer is the salary in that one year, not everything earned up to it.

CHECK YOURSELF

27. A job's monthly salary starts at ₹18000 with an annual increment of ₹1200. Using the same method as the salary example in this chapter, find the salary in the 6th year.

- **A.** ₹24000
- **B.** ₹25200
- **C.** ₹7200
- **D.** ₹108000

28. A second job offers the same ₹18000 start but grows by a fixed 5% each year instead of a fixed rupee amount. Why does the AP method from this chapter no longer apply directly to that second job?

- **A.** because percentages cannot be used in any mathematics chapter
- **B.** because ₹18000 is too large a starting salary for an AP
- **C.** a percentage rise is not a fixed rupee step

1 more in this set online.

WORKED EXAMPLE · Finding a and d from two given terms

1.	$a + 3d = 11$	*The 4th term, using $a_n = a + (n-1)d$ with $n = 4$.*
2.	$a + 8d = 26$	*The 9th term, the same formula with $n = 9$.*
3.	$5d = 15$	***Subtracting removes a immediately and leaves one equation in d alone.***
4.	$d = 3$	*Divide both sides by 5.*
5.	$a = 11 - 3 \cdot 3 = 2$	*Substitute $d = 3$ back into the first equation.*
6.	the AP is $2, 5, 8, 11, 14, \cdots$	*List the terms starting from $a = 2$, adding $d = 3$ each time.*
7.	$2 + 3 \cdot 3 = 11$ and $2 + 8 \cdot 3 = 26$	**CHECK** *Check both original equations by substituting $a = 2$ and $d = 3$ back in: both come out right, the 4th term 11 and the 9th term 26.*

BACK
Finding a and d from two given terms · p. 152

TRY
In an AP, the 3rd term is 7 and the 7th term is 19. Find a and d.

Answer: $d = 3$, $a = 1$.

RULE
Subtracting the two equations removes a immediately: $(a + 8d) - (a + 3d) = 26 - 11$ gives $5d = 15$, so $d = 3$.

FIND THE FIRST TERM AND THE COMMON DIFFERENCE, FROM ANY TWO GIVEN TERMS OF AN AP.

1. **Write both terms as equations** Use $a_n = a + (n-1)d$ for each given term number.
2. **Subtract the equations** This removes a and leaves one equation in d alone.
3. **Solve for d** Divide to find d.
4. **Substitute back for a** Put d into either equation and solve for a.
5. **List the AP and check** Start at a and add d each time. Check that the two given terms come out right.

CHECK YOURSELF

29. In an AP, the 2nd term is 7 and the 6th term is 23. Using the same two-equation method as this chapter's worked example, find a and d.

- **A.** $a = 7, d = 23$
- **B.** $a = 3, d = 5$
- **C.** $a = 4, d = 4$
- **D.** $a = 3, d = 4$

30. Which order of steps correctly solves $a + 3d = 11$ and $a + 8d = 26$ for a and d?

- **A.** subtract to find d first, then substitute back to find a
- **B.** substitute a guessed value of a into both equations to check which one fits d
- **C.** solve each equation separately for a, since d is not needed to find it

1 more in this set online.

WORKED EXAMPLE · The 8th term from the end

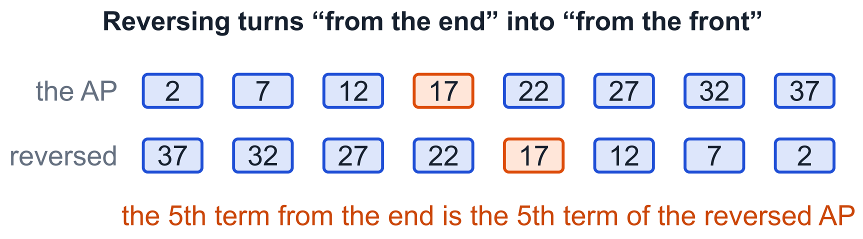
1.	the common difference is $d = -3$	*$12 - 15 = -3$, confirmed by $9 - 12 = -3$.*
2.	the reversed AP has first term -21 and common difference 3	***Reversing the AP turns a "from the end" question into an ordinary nth-term question.*** *The reversed list starts from the last term $l = -21$, and $-d$ becomes 3.*
3.	the 8th term from the end is the 8th term of the reversed AP	*Reversing restarts the count at l.*
4.	$-21 + (8-1)\cdot 3 = -21 + 21 = 0$	*Apply the ordinary nth-term formula to the reversed AP, with $r = 8$.*
5.	$-21, -18, -15, -12, -9, -6, -3, 0$	**CHECK** *List the reversed AP out and count to the 8th term: it lands on 0 again, the same answer the formula gave above.*

BACK
The rth term from the end · p. 153

TRY
A finite AP is $10, 8, 6, \cdots, -10$. Find the 5th term from the end.

Answer: Reverse: $a = -10,\ d = 2$. $-10 + (5-1)\cdot 2 = -2$.

KEY
Reversing the AP turns "from the end" into an ordinary nth-term question, starting from l.



The fourth chip from the end and the fifth chip from the front hold the same value in the reversed list.

FIND A TERM COUNTED FROM THE END OF A FINITE AP.

1. **Find d** Subtract consecutive terms to get the common difference.
2. **Reverse the AP** The reversed list starts at the last term l, with common difference $-d$.
3. **Apply the ordinary formula** Use $a_n = a + (n-1)d$ on the reversed AP, with the position counted from the end.
4. **Read off the answer** The result is the term you wanted, counted from the end.

CHECK YOURSELF

31. A finite AP begins $40, 35, 30, \cdots$ **and ends at the last term** -15**. Find the 5th term from the end.**

- **A.** 10
- **B.** 5
- **C.** -40
- **D.** -15

32. Why does the common difference become $-d$**, not** d**, when a finite AP is read from its last term backward?**

- **A.** because the last term l is always negative
- **B.** because counting from the end always uses subtraction instead of a formula
- **C.** because r must always be larger than n
- **D.** reversing direction flips the sign of every step

1 more in this set online.

WORKED EXAMPLE · **Deriving the sum formula, Gauss's reverse-and-add trick**

1.	$S_n = a + (a+d) + (a+2d) + \cdots + [a + (n-1)d]$	*Write the sum forwards, term by term.*
2.	$S_n = [a + (n-1)d] + [a + (n-2)d] + \cdots + (a+d) + a$	*Write the identical sum backwards, term by term.*
3.	every matched pair totals $2a + (n-1)d$	***What one line gains going forward, the other loses going backward: every pair totals the same amount.***
4.	$2S_n = n[2a + (n-1)d]$	*There are n such pairs, one for every term in the sum.*
5.	$S_n = n/2 \cdot [2a + (n-1)d]$	*Divide both sides by 2.*

WORKED EXAMPLE · **Deriving the sum formula, Gauss's reverse-and-add trick**

6. $S_4 = 4/2 \cdot [2 \cdot 5 + (4-1) \cdot 6] = 56$

CHECK *Check the formula against a small case already seen: the AP* $5, 11, 17, 23$ *has* $a = 5$ *and* $d = 6$. *Adding directly,* $5 + 11 + 17 + 23 = 56$ *too.*

RULE
n pairs, each totalling $2a + (n-1)d$, give $2S_n$. Halve it for S_n.

KEY
Writing the sum forwards AND backwards, then adding, makes every pair total the same amount.

Write the sum twice, the second time backwards

$$
\begin{array}{rccccc}
S_5 = & \boxed{3} & \boxed{7} & \boxed{11} & \boxed{15} & \boxed{19} \\
S_5 = & \boxed{19} & \boxed{15} & \boxed{11} & \boxed{7} & \boxed{3} \\
+ & & & & & \\
\hline
2S_5 = & \boxed{22} & \boxed{22} & \boxed{22} & \boxed{22} & \boxed{22}
\end{array}
$$

$$2S_5 = 5 \times 22 = 110, \text{so } S_5 = 55$$

$$S_n = n/2(a + l)$$

The forward row and the backward row pair up so that every one of the five columns adds to 22.

CHECK YOURSELF

33. In the Gauss-style derivation, why does every forward-backward pair total the same value, $2a + (n-1)d$**?**

- **A.** forward gain always matches backward loss, term for term
- **B.** because every term in the AP is secretly equal
- **C.** because n is always an even number in this derivation
- **D.** because the first and last terms are always equal to each other

34. The derivation ends with $2S_n = n[2a + (n-1)d]$**. Why divide both sides by** 2 **at this last step?**

- **A.** because n is always an even number
- **B.** because d must be halved before it can be used in the formula
- **C.** because only half the terms were actually added together
- **D.** forward-plus-backward counts every term twice

1 more in this set online.

WORKED EXAMPLE · **Total savings after 18 months**

1. the monthly deposits form an AP with $a = 200$ and $d = 40$

The first month's deposit is ₹200 and each later one is a fixed ₹40 more than the month before. It is the DEPOSITS that form the AP; the total is their sum.

2. $S_{18} = 18/2 \cdot [2 \cdot 200 + (18-1) \cdot 40]$

Apply $S_n = n/2 \cdot [2a + (n-1)d]$ *with* $n = 18$.

3. $S_{18} = 9 \cdot [400 + 680] = 9 \cdot 1080$

Finish the arithmetic inside the bracket, then multiply.

4. $S_{18} = 9720$

Complete the multiplication.

WORKED EXAMPLE · Total savings after 18 months

5. the total saved after 18 months is ₹9720

This asks for a running total, S_n, not the amount added in the 18th month alone, which would be a_{18}.

6. $l = 200 + (18 - 1) \cdot 40 = 880,$ then $S_{18} = 18/2 \cdot (200 + 880) = 9720$

CHECK *Check the total a second way, using the l-form of the sum formula, where l is the 18th month's own deposit: it lands on ₹9720 again, the same total found above.*

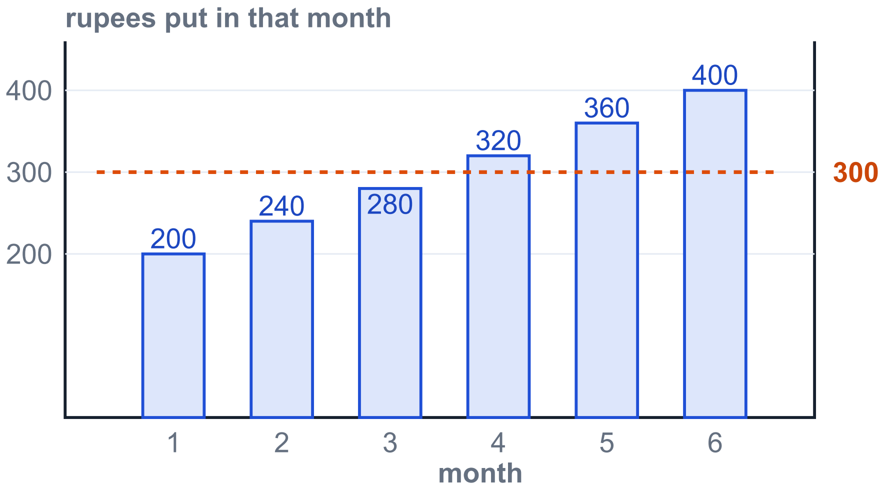
CAUTION
This asks for a RUNNING TOTAL, S_n, not the amount added in the 18th month alone, which would be a_{18}.

BACK
The sum of the first n terms · p. 151

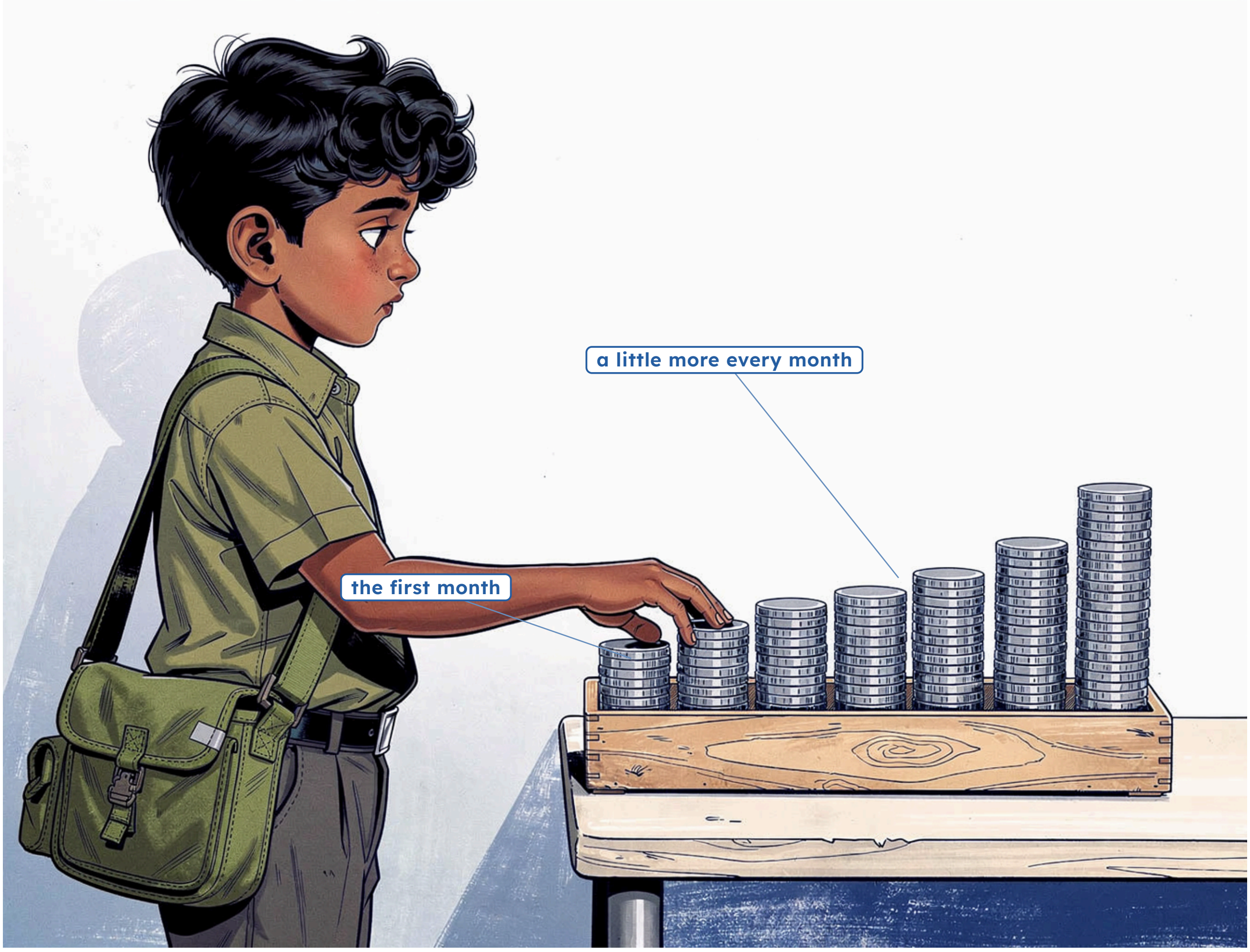
TRY
₹100 goes into a savings box in month 1, and ₹20 more each month than the month before. Find the total after 10 months.

Answer: $S_{10} = 10/2 \cdot [2 \cdot 100 + (10 - 1) \cdot 20] = 1900.$ ₹1900.

Six months, one running total



Each month's deposit is ₹40 more than the last, and the six pair evenly above and below the dashed average line.



Kabir keeps each month's coins in his savings box, and every month's stack is a little taller than the one before.

FIND THE TOTAL SAVED, WHEN A FIXED AMOUNT GROWS BY A FIXED AMOUNT EVERY MONTH.

1. **Find a and d** The first month's deposit is a. The fixed amount each later deposit adds on top of the one before is d.
2. **Write the sum formula** Use $S_n = n/2 \cdot [2a + (n-1)d]$ for the number of months given.
3. **Do the arithmetic** Work out the bracket first, then multiply.
4. **Read it as a total, not a term** The answer is everything saved so far, not the amount added in the last month alone.

CHECK YOURSELF

35. A savings box takes a deposit of ₹150 in the first month, and each following month's deposit is ₹30 more than the one before it. Using the same method as this chapter's savings example, find the total saved after 12 months.

- **A.** ₹480
- **B.** ₹3780
- **C.** ₹5760
- **D.** ₹4140

36. The same S_n reasoning used for a savings box also answers 'what is the total distance covered by a runner who covers 2 km in the first lap and 0.5 km more in each following lap, over 10 laps?' Why does the same formula apply to both?

- **A.** because both situations involve money, even though laps are measured in kilometres
- **B.** both add a starting amount and a fixed extra amount at every repeated step
- **C.** because both situations always run for exactly 12 steps

1 more in this set online.

WORKED EXAMPLE · How many terms give a sum of 60

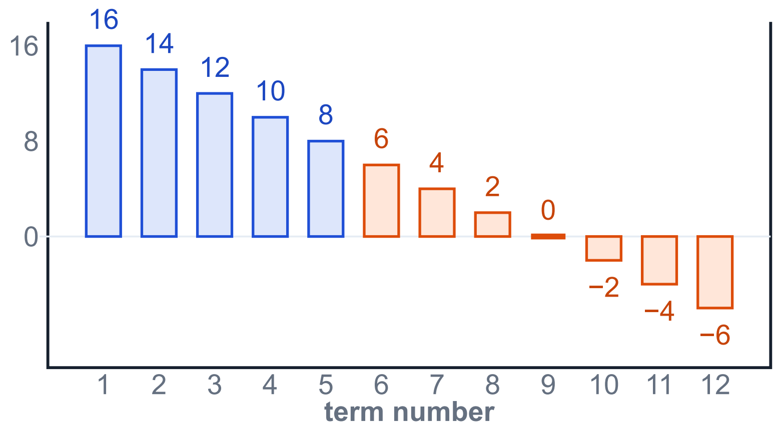
1.	$a = 16, d = -2, S_n = 60$	*Read the first term, common difference, and target sum off the AP and the question.*
2.	$60 = n/2 \cdot [2 \cdot 16 + (n-1) \cdot (-2)]$	*Substitute the known values into $S_n = n/2 \cdot [2a + (n-1)d]$.*
3.	$n^2 - 17n + 60 = 0$	*Expand and simplify, a quadratic in n.*
4.	$(n-5)(n-12) = 0$	*Factorise the quadratic.*
5.	$n = 5$ or $n = 12$	*Either factor can be zero.*
6.	both answers are valid	***Since a is positive and d is negative, the terms from the 6th through the 12th add to zero, so the sum reaches 60 twice.***
7.	$S_5 = 5/2 \cdot [32 + 4 \cdot (-2)] = 60$ and $S_{12} = 12/2 \cdot [32 + 11 \cdot (-2)] = 60$	**CHECK** *Check both answers by substituting back into the sum formula: $n = 5$ and $n = 12$ both give a sum of 60.*

TRY
An AP begins $3, 5, 7, \cdots$. How many terms give a sum of 48?

Answer: $n = 6$.

CAUTION
Two positive values of n can both be correct: it means the terms between them cancel out.

Terms 6 through 12 cancel to zero



The last seven bars add to zero, so the first five bars and the first twelve bars reach the same total.

FIND HOW MANY TERMS OF AN AP MUST BE ADDED TO REACH A GIVEN SUM.

1. **Read off a, d and the target sum** Take these straight from the AP and the question.

2. **Substitute into the sum formula** Put a, d and S_n into $S_n = n/2 \cdot [2a + (n-1)d]$.
3. **Simplify to a quadratic** Expand and collect terms to get a quadratic in n.
4. **Factorise and solve** Find the two values of n that make the quadratic zero.
5. **Check both answers** Substitute each value of n back into the sum formula to see if both are valid.

CHECK YOURSELF

37. Solving $S_n = n/2 \cdot [2a + (n-1)d]$ for n, given a, d and S_n, generally produces what kind of equation in n?

- **A.** a linear equation, since n appears only once in the formula
- **B.** an equation with no solution for n at all
- **C.** $(n-1)$ times n in the formula gives an n^2 term
- **D.** an equation only involving d, with n cancelling out

38. An AP begins $20, 17, 14, \cdots$. How many terms must be added to give a sum of 65?

- **A.** 6
- **B.** 4
- **C.** 5
- **D.** $65/20 = 3.25$, rounded to 3

1 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap

- COMMON DIFFERENCE: d is the fixed step from one term to the next. From 5 to 11, $d = 11 - 5 = 6$.
- NTH TERM: $a_n = a + (n-1)d$. The 4th term of that same AP is $a_4 = 5 + (4-1) \cdot 6 = 23$.
- SUM: $S_n = n/2 \cdot [2a + (n-1)d]$. Its first four terms add to $S_4 = 4/2 \cdot [10 + 18] = 56$.
- TERM VS SUM: $a_n = S_n - S_{n-1}$. Here $a_4 = 56 - 33 = 23$, matching the term above.

Let us step back over what we have built. Every arithmetic progression is fixed by exactly two numbers: the first term a and the common difference d. Once you know both, nothing else is needed to write out the whole list.

From those two numbers, two formulas follow, and they answer two different questions. The nth term, $a_n = a + (n-1)d$, is one single value: a place on the list, and nothing more. The sum of the first n terms is $S_n = n/2 \cdot [2a + (n-1)d]$. It is a running total: everything added together, up to that point.

A salary in a given year is one term. Total savings after several months is a sum. Before you answer a question about an AP, check which one it is asking for. Your turn: a question asks for the total after 4 years. Term, or sum? (Answer: sum, S_4.) Confusing the two sends a calculation to the wrong formula, no matter how carefully the arithmetic is done afterward.

BACK
The nth term formula · p. 150

KEY
One idea, two readings: a single term, or a running total. Never confuse which one a question asks for.

CHECK YOURSELF

39. Which correctly summarises how a_n and S_n relate, for the same AP?

- **A.** a_n and S_n are two different formulas for the same quantity
- **B.** a_n is always larger than S_n for the same AP
- **C.** S_n only exists for infinite APs, never finite ones
- **D.** one term, one running total, linked only by subtraction

40. Thinking of an AP as evenly spaced points along a straight line helps explain which fact about d?

- **A.** d can be positive, negative, or zero, since a line can rise, fall, or stay flat
- **B.** d must always be a whole number, since points on a line sit at whole-number marks
- **C.** d must always be positive, since a line always moves forward
- **D.** d has no connection to the line picture at all

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

41. The first term of a sequence is 7, and 5 is added to each term to get the next one. Which of these is the correct start of this sequence?

- **A.** 7, 35, 175, 875
- **B.** 7, 12, 17, 22
- **C.** 7, 2, −3, −8
- **D.** 7, 12, 24, 48

42. An AP has first term $a = 4$ and common difference $d = 6$. Using $a_n = a + (n - 1)d$, what is the 5th term?

- **A.** 34
- **B.** 10
- **C.** 28
- **D.** 22

3 more in this set online.

IN EVERY CHAPTER

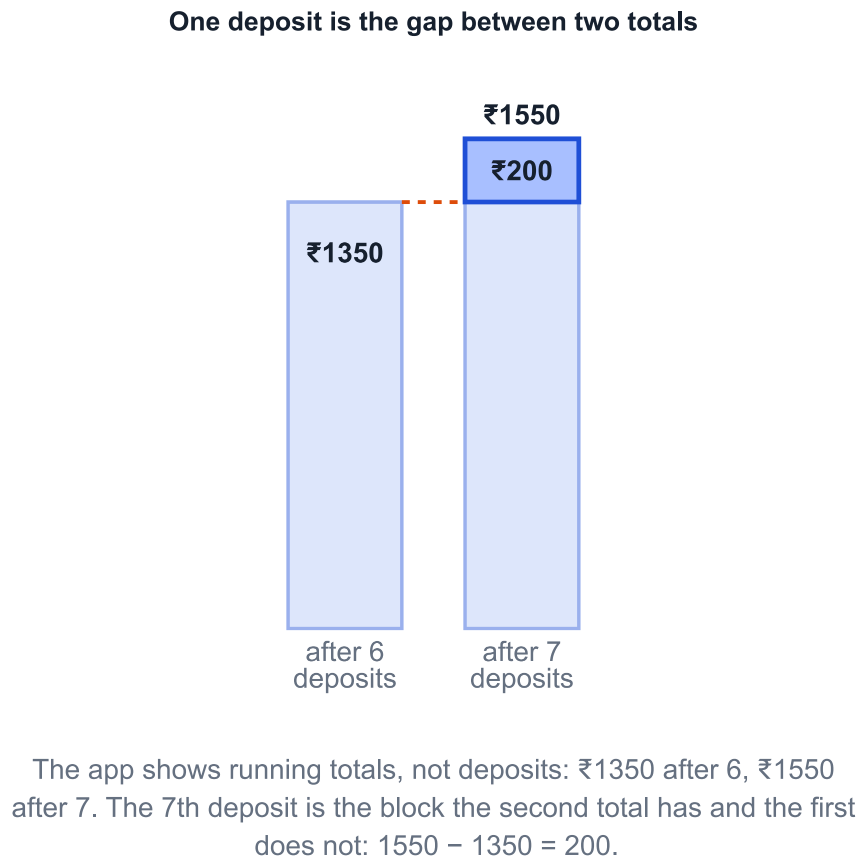
Where you will meet this

You will meet a fixed step almost every month of your life: a price, a saving, a distance. Here are seven of those places.

- **Tracking a rising allowance.** Your pocket money is ₹200 this month. It goes up by a fixed ₹50 every month after that.
  The maths: the fixed ₹50 added each time is the common difference of the AP.
  From month 1 to month 2: $250 - 200 = 50$. From month 2 to month 3: $300 - 250 = 50$.

- **Checking if stair heights are even.** A staircase has 6 steps at heights $15, 30, 45, 60, 75, 90$ cm from the ground.
  The maths: the list is an AP only if the gap between consecutive heights stays the same throughout.
  Gaps: $30 - 15 = 15$, $45 - 30 = 15$, $60 - 45 = 15$. Every gap is 15 cm.
- **Pricing a tailor's extra pieces.** A tailor's bill is ₹150 for one piece, and ₹20 more for each extra piece. Two pieces cost ₹170 and three cost ₹190.
  The maths: the bill for n pieces is the nth term of an AP, $a_n = a + (n-1)d$.
  For 10 pieces: $a_{10} = 150 + (10-1) \cdot 20 = 330$ rupees.
- **Adding up ten days of jogging.** You jog 2 km on day 1. You add 0.5 km more each day after that.
  The maths: the total distance over n days is the sum of the first n terms of an AP.
  Over 10 days: $S_{10} = 10/2 \cdot [2 \cdot 2 + 9 \cdot 0.5] = 5 \cdot 8.5 = 42.5$ km.
- **Reading a savings app's running total.** Your savings app shows a running total of ₹1350 after 6 deposits and ₹1550 after 7.
  The maths: a single deposit is a term, found as the difference between two consecutive running sums.
  The 7th deposit: $a_7 = 1550 - 1350 = 200$ rupees.

BACK
The common difference of an AP · p. 146



Any single term of an AP is the difference between two neighbouring sums, so a running total is enough to recover it.

- **Finding a print shop's rate from two invoices.** A print shop's price list is an AP. Printing 4 posters costs ₹700, and printing 9 costs ₹1200.

The maths: two known terms give two equations in a and d, and solving them finds both. $a + 3d = 700$ and $a + 8d = 1200$. Subtracting: $5d = 500$, so $d = 100$ and $a = 400$.

- **Counting bib numbers from the back.** A marathon assigns bib numbers as an AP: $101, 103, 105, \ldots$ up to 199 for the last runner.
  The maths: counting from the end reverses the AP. The rth bib from the back uses the last term as the new start.
  5th bib from the end: $l + (r-1)(-d) = 199 + 4 \cdot (-2) = 191$.

Your turn. A sapling is 20 cm tall now. It grows a fixed 4 cm every week. How tall will it be after 12 more weeks? *Answer:* Term 13 (now plus 12 weeks): $a_{13} = 20 + (13-1) \cdot 4 = 68$ cm.

Practice: Exercise 5.1

PRACTICE SET

1. PRACTICE Is $7, 11, 15, 19, \cdots$ an AP? If so, find d and the next two terms. (Worked in full below — read it, then do the next two the same way.)

   SOLUTION — Q1 · **Worked in full**

1.	$11 - 7 = 4$	*test a list by subtracting each term from the one after it*
2.	$15 - 11 = 4$ and $19 - 15 = 4$	*every gap must be tested, not just the first — one constant gap proves nothing*
3.	the gap is always 4, so this is an AP with $d = 4$	*an AP is exactly a list whose consecutive differences are all equal*
4.	$19 + 4 = 23$ and $23 + 4 = 27$	*the check — keep adding d to extend the list*

2. PRACTICE Is $2, 6, 18, 54, \cdots$ an AP? Give a reason. (Start the same way — subtract each term from the one after it, and compare the gaps.)

3. PRACTICE Write the first four terms of the AP with $a = 6$ and $d = -4$. (The first term is a; each term after it adds d.)

4. PRACTICE A taxi charges ₹25 for the first kilometre and ₹8 for each additional kilometre after that. Do the total fares for 1, 2, 3, 4 kilometres form an AP? If so, what is d?

5. PRACTICE Write the first four terms of the AP with $a = 10$ and $d = -3$.

6. PRACTICE For the AP $2, 5, 8, 11, \cdots$, write a and d.

7. PRACTICE Is $3, 6, 12, 24, \cdots$ an AP? Give a reason.

8. PRACTICE Is $-1, -3, -5, -7, \cdots$ an AP? If so, find d and the next two terms.

9. PRACTICE A hall has 30 seats in the first row, 32 in the second, 34 in the third, and so on. Do the number of seats per row form an AP? If so, find d.

ANSWERS **1.** Yes, it is an AP with $d = 4$. The next two terms are 23 and 27.
2. No. The gaps are 4, 12 and 36, which are not equal, so the list is not an AP.
3. $6, 2, -2, -6$. **4.** Yes — the fares are ₹25, ₹33, ₹41, ₹49, an AP with $d = 8$.
5. $10, 7, 4, 1$. **6.** $a = 2$, $d = 3$.
7. No — the differences $6 - 3 = 3$ and $12 - 6 = 6$ are not equal.
8. Yes, $d = -2$; the next two terms are -9 and -11. **9.** Yes, an AP with $d = 2$.
Full solutions: **manthika.com/learn/math10-ch05**

Further practice

PRACTICE SET

10. PRACTICE Write the first four terms of the AP with $a = -7$ and $d = 5$.

SOLUTION — Q10 · **Worked in full**

1. $a_2 = -7 + 5 = -2$ — *Add d to the first term to get the second term.*

2. $a_3 = -2 + 5 = 3$ — *Add d again for the third term.*

3. $a_4 = 3 + 5 = 8$ — *One more step of d gives the fourth term.*

11. PRACTICE The cost of digging a well is ₹150 for the first metre and rises by ₹50 for each additional metre after that. Do the costs of digging 1, 2, 3, 4 metres form an AP? If so, find d.

12. PRACTICE A tank holds 6000 litres of water. Every hour, 20% of the water currently in the tank is drained out. Do the amounts of water left in the tank at the end of each hour form an AP? Give a reason.

13. PRACTICE Vikram saves ₹500 in the first month of a year and increases his saving by a fixed ₹200 every month after that. Do his monthly savings for the first four months form an AP? If so, find d.

14. PRACTICE Is $46, 39, 32, 25, \cdots$ an AP? If so, find d and the next two terms.

15. PRACTICE Is $5, 10, 20, 40, \cdots$ an AP? Give a reason.

16. PRACTICE For the AP $-0.8, -0.3, 0.2, 0.7, \cdots$, write a and d, and find the next term.

ANSWERS **10.** $-7, -2, 3, 8$.
11. Yes — the costs are ₹150, ₹200, ₹250, ₹300, an AP with $d = 50$.
12. No — the amount drained each hour depends on how much water remains (a fixed fraction, not a fixed amount), so the differences between successive amounts are not equal.
13. Yes — the savings are ₹500, ₹700, ₹900, ₹1100, an AP with $d = 200$.
14. Yes, $d = -7$; the next two terms are 18 and 11.
15. No — $10 - 5 = 5$ but $20 - 10 = 10$; the differences are not equal, since each term is double the one before it, not a fixed amount more.
16. $a = -0.8$, $d = 0.5$; the next term is 1.2.
Full solutions: **manthika.com/learn/math10-ch05**

Practice: Exercise 5.2

PRACTICE SET

1. PRACTICE In an AP, $a = 4$, $d = 6$ and $n = 10$. Find a_n. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	$a_n = a + (n-1)d$	*write the formula down before substituting anything*
2.	$a_{10} = 4 + (10-1) \cdot 6$	*substitute all three: $a = 4$, $d = 6$, $n = 10$*
3.	$a_{10} = 4 + 9 \cdot 6 = 4 + 54 = 58$	*it is $n - 1$ steps, not n — the first term takes no step at all*
4.	counting up from 4 in sixes, the tenth entry is 58	*the check — the formula and the counting must agree*

2. PRACTICE In an AP, $a = 3$, $d = 7$ and $a_n = 59$. Find n. (Start the same way — write $a_n = a + (n-1)d$ and put in what you know. This time n is what is missing.)

3. PRACTICE Which term of the AP $5, 9, 13, \cdots$ is 101? (Read a and d off the list first.)

4. PRACTICE In an AP, $a = 7$, $d = 3$, $n = 8$. Find a_n.

5. PRACTICE In an AP, $a = 5$, $d = 4$, and $a_n = 45$. Find n.

6. PRACTICE Which term of the AP $21, 18, 15, \cdots$ is -81?

7. PRACTICE Find three numbers between 8 and 28 that, together with 8 and 28, form an AP of five terms.

8. PRACTICE The 3rd term of an AP is 12 and the 8th term is 37. Find a and d.

9. PRACTICE A finite AP has 60 terms. Its last term is 10 and its common difference is -2. Find the 5th term from the end.

10. PRACTICE A company's monthly water bill starts at ₹1200 and rises by a fixed ₹150 every month. Find the bill in the 10th month.

ANSWERS **1.** $a_{10} = 58$. **2.** $n = 9$. **3.** The 25th term. **4.** $a_8 = 7 + (8-1) \cdot 3 = 28$.
5. $45 = 5 + (n-1) \cdot 4$, so $n - 1 = 10$ and $n = 11$.
6. $a = 21$, $d = -3$. $-81 = 21 + (n-1) \cdot (-3)$, so $n - 1 = 34$ and $n = 35$.
7. $d = (28-8)/4 = 5$; the three numbers are $13, 18, 23$.
8. $a + 2d = 12$ and $a + 7d = 37$; subtracting gives $5d = 25$, so $d = 5$ and $a = 2$.
9. $l = 10$; reversing gives common difference $-d = 2$; the 5th term from the end is $10 + (5-1) \cdot 2 = 18$.
10. $a_{10} = 1200 + (10-1) \cdot 150 = 2550$; the bill in the 10th month is ₹2550.
Full solutions: **manthika.com/learn/math10-ch05**

Further practice

Two ends given, three terms found

given — given

8 | 13 | 18 | 23 | 28

found, from $d = (28 - 8) \div 4 = 5$

Once the two end chips fix the step, every chip between them can be read off, not solved for one by one.

PRACTICE SET

11. PRACTICE In an AP, $a = -12$, $d = 5$, $n = 15$. Find a_n.

SOLUTION — Q11 · Worked in full

1.	$a_{15} = -12 + (15 - 1) \cdot 5$	*Substitute a, d and n into the nth-term formula.*
2.	$a_{15} = -12 + 70 = 58$	*Multiply and add to find the answer.*

12. PRACTICE In an AP, $a = 6$, $d = -4$, and $a_n = -58$. Find n.

13. PRACTICE Which term of the AP $8, 14, 20, 26, \cdots$ is 146?

14. PRACTICE Which of these is the 16th term of the AP $50, 45, 40, \cdots$?

A. -15
B. -20
C. -25
D. -30

15. PRACTICE Three numbers are to be inserted between 10 and 34 so that the five numbers together form an AP. Find them.

16. PRACTICE The first and fourth terms of an AP are 9 and 27. Find the second and third terms.

17. PRACTICE Is 250 a term of the AP $7, 12, 17, 22, \cdots$? Give a reason.

18. PRACTICE The 5th term of an AP is 18 more than its 2nd term. If the 9th term is 70, find the AP.

19. PRACTICE A finite AP has 42 terms. Its first term is -8 and its common difference is 3. Find the 6th term from the end.

20. PRACTICE A theatre has 20 seats in the first row and 4 more seats in each row after that. Find the number of seats in the 15th row.

21. PRACTICE How many two-digit numbers leave remainder 1 when divided by 6?

ANSWERS **11.** $a_{15} = 58$. **12.** $n = 17$. **13.** $n = 24$, the 24th term. **14.** C — -25.
15. $16, 22, 28$. **16.** $15, 21$.
17. No — $n - 1 = 48.6$ is not a whole number, so 250 is not a term of this AP.
18. $a = 22$, $d = 6$; the AP is $22, 28, 34, 40, \cdots$ **19.** 100. **20.** 76 seats in the 15th row.
21. 15 two-digit numbers.
Full solutions: **manthika.com/learn/math10-ch05**

Practice: Exercise 5.3

PRACTICE SET

1. PRACTICE Find the sum of the first 12 terms of the AP $4, 9, 14, \cdots$ (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · Worked in full

1.	$a = 4$, $d = 9 - 4 = 5$, $n = 12$	*read the first term and the gap off the list, and the count off the question*
2.	$S_n = n/2[2a + (n - 1)d]$	*this is the sum formula, and it needs no last term*
3.	$S_{12} = 12/2[2 \cdot 4 + 11 \cdot 5]$	*substitute: again $n - 1$ steps, so 11 and not 12*
4.	$S_{12} = 6[8 + 55] = 6 \cdot 63 = 378$	*work the bracket out first, then multiply*
5.	the 12th term is $4 + 11 \cdot 5 = 59$, and $12/2(4 + 59) = 6 \cdot 63 = 378$	*the check — the other sum formula, $n/2(a + a_n)$, must give the same answer*

2. PRACTICE In an AP, $a = 6$, $d = 3$ and $S_n = 195$. Find n. (Start the same way — write the sum formula and substitute. What is left is a quadratic in n.)
3. PRACTICE A person saves ₹20 in the first week and ₹5 more in each following week. Find the total saved over 10 weeks. (Name a, d and n from the words before reaching for the formula.)
4. PRACTICE Find the sum of the first 15 terms of the AP $3, 7, 11, \cdots$
5. PRACTICE In an AP, $a = 8$, $d = 5$, and $S_n = 305$. Find n.
6. PRACTICE How many terms of the AP $24, 21, 18, \cdots$ must be taken to give a sum of 78?
7. PRACTICE A person saves ₹32 in the first week and ₹4 more every following week. Find the total savings over 12 weeks.
8. PRACTICE A construction contract has a penalty clause: ₹200 for the first day of delay, increasing by ₹50 for each additional day. Find the total penalty for a 10-day delay.
9. PRACTICE A prize fund of ₹7000 is to be split among 7 winners so that each winner receives ₹100 less than the winner ranked just above. Find the amount the first-ranked winner receives.
10. PRACTICE Section A of a class plants 3 saplings on the first day of a plantation drive and 2 more each following day. Find the total number of saplings Section A plants in 5 days.

ANSWERS **1.** $S_{12} = 378$. **2.** $n = 10$. **3.** ₹425.
4. $S_{15} = 15/2 \cdot [2 \cdot 3 + 14 \cdot 4] = 15/2 \cdot 62 = 465$.
5. $305 = n/2 \cdot [16 + 5(n - 1)]$, which gives $n = 10$.
6. $a = 24$, $d = -3$. $78 = n/2 \cdot [48 - 3(n - 1)]$ simplifies to $n^2 - 17n + 52 = 0$, which factors as $(n - 4)(n - 13) = 0$; both $n = 4$ and $n = 13$ are valid, since a is positive and d is negative and the terms between them cancel.
7. $S_{12} = 12/2 \cdot [64 + 11 \cdot 4] = 6 \cdot 108 = 648$; total savings ₹648.
8. $S_{10} = 10/2 \cdot [400 + 9 \cdot 50] = 5 \cdot 850 = 4250$; total penalty ₹4250.

9. $7000 = 7/2 \cdot [2a + 6 \cdot (-100)]$, which gives $a = 1300$; the first-ranked winner receives ₹1300.

10. $S_5 = 5/2 \cdot [6 + 4 \cdot 2] = 5/2 \cdot 14 = 35$; Section A plants 35 saplings in total.

Full solutions: **manthika.com/learn/math10-ch05**

BACK
The sum of the first n terms · p. 151

Further practice

PRACTICE SET

11. PRACTICE Find the sum of the first 20 terms of the AP $5, 9, 13, \cdots$

SOLUTION — Q11 · **Worked in full**

1.	$S_{20} = 20/2 \cdot [2 \cdot 5 + 19 \cdot 4]$	*Substitute a, d, n into the sum formula.*
2.	$S_{20} = 10 \cdot 86 = 860$	*Simplify inside the bracket and multiply.*

12. PRACTICE In an AP, $a = 4$, $d = 5$, and $S_n = 216$. Find n.

13. PRACTICE In an AP, $d = 4$, $n = 12$, and $S_n = 288$. Find a.

14. PRACTICE In an AP, $S_{10} = 250$ and $S_9 = 207$. Find a_{10}.

15. PRACTICE The sum of the first 10 terms of the AP $2, 4, 6, 8, \cdots$ is

A. 90
B. 100
C. 110
D. 120

16. PRACTICE The first term of an AP is -5, the last term is 46, and the sum is 205. Find the number of terms.

17. PRACTICE Find the sum of the first 25 terms of the AP $40, 35, 30, \cdots$

18. PRACTICE How many terms of the AP $20, 18, 16, \cdots$ must be taken to give a sum of 90?

19. PRACTICE A charity fair sells tickets over several days: 20 tickets on the first day, and 5 more each day than the day before. If 570 tickets are sold in total, how many days did the sale run?

20. PRACTICE Bricks are stacked in rows so that there are 24 bricks in the bottom row, and each row above has 2 fewer bricks than the row below it. If the stack has 9 rows, find the number of bricks in the top row and the total number of bricks in the stack.

21. PRACTICE A swimming pool is drained so that 500 litres are removed in the first minute, and each following minute 40 fewer litres are removed than the minute before, until a total of 2400 litres have been removed. Find the number of minutes needed, given that the amount removed each minute cannot be negative.

ANSWERS **11.** $S_{20} = 860$. **12.** $n = 9$. **13.** $a = 2$. **14.** $a_{10} = 43$. **15.** C — 110. **16.** $n = 10$.
17. $S_{25} = -500$.
18. $n = 6$ or $n = 15$ — both are valid, since a is positive and d is negative and the terms between them cancel.
19. 12 days. **20.** The top row has 8 bricks; the stack has 144 bricks in total.

21. $n = 6$ minutes — the other root, $n = 20$, is rejected because it would require removing a negative amount of water in some minute.
Full solutions: **manthika.com/learn/math10-ch05**

Practice: Exercise 5.4 (Optional)

PRACTICE SET

1. PRACTICE An AP has $a = 60$ and $d = -7$. Find its first negative term. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	$a_n = 60 + (n-1)(-7) = 67 - 7n$	*put a and d into the formula and simplify, so one expression covers every term*
2.	the first negative term is the first n with $67 - 7n < 0$	*turn the words into an inequality rather than listing terms one by one*
3.	$67 < 7n$, so $n > 9.57\cdots$	*solve it as an ordinary inequality*
4.	n counts terms, so the first whole n past 9.57 is $n = 10$	*n cannot be a fraction — this is the step where a right inequality gives a wrong term*
5.	$a_{10} = 67 - 70 = -3$	*the check — and $a_9 = 67 - 63 = 4$ is still positive, so the 10th really is the first*

2. PRACTICE An AP has $a = 90$ and $d = -8$. Find its first term that is less than 20. (Start the same way — simplify a_n first, then ask which n makes it drop below 20.)

3. PRACTICE In an AP, $a_4 = 17$ and $a_9 = 42$. Find a and d. (Write each of the two terms with the formula, then subtract one equation from the other.)

4. PRACTICE An AP has $a = 105$ and $d = -6$. Find its first negative term.

5. PRACTICE A ladder has 11 rungs. The bottom rung is 2500 mm wide and the top rung is 1000 mm wide, with the width decreasing uniformly rung by rung. Find the total length of wood needed for all the rungs together.

6. PRACTICE House numbers on one side of a street run $1, 2, 3, \cdots, 49$. Find the house number x such that the sum of the house numbers before x equals the sum of the house numbers after it.

7. PRACTICE A stepped terrace has 15 steps. The first step is 25 cm above the ground, and each step is 25 cm higher than the one before it. Find the height of the top step above the ground.

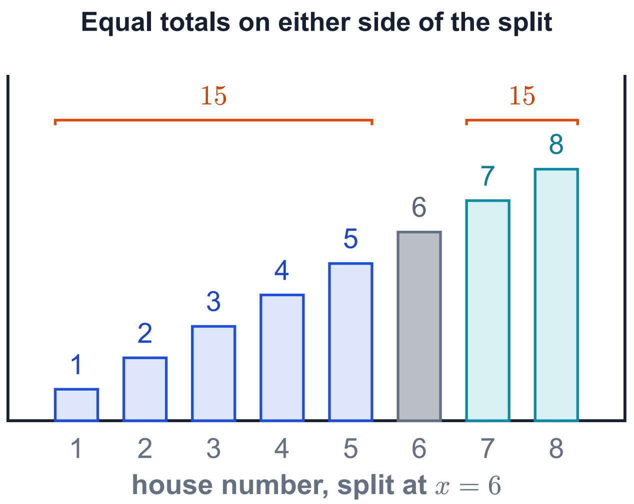
8. PRACTICE In an AP, the sum of the first 7 terms is 49 and the sum of the first 17 terms is 289. Find the sum of the first n terms.

ANSWERS **1.** The 10th term, $a_{10} = -3$. **2.** The 10th term, $a_{10} = 18$. **3.** $d = 5$ and $a = 2$.

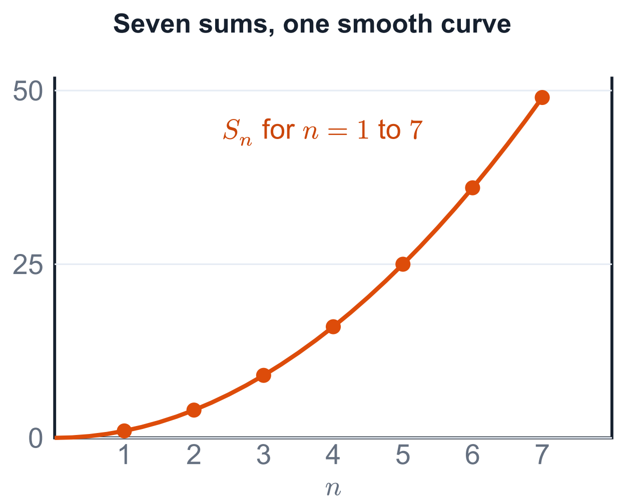
4. $a_n = 105 - 6(n-1)$; setting $a_n < 0$ gives $n > 18.5$, so $n = 19$ and $a_{19} = 105 - 6 \cdot 18 = -3$ is the first negative term.
5. $S_{11} = 11/2 \cdot (2500 + 1000) = 19250$, in millimetres — 19.25 metres of wood.
6. The total of all 49 house numbers is 1225; setting the sum before x equal to the sum after x gives $x^2 = 1225$, so $x = 35$.
7. $a_{15} = 25 + (15-1) \cdot 25 = 375$; the top step is 375 cm (3.75 m) above the ground.
8. Solving $a + 3d = 7$ and $a + 8d = 17$ gives $d = 2$ and $a = 1$; then $S_n = n/2 \cdot [2 + 2(n-1)] = n^2$.

Full solutions: **manthika.com/learn/math10-ch05**

Further practice



The five houses before house six add to fifteen, and the two houses after it add to fifteen too.



Seven marked points, one for each term count from 1 to 7, all sit exactly on one smooth upward curve.

PRACTICE SET

9. PRACTICE In an AP, $a_3 = 10$ and $a_7 = 26$. Which of these is its common difference d?

A. 3
B. 4
C. 5
D. 6

SOLUTION — Q9 · **Worked in full**

1.	$a_7 - a_3 = 4d$	*The 7th term is four steps of d beyond the 3rd term.*
2.	$4d = 26 - 10 = 16$	*Substitute the given terms.*
3.	$d = 16/4 = 4$	*Divide to find the common difference.*

10. PRACTICE An AP has $a = 88$ and $d = -7$. Find its first term that is less than 10.

11. PRACTICE A hall has 25 seats in the front row, and each row behind has 3 more seats than the row in front. There are 18 rows in the hall. Find the number of seats in the last row and the total seating capacity of the hall.

12. PRACTICE A supermarket stacks tin cans in a pyramid display with 15 cans in the bottom row, 2 fewer cans in each row above it, and a single can on top. How many rows are there, and how many cans are used in total?

13. PRACTICE The sum of the 4th and 8th terms of an AP is 20, and their product is 96. Find the first three terms of the AP, given that the AP is increasing.

14. PRACTICE In an AP, the sum of the first 6 terms is 72 and the sum of the first 16 terms is 512. Find the sum of the first n terms.

ANSWERS **9.** B — 4. **10.** $a_{13} = 4$.
11. The last row has 76 seats; the hall seats 909 in total. **12.** 8 rows; 64 cans in total.
13. $5, 6, 7$. **14.** $S_n = 2n^2$.
Full solutions: **manthika.com/learn/math10-ch05**