6

CHAPTER 6

Triangles

WHAT THIS CHAPTER BUILDS

You will learn when two triangles have the same shape, prove it with three short tests, and use the ratio between them to find lengths you cannot measure.

GEOMETRY UNIT, PROOFS

IN THIS CHAPTER

Priya points to a landscape print pinned on the board while Dhruv holds a smaller copy of the same photograph.

Getting started

Same shape, any size

A STORY

The print and the copy

Priya points at a landscape print pinned to the notice board. Dhruv holds a smaller copy of the same photograph.

"Same hills, same clouds," Priya says. "Only the size has changed."

"But the big one is wider and taller," Dhruv says. "How is that the same shape?"

"Measure them," Priya says.

Dhruv lays a ruler along his copy. "15 cm wide, 10 cm tall." He measures the print on the board. "30 cm wide, 20 cm tall."

"Every length is exactly double," Priya says. "And every corner is still a right angle. Equal angles, and every length grown by the same number: that is what same shape means."

"Two figures like that are called similar," Dhruv says.

"Similar," Priya says. "Same shape, any size."

"So if I know one length in my copy," Dhruv asks, "can I find the matching length on the board without measuring it?"

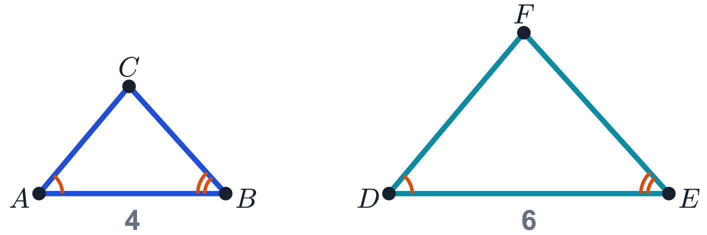
You can. And for triangles you will find something stronger: the angles alone are enough.

Take a photograph. Enlarge it to poster size. Every angle in the picture stays exactly the same. Only the size changes. That is what SIMILARITY means for two triangles. Two triangles are similar when their corresponding angles are equal and their corresponding sides are in the same ratio. Check only one of the two conditions, and you can be fooled. Neither condition alone is enough. Let us look at one result that carries almost every shortcut in this chapter. **A line inside a triangle, drawn parallel to one side, divides the other two sides in exactly the same ratio.** This is the Basic Proportionality Theorem. We will build AA, SSS and SAS on it, one after another. Congruence is this chapter's other main idea. It turns out to be similarity's special case: the same shape, with the ratio locked at 1 : 1.

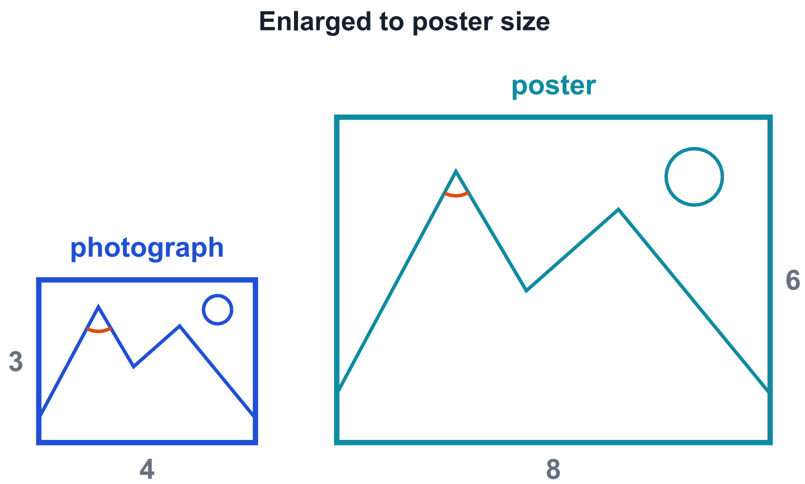
RULE
When two angles are given, reach for AA first; save SSS and SAS for when angles are not given.

KEY
Every criterion ahead is a shortcut: proving a few matching parts is enough, without measuring all six.

Same shape, any size



Two triangles show the same shape at two sizes, with matching angle marks and their bases printed as 4 and 6.



The poster is the photograph doubled, so every length is twice as long and the marked angle has not moved at all.

IN THE WILD

A photo on your phone and the same photo printed poster-size are similar figures. Every angle stays the same. Every length is multiplied by one and the same factor. That is exactly what makes two triangles similar too. Once we know that, two or three matching parts are enough to prove two triangles are the same shape. One known length then gives every other, without measuring it.

CHECK YOURSELF

1. Two triangles are similar when

- **A.** their corresponding angles are equal and their corresponding sides are in the same ratio
- **B.** their areas are equal, whatever their shape
- **C.** their corresponding sides are equal in length
- **D.** their corresponding sides are in the same ratio, whatever their angles happen to measure at each vertex

2. The Basic Proportionality Theorem is the result the AA, SSS and SAS similarity criteria are built on. What does BPT actually guarantee?

- **A.** a line parallel to one side of a triangle is always exactly half its length, whatever the triangle's shape
- **B.** any two sides of a triangle are always in the same ratio
- **C.** a triangle's three angles always sum to 180°
- **D.** a line parallel to one side of a triangle divides the other two sides in the same ratio

1 more in this set online.

IN EVERY CHAPTER

Before you start

DO THIS FIRST

You have proved triangles congruent before. Now compare triangles that only look alike. Try each check below. It takes a minute.

- **The triangle congruence tests** (Class 9, Ganita Manjari, page 99).
  Here: a similarity test matches parts in order, as these tests do.
  Check: Sides 3, 4, 5 and 5, 3, 4 pair up, so SSS applies.

- **RHS congruence** (Class 9, Ganita Manjari, page 104).
  Here: the hypotenuse and one side fix the whole right triangle.
  Check: Hypotenuse 10 and a leg 6 give the other leg $\sqrt{10^2 - 6^2} = 8$.
- **Sides in the same ratio** (Class 8, Ganita Prakash Part 1, page 161).
  Here: similar triangles need their sides in this same ratio.
  Check: $2 : 4$ and $5 : 10$ are both $1 : 2$, in proportion.
- **Area of a triangle** (Class 9, Ganita Manjari, page 154).
  Here: the ratio theorem's proof compares areas over a common height.
  Check: A triangle with base 6 and height 4 has area $1/2 \cdot 6 \cdot 4 = 12$.
- **Angles of a triangle** (Class 8, Ganita Prakash Part 1, page 94).
  Here: match two angles and the third matches, since all three make $180°$.
  Check: a triangle with angles $70°, 60°, 50°$ sums to $180°$.

If any of these felt new, read the page named before going on.

Similar figures, defined

Similarity is not only for triangles. Take any two polygons with the same number of sides. We call them similar when two conditions both hold. Every pair of corresponding angles is equal. Every pair of corresponding sides is in the same ratio. Check only one of those conditions, and you can be fooled. A square and a rectangle share four equal corresponding angles, all $90°$. But a 2 cm square's sides are not in the same ratio as a 2 cm by 4 cm rectangle's sides. Equal angles, unequal ratios: not similar. Now run the test the other way, and it still fails. A square and a rhombus can have all four sides in the identical ratio $1 : 1 : 1 : 1$. Yet a rhombus can be squashed to any angle at all, so its own angles need not be $90°$. Equal ratios, unequal angles: still not similar. Try it yourself: a square and a rhombus have sides in the same ratio. Are they similar? (Answer: not necessarily. Their angles need not match.)

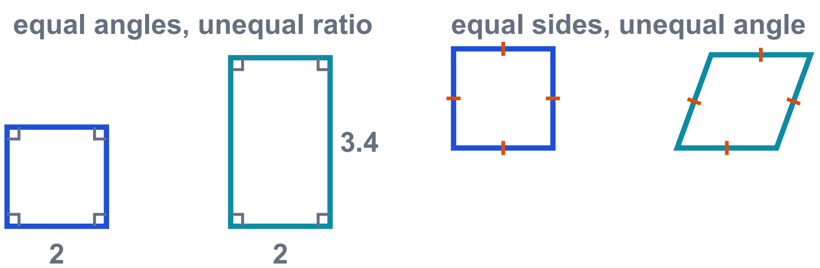
TRY
A square and a rhombus have sides in the same ratio. Are they similar?

Answer: Not necessarily. Their angles need not be equal, and both conditions are needed.

BACK
Same shape, any size · p. 179

RULE
Match corresponding vertices in the same order before comparing angles or sides.

Neither condition alone is enough



Two shape pairs each pass only one similarity test, equal angles alone or equal side ratios alone.

CHECK YOURSELF

3. Two polygons with the same number of sides are similar when

- **A.** their corresponding angles are equal
- **B.** equal corresponding angles and proportional corresponding sides
- **C.** their corresponding sides are in the same ratio, whatever their corresponding angles happen to be
- **D.** they have the same number of sides

4. A student says a square and a rectangle are always similar, because every angle in both is $90°$. What is wrong with this claim?

- **A.** nothing is wrong — matching all four angles is always enough for polygons
- **B.** equal angles alone do not guarantee the sides are in the same ratio
- **C.** a square and a rectangle can never be similar, whatever their sides
- **D.** the angles should be compared in degrees, not right angles

1 more in this set online.

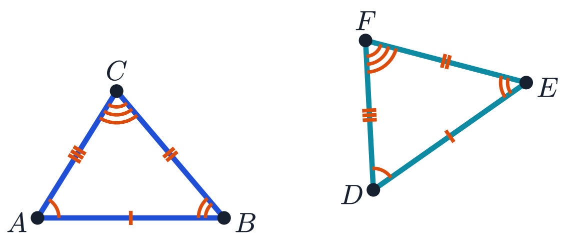
Congruent triangles, defined

Congruence asks for more than similarity does. Two triangles are congruent when they have exactly the same shape and the same size. Every pair of corresponding sides is equal in length. Every pair of corresponding angles is equal in measure. Think of congruence as similarity with the scale factor pinned down. A similar pair can sit at any ratio at all: $1:2$, $1:3$, or any other value the problem hands you. A congruent pair is a similar pair whose ratio happens to be exactly $1:1$. A triangle and its exact copy are congruent. A triangle and a poster-sized enlargement of it are similar but not congruent. The shape survived the enlargement. The size did not. Ask yourself one question to keep the two words apart. Has the size changed, or only the scale of the picture? (Answer: if the size changed, you can say only similar, never congruent.)

CAUTION
"Same shape" alone only gives similarity: "same shape and same size" gives congruence.

KEY
Congruent = similar AND the same size. Every congruent pair is similar; not every similar pair is congruent.

Congruent locks the scale at one



Two triangles, one turned and moved, have all three sides and all three angles marked equal to the other's.

CHECK YOURSELF

5. Two triangles are congruent when

- **A.** their corresponding angles are equal, whatever their side lengths
- **B.** every corresponding side and angle is equal
- **C.** their areas are equal, however their shapes differ
- **D.** their corresponding sides are in the same ratio, at any scale

6. Congruence is described as the special case of similarity where the scale factor is $1:1$. What does this mean in practice?

- **A.** every similar pair of triangles is automatically a congruent pair too
- **B.** every congruent pair of triangles is automatically a similar pair too
- **C.** similarity and congruence describe the same relationship, worded differently
- **D.** a scale factor of $1:1$ turns any two triangles into similar ones

1 more in this set online.

The ideas

Similarity, applied to triangles

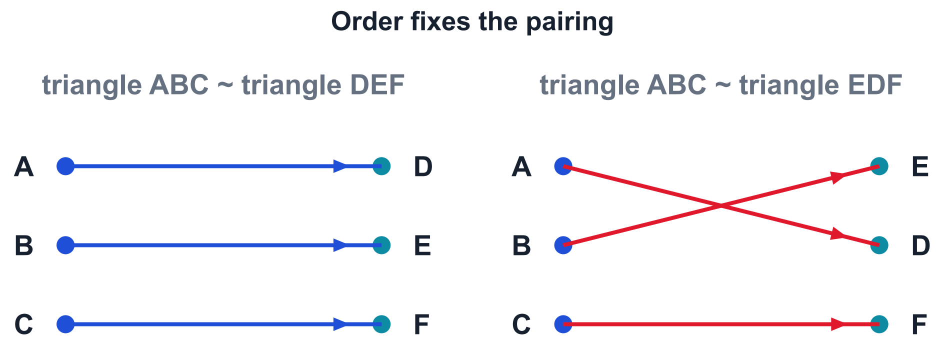
Apply the general similarity test to triangles, and one more discipline gets added: CORRESPONDENCE ORDER. Write triangle ABC is similar to triangle DEF. That sentence already pairs the vertices. A pairs with D, B with E, C with F. Every statement that follows reads off that pairing. Angle A equals angle D, not angle E or angle F. Side AB corresponds to side DE, not to EF or FD. **Change the order of the letters, and the statement changes meaning.** Triangle ABC similar to triangle EDF describes a different correspondence entirely, for the very same two triangles. It can be false where the first statement was true. Before you compare a single angle or side across two triangles said to be similar, write the correspondence out, vertex by vertex. Try it now: triangle ABC is similar to triangle DEF. Which angle equals angle B? (Answer: angle E, the second-listed vertex on each side.) Guessing which vertex matches which, midway through a problem, is the easiest way to compare the wrong pair of angles.

BACK
Similar figures, defined · p. 181

AHEAD
The ratios depend only on the angle · p. 256

TRY
Triangle ABC is similar to triangle DEF. Which angle equals angle B?

Answer: Angle E, the second-listed vertex on each side.



Two panels join the same triangle to D, E, F in one order and E, D, F in the other, crossing the arrows.

CHECK YOURSELF

7. For triangle ABC and triangle DEF, writing $\triangle ABC \sim \triangle DEF$ means the correspondence is

- **A.** A with D, B with E, C with F, in that matched order
- **B.** any pairing of the two triangles' vertices, since similarity does not depend on order
- **C.** A with F, B with E, C with D, the reverse order
- **D.** only the first letters, A with D; the rest can be any order

8. Why does writing $\triangle ABC \sim \triangle DEF$ with the vertices in that exact order matter?

- **A.** it fixes which sides and angles the ratios and equalities in the statement refer to
- **B.** it only affects how the statement is read aloud, not its meaning
- **C.** it is required only for congruent triangles, not for similar ones
- **D.** it lets the two triangles be drawn in exactly the same orientation and position on the page

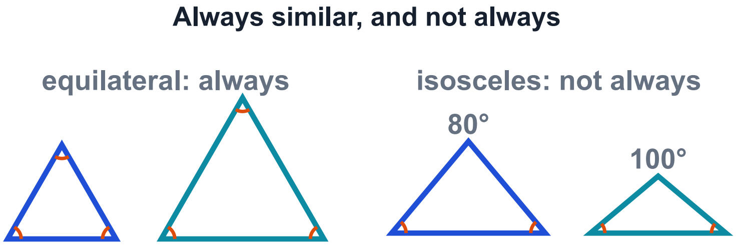
1 more in this set online.

Examples and counterexamples

Two quick tests separate genuine similarity from a resemblance that only looks like it. Let us check equilateral triangles first, then isosceles ones. Every equilateral triangle is similar to every other one, always, with no further checking needed. Each of its three angles measures exactly 60°, no matter the triangle's actual size. So the angle condition is satisfied automatically. Isosceles triangles get no automatic pass like that. Check two examples yourself: a triangle with angles 50°, 50°, 80° is isosceles, since it has two equal angles. So is a triangle with angles 40°, 40°, 100°. Yet the two sets of angles disagree, so the two triangles are not similar. Equilateral happens to force the angles too, which is why it always works. Isosceles only fixes that two angles match each other within one triangle. It says nothing about how that triangle compares to one you have not measured yet.

CAUTION
Two isosceles triangles need not be similar: check the actual angles, not just the isosceles label.

KEY
Equilateral triangles are always similar to each other: every angle is fixed at 60°.



Two equilateral triangles share identical angle marks, but two isosceles triangles have different angles and are not similar.

A SCHOLAR INDIA REMEMBERS

A square and a rectangle both have four right angles. Are they similar? What is your reason? Take a square with sides 1 and 1, and a rectangle with sides 1 and 2. The angles match. The sides are not in one ratio. So they are not similar. For four-sided shapes, equal angles alone are not enough.

Gautama · the sarus crane

CHECK YOURSELF

9. Are all equilateral triangles similar to each other?

- **A.** no, not unless their side lengths are also equal
- **B.** yes, since every angle measures 60°, at any size
- **C.** only if they are also congruent
- **D.** it depends on whether the triangles are drawn the same way up

10. Unlike equilateral triangles, two isosceles triangles are not always similar. Why not?

- **A.** isosceles triangles never have any equal angles to compare
- **B.** isosceles triangles can have more than three sides
- **C.** being isosceles fixes equal sides, not the angle measures
- **D.** similarity does not apply to any triangle with equal sides

1 more in this set online.

SSS congruence, reviewed

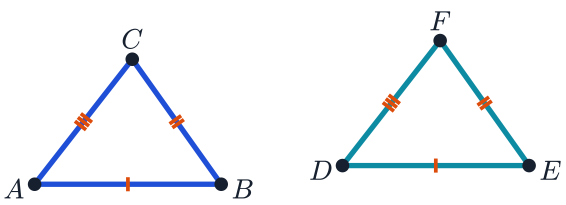
Take two triangles and compare them side by side. Measure every one of the three sides of the first. Match each length against a side of the second. If all three matched pairs come out equal, the two triangles are congruent. This is the SSS congruence criterion, Side-Side-Side. A triangle with sides 5 cm, 6 cm and 7 cm is congruent to a second triangle with the same three sides, no matter what angles happen to sit between them. You do not need to measure a single angle. Hinge three straight lengths together at their ends, in any order. Only one triangle can result. There is no second, differently shaped triangle hiding behind the same three lengths. Check the correspondence carefully when you use SSS. Side AB must match the side reported equal to it in the second triangle, not merely any side of the same length lying elsewhere in it. Your turn: one triangle has sides 3 cm, 4 cm, 5 cm, and a second has sides 3 cm, 4 cm, 5 cm too. Are they congruent by SSS? (Answer: yes. Every side matches its corresponding side.)

BACK
Congruent triangles, defined · p. 182

Class 9 — Proved by construction in Class 9; reviewed here as background, not re-derived.

KEY
Three matching sides is always enough to fix a triangle completely.

SSS: three sides are enough



Two triangles, moved but not resized, have all three side pairs tick-marked equal, with no angle marked.

CHECK YOURSELF

11. The SSS congruence criterion, proved in Class 9, states that two triangles are congruent when

- **A.** all three sides of one are in the same ratio as all three corresponding sides of the other
- **B.** all three sides of one equal all three corresponding sides of the other
- **C.** two sides and the angle between them match
- **D.** any three matching parts, sides or angles, are equal

12. Why is matching all three sides always enough to make two triangles congruent?

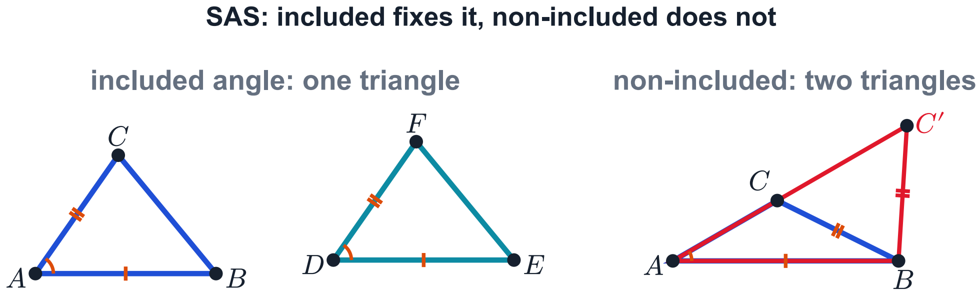
- **A.** three sides are simply easier to measure in a classroom than three angles would be
- **B.** three sides guarantee the triangle is equilateral
- **C.** three sides fix the triangle's angles but not its size
- **D.** a triangle's three side lengths fix its shape and size completely

1 more in this set online.

SAS congruence, reviewed

SAS congruence needs three matched parts too, in a specific arrangement. Take two sides of one triangle and the angle sandwiched between them, the INCLUDED angle. Match all three against a second triangle's corresponding two sides and included angle. Picture a 5 cm side, a 60° included angle and a 7 cm side in one triangle. Match them, in that order, against the same three measurements in another triangle. The two triangles are then congruent by SAS. That single word, INCLUDED, decides whether SAS applies at all. Match two sides and some other angle of the triangle, one not sitting between them, and the criterion does not apply. A different triangle can still share those same three measurements. Let us picture the construction together. Draw the two sides first, joined at a point. Then fix the angle between them before you draw the sides' far ends. The order only works because the angle sits between the two sides being drawn. Your turn: does SAS apply to a 5 cm side, a 7 cm side and a 60° angle that is NOT between them? (Answer: no. The angle must be included.)

CAUTION
The angle must sit BETWEEN the two matched sides: a non-included angle does not work.



The included angle fixes one triangle on the left, but the same two sides with a non-included angle allow two triangles on the right.

CHECK YOURSELF

13. The SAS congruence criterion states that two triangles are congruent when

- **A.** two sides and any one angle of the triangle match two corresponding sides and any one corresponding angle
- **B.** two sides and the angle included between them match two corresponding sides and their included angle
- **C.** one side and the two angles adjacent to it match
- **D.** two angles and any side match two corresponding angles and any side

14. Triangle ABC and triangle DEF have $AB = DE$, $BC = EF$, and angle A equal to angle D. A student claims SAS congruence applies. What is wrong?

- **A.** nothing is wrong — matching any two sides and any one angle of the triangle is always enough
- **B.** the two sides should have been AB and AC instead
- **C.** SAS requires all three sides to be checked, not just two
- **D.** angle A is not the angle included between the two matched sides AB and BC

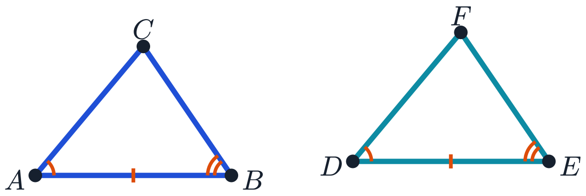
1 more in this set online.

ASA congruence, reviewed

ASA congruence swaps the roles SAS uses. Two angles now anchor the match, and a side sits between them. Take two angles of one triangle and the side joining them, the INCLUDED side. Match all three against a second triangle's corresponding two angles and included side. Take a triangle with angles 50° and 60° and a 6 cm side between them. It is congruent by ASA to any triangle carrying that same 50° angle, 6 cm side and 60° angle, in that order. SAS asks the same discipline of its angle. ASA asks it of a side instead. An angle at each end of the wrong side breaks the criterion, even where every measurement is individually correct. Only the side actually wedged between two matched angles counts. Once we fix two angles of a triangle, the third is fixed too, by the angle sum property. But check that ASA still names the side explicitly. A side's own length is never implied by angles alone. Your turn: a triangle has angles 50° and 60°, but the 6 cm side given is NOT between them. Does ASA apply? (Answer: no. The side must be included.)

CAUTION
The side must sit BETWEEN the two matched angles, exactly as SAS needs the angle between the two matched sides.

ASA: the side must be included



Two triangles, moved but not resized, have two matching angles and the side between them marked equal.

CHECK YOURSELF

15. The ASA congruence criterion states that two triangles are congruent when

- **A.** two angles and any side match two corresponding angles and any side
- **B.** two sides and the angle between them match
- **C.** two angles and their included side match
- **D.** all three angles match, with no side needed

16. Knowing two angles of a triangle already fixes the third, by the angle sum property — that alone only gives similarity (AA). What does the included side add, to upgrade this to ASA congruence?

- **A.** it fixes the actual size of the triangle, not just its shape
- **B.** it fixes a second pair of equal angles, on top of the two already given
- **C.** it removes the need for the two angles to be equal
- **D.** it only matters for triangles that are also isosceles

1 more in this set online.

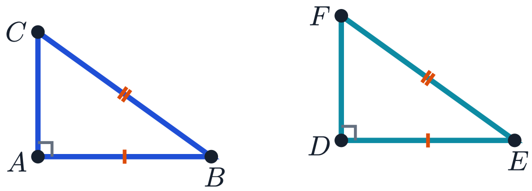
RHS congruence, reviewed

RHS congruence is the one criterion reserved for right triangles alone. The name is a checklist: Right angle, Hypotenuse, Side. Confirm both triangles carry a right angle first. Then match the HYPOTENUSE, the side opposite the right angle and always the longest side in a right triangle. Match one more side, any one, between the two triangles. Once the right angle is confirmed in both, matching those two lengths is enough to make the triangles congruent. Take a right triangle with hypotenuse 10 cm and one leg 6 cm. It is congruent, by RHS, to any right triangle sharing that same hypotenuse and leg. The third side is then forced to match too. You never need to measure it directly. Check that first, always, before you reach for it. This is also the one congruence criterion the book returns to directly, in a later chapter's proof about tangent lengths from a point outside a circle. Your turn: two triangles are not right triangles. Can RHS ever apply to them? (Answer: no. RHS needs a right angle in both triangles first.)

AHEAD
RHS: a shortcut for right triangles · p. 330

KEY
RHS needs a right angle in BOTH triangles first: check that before matching sides.

RHS: hypotenuse, then one leg



Two right triangles have matching right angles, hypotenuses and one leg marked, with the remaining leg left unmarked.

CHECK YOURSELF

17. The RHS congruence criterion states that in two right triangles, matching the hypotenuse and

- **A.** one other angle makes the two triangles congruent
- **B.** the two acute angles makes the two triangles congruent
- **C.** the triangle's area makes the two triangles congruent
- **D.** one other side makes the two triangles congruent

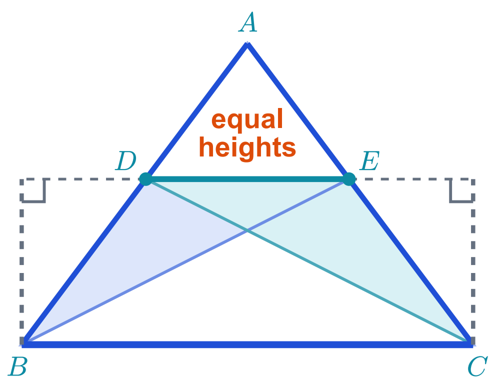
18. RHS applies only to right triangles. Why can it not be used for a triangle with no right angle?

- **A.** without a right angle, the hypotenuse and one side no longer fix the third side by the Pythagorean relation
- **B.** a triangle with no right angle does not have a longest side to call the hypotenuse
- **C.** RHS is really just SSS in disguise, so it works for any triangle
- **D.** it can be used for any triangle at all, the word "right" in RHS is just a naming convention, nothing more than that

1 more in this set online.

The Basic Proportionality Theorem

Same base, same parallels, same area



same base DE, same parallels, so the same area

The whole proof hangs on this one fact. Triangles BDE and DEC stand on the same base DE and sit between the same two parallel lines, so their heights, and so their areas, are equal.

Equal heights are why the two areas match, and that equality is the step every proof of the theorem passes through.

THEOREM (Basic Proportionality Theorem). If a line is drawn parallel to one side of a triangle, it divides the other two sides in the same ratio. Picture triangle ABC with a line through point D on AB and point E on AC, drawn parallel to BC. The theorem says $AD : DB = AE : EC$. Let us see why. The proof compares the AREAS of triangles built from this configuration. It uses one fact about area: triangles standing on the same base, between the same two parallel lines, always have equal area. Follow the numbered proof below, one justification per step, and check each step against the figure as you go.

TRY
If D and E are midpoints, what must $AE : EC$ equal?

Answer: $1 : 1$. The theorem forces $AD : DB = AE : EC$, so if D is a midpoint, E must be one too (Exercise 6.2).

RULE
Two triangles on the same base, between the same parallels, always have equal area.

BACK
Similarity, applied to triangles · p. 183

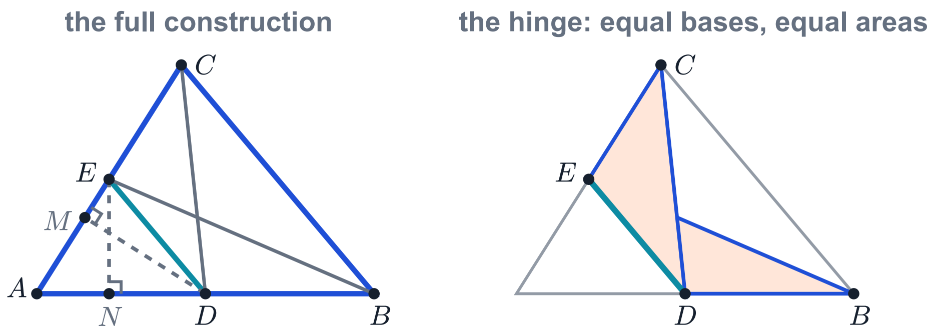
Given: Triangle ABC has a line through D on AB and E on AC, parallel to BC.
To Prove: $AD : DB = AE : EC$.

Proof:

1. let DE be drawn parallel to BC, with D on AB and E on AC — *This is the exact configuration the theorem describes. We start by naming it.*
2. join BE and CD; draw DM perpendicular to AC and EN perpendicular to AB — *We add these four lines ourselves. They are not given. We draw them so we can compare triangle areas using perpendicular heights.*
3. the area of triangle ADE equals $1/2 \cdot AD \cdot EN$ — *This is the standard area formula for a triangle. Here AD is the base and EN is the height.*
4. the area of triangle BDE equals $1/2 \cdot DB \cdot EN$ — *The same formula again, with the same height EN and base DB this time.*
5. area of triangle ADE : area of triangle $BDE = AD : DB$ — *In this ratio of the two areas, the common factor $1/2 \cdot EN$ cancels. Only $AD : DB$ is left.*
6. the area of triangle ADE equals $1/2 \cdot AE \cdot DM$, and the area of triangle DEC equals $1/2 \cdot EC \cdot DM$ — *The same reasoning again, this time using base AE or EC and height DM.*
7. area of triangle ADE : area of triangle $DEC = AE : EC$ — *Here too the common factor $1/2 \cdot DM$ cancels. Only $AE : EC$ is left.*
8. the area of triangle BDE equals the area of triangle DEC — ***Both triangles stand on the same base DE, between the same two parallel lines DE and BC.** So their heights from DE are equal, and so their areas are equal too.*
9. $AD : DB = AE : EC$ — *Triangle ADE's area is common to both ratios above. Its partner triangles are equal in area, by the last step. So the two ratios must be equal. That is what we set out to prove.*

■

The parallel line and its two triangles



The left panel shows the full construction, and the right panel isolates just the two triangles sharing the joining segment.

A SCHOLAR INDIA REMEMBERS

Gautama lived in north India around the 2nd century BCE. By tradition, he wrote the Nyaya-sutras. That book sets out an argument as a public procedure. You state your reason. You name a general pattern. You apply the pattern to the case in front of you. Anyone watching can check each step. The proof of the Basic Proportionality Theorem is set out the same way.

Gautama · the sarus crane

CHECK YOURSELF

19. In the BPT proof, after drawing DM perpendicular to AC and EN perpendicular to AB, what is this construction actually for?

- **A.** so that triangle ABC itself becomes a right triangle once the lines are drawn
- **B.** so that DE is confirmed parallel to BC
- **C.** so that AD and AE can be shown equal
- **D.** so that triangle areas can be compared using a common, matching height

20. The proof states that triangle BDE and triangle DEC have equal area. What makes this true?

- **A.** they share base DE between the same two parallels
- **B.** they are both right triangles, so their areas must match
- **C.** BD and DC happen to be equal in this figure
- **D.** both triangles share the same three vertices as triangle ADE

2 more in this set online.

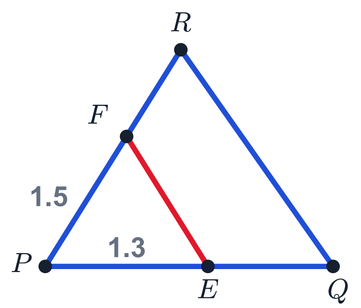
Converse of the Basic Proportionality Theorem

Let us run the Basic Proportionality Theorem in the opposite direction. A new, equally useful statement appears. The theorem itself starts from a parallel line and concludes a ratio. The CONVERSE starts from a ratio and concludes a parallel line. If a line divides two sides of a triangle in the same ratio, that line is parallel to the third side. The syllabus states this converse without proof. We use it directly, the same way we will use the AA, SSS and SAS similarity criteria a little further on. Checking a ratio answers whether a line is parallel. Knowing a line is parallel lets you write down a ratio. Your turn: a line divides two sides of a triangle in the ratio 2 : 3 on both sides. Is the line parallel to the third side? (Answer: yes, by the converse.)

RULE
Compare the ratio on BOTH sides the line meets before concluding it is parallel to the third side.

KEY
The theorem goes ratio-from-parallel; the converse goes parallel-from-ratio.

Ratios that do not match are not parallel



A line drawn at two unequal ratios, 1.3 and 1.5, tilts visibly away from being parallel to the base.

CHECK YOURSELF

21. The converse of the Basic Proportionality Theorem states that

- **A.** if a line divides two sides of a triangle in the same ratio, the line is parallel to the third side
- **B.** if a line is parallel to one side of a triangle, it divides the other two sides in the same ratio
- **C.** if two sides of a triangle are equal, the triangle is isosceles
- **D.** if a line divides two sides of a triangle in the same ratio, the two triangles formed are congruent

22. In triangle PQR, points E on PQ and F on PR are given. Which two ratios must be compared to decide, by the converse of BPT, whether EF is parallel to QR?

- **A.** $PE : EQ$ and $PF : FR$
- **B.** $PE : PQ$ and $PF : PR$
- **C.** $EQ : FR$ and $PE : PF$
- **D.** QR and EF directly, as lengths

2 more in this set online.

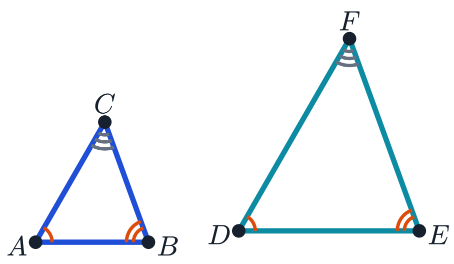
The AA similarity criterion

Two matching angles turn out to be all that similarity ever needs, for triangles. The AA criterion, Angle-Angle, states it plainly. If two angles of one triangle equal two angles of another, the two triangles are similar. You do not need to measure a single side. This is not an accident. Every triangle's three angles add to $180°$. So once two angles are fixed, the third is fixed too, automatically equal in both triangles. Three matching angles, in turn, force the sides into the same ratio. AA never has to show that step itself. We state it here without proof, exactly as we stated the converse of the Basic Proportionality Theorem. Checking a third angle anyway wastes a step. The third angle will always match once the first two do. Your turn: two triangles share two matching angles, $50°$ and $70°$. Are they similar? (Answer: yes, by AA. No side needs checking.)

RULE
Check any two angles; never assume you must check all three.

KEY
Two matching angles are enough: the third angle is forced by the angle sum property.

Two angles are enough for AA



Two triangles of different sizes have two given angle pairs marked, with the third pair marked in a different colour to show it follows automatically.

A SCHOLAR INDIA REMEMBERS

You say two angles are enough. What was the reason again? Walk me through it. The three angles of a triangle add up to 180°. Say two angles are 50° and 60° in both triangles. Then the third angle is 70° in both. So all three angles match, and the triangles are similar.

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CHECK YOURSELF

23. The AA similarity criterion states that two triangles are similar when

- **A.** two angles of one triangle equal two angles of the other
- **B.** two angles of one triangle equal two angles of the other, and their included side also matches
- **C.** one angle of one triangle equals one angle of the other
- **D.** all three angles of one triangle equal all three angles of the other

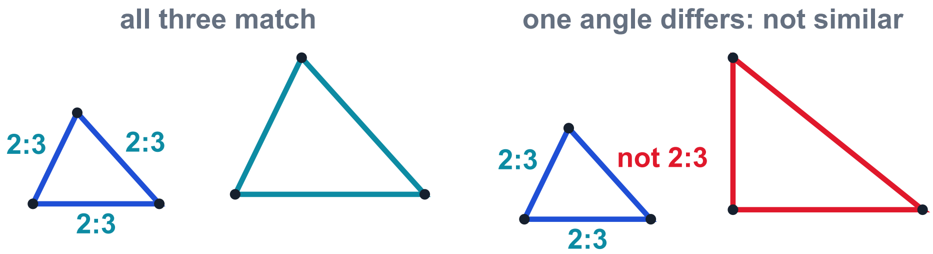
24. Why does matching only two angles, not three, already prove two triangles similar?

- **A.** because two matched angles always determine a triangle's exact side lengths directly, without any scaling
- **B.** because most triangles only have two distinct angle values anyway
- **C.** the third pair of angles is forced to match too, by the angle sum property
- **D.** because the third angle is irrelevant to a triangle's shape

2 more in this set online.

The SSS similarity criterion

Three ratios must all agree



On the left all three side ratios agree, so the triangles are similar, but on the right a changed angle breaks the third ratio.

SSS similarity asks the opposite question from SSS congruence. It does not ask whether the sides are EQUAL. It asks whether they are in the SAME RATIO. Take three sides of one triangle in the same ratio as the three corresponding sides of another, and the two triangles are similar. $AB : DE$, $BC : EF$ and $CA : FD$ might all equal $2 : 3$, for example. The syllabus states this without proof. **All three ratios must agree, in correspondence order, not just two of them.** Check this yourself: a triangle with sides in ratio $2 : 3$, $2 : 3$ and $2 : 4$ against another is not similar by SSS. Two ratios matching and a third one drifting is not the same thing as all three matching. Once you confirm SSS similarity, every pair of corresponding angles comes out equal too, with no separate angle check required. Your turn: three side ratios all equal $3 : 4$. Do the corresponding angles need checking separately? (Answer: no. They are equal automatically.)

CAUTION
All three ratios must match, in correspondence order: two matching and one different is not SSS.

KEY
Three matching side ratios are enough: the angles follow automatically.

CHECK YOURSELF

25. The SSS similarity criterion states that two triangles are similar when

- **A.** all three sides of one equal all three corresponding sides of the other
- **B.** all three sides of one are in the same ratio as all three corresponding sides of the other
- **C.** two of the three sides are in the same ratio, and the third side's ratio can be anything at all
- **D.** the triangles' perimeters are in the same ratio

26. Triangle ABC has sides $AB = 4$, $BC = 6$, $CA = 9$. Triangle DEF has sides $DE = 8$, $EF = 12$, $FD = 15$. Two of the three ratios, $AB : DE$ and $BC : EF$, both equal $1 : 2$. Is SSS similarity satisfied?

- **A.** no, since $CA : FD = 9 : 15$, which is $3 : 5$, not $1 : 2$
- **B.** yes, since two matching ratios out of three is already enough
- **C.** yes, since the triangles share the same ratio on their longest sides
- **D.** it cannot be decided without knowing the angles

2 more in this set online.

The SAS similarity criterion

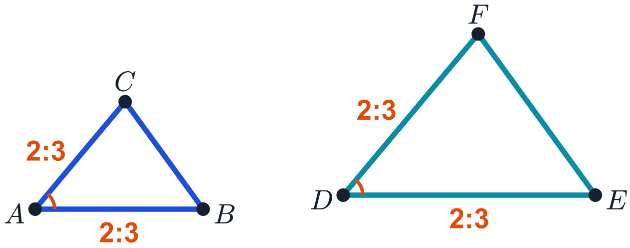
SAS similarity is SAS congruence's looser cousin. A ratio stands in for an equality. If one angle of a triangle equals one angle of another, and the two sides INCLUDING that angle are in the same ratio in both triangles, the two triangles are similar. The syllabus states this without proof, the same way it states AA and SSS similarity. That is exactly the discipline SAS congruence already demanded. We carry it over unchanged into the similarity version. An angle that does not sit between the two sides being compared does not satisfy SAS similarity, even where the ratio happens to hold. Compare this to SAS congruence in your own words. Congruence needs the two sides EQUAL and the angle EQUAL. Similarity needs the two sides in the SAME RATIO and the angle EQUAL. Only the side condition loosens. The angle condition never does. Your turn: two triangles have one matching angle and two sides in ratio $3 : 5$ including that angle. Similar by SAS? (Answer: yes.)

BACK
Similarity, applied to triangles · p. 183

CAUTION
A matching angle that is NOT between the two matched sides does not satisfy SAS.

KEY
The matched angle must be the one INCLUDED between the two matched sides.

One angle, two sides in ratio



Two triangles of different sizes share one marked angle and two matching side ratios, with the third side left unmarked.

CHECK YOURSELF

27. The SAS similarity criterion states that two triangles are similar when

- **A.** one angle of one equals one angle of the other, and the two sides including that angle are in the same ratio
- **B.** one angle of one equals one angle of the other, and any two of the sides in the triangle are in the same ratio at all
- **C.** two sides of one equal two sides of the other, with no angle condition
- **D.** the two triangles' areas are in the same ratio

28. Triangle ABC and triangle DEF have $(AB)/(DE) = (BC)/(EF)$, and angle A equal to angle D. A student applies SAS similarity directly. What is the problem?

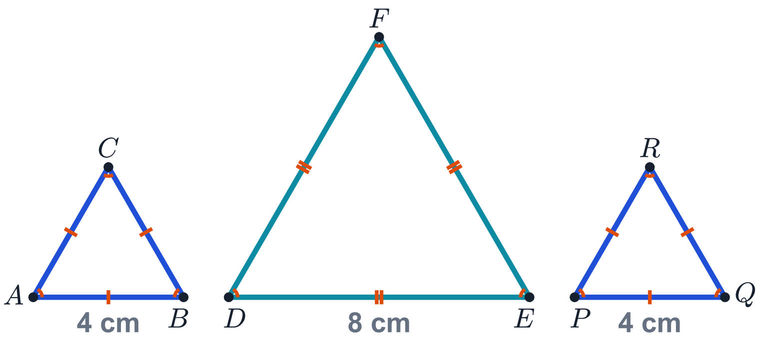
- **A.** nothing is wrong — matching one angle and any two sides in ratio is always enough
- **B.** angle A is not the angle included between sides AB and BC
- **C.** the ratio should have used CA instead of AB
- **D.** SAS similarity requires all three sides in ratio, not just two

2 more in this set online.

Common mistakes

Similar is not congruent

Similar is not congruent



Three equilateral triangles share identical angle marks, but only the first and third also share the same tick count.

Here is the mix-up this chapter exists partly to prevent. Two triangles with the same three angles look, at a glance, like the same triangle. It is tempting to call them congruent on the strength of that alone. They are not, necessarily. A triangle with angles 60°, 60°, 60° and sides 4 cm each is similar to a triangle with the identical three angles and sides 8 cm each. But the two are not congruent. One is simply the other, doubled in size. Congruent needs the scale locked at exactly $1:1$: same shape AND same size, not merely the same shape at any size that happens to turn up. Treating similar and congruent as interchangeable words costs marks, specifically on questions that name which one applies. We can catch this trap with one question: has an actual side length been fixed, or only the angles? A question stating equal angles alone is asking about similarity. A question also fixing an actual side length is asking about congruence. Your turn: a question gives two triangles with equal angles only. Similar, congruent, or both? (Answer: similar only. No side length was fixed.)

TWO TRIANGLES HAVE THE SAME THREE ANGLES. ARE THEY CONGRUENT?

WEAK
A triangle with angles 60°, 60°, 60° has sides 4 cm each. A second triangle has the identical three angles, with sides 8 cm each. Same angles, so it is tempting to call them congruent.

But check the sides: 4 cm against 8 cm. They do not match, so the two triangles are not congruent.

STRONG
The two triangles share the same three angles, so they are similar, with sides in ratio $1:2$.

Congruent needs more than equal angles: the same shape AND the same size, scale $1:1$. Here the scale is $1:2$, not $1:1$, so the triangles are similar but not congruent.

BACK
Similar figures, defined · p. 181

CAUTION
Equal angles alone prove similarity, never congruence: check the sides' actual lengths too.

KEY
Congruent is the ONE-to-one case of similar: same shape at scale factor 1.

A SCHOLAR INDIA REMEMBERS

Tell me the difference in one line each. Similar means the same shape: the angles match, and the sides are in one ratio. Congruent means the same shape and the same size: that ratio is 1. *So every congruent pair is similar. A similar pair is congruent only when the ratio is* 1.

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WATCH OUT

The trap: Seeing two triangles with equal angles but visibly different sizes, the student marks the pair congruent.

The correction: Equal angles alone make two triangles similar, not congruent: congruence also needs the sides to match at scale $1:1$.

CHECK YOURSELF

29. A student sees two triangles with exactly the same three angles and says they must be congruent, since "similar" and "congruent" feel like the same word. What is the real relationship?

- **A.** the triangles are always congruent whenever their angles all match
- **B.** the triangles are neither similar nor congruent until the side lengths are checked and compared first
- **C.** the triangles are congruent only if drawn in the same orientation
- **D.** the triangles are similar, but not necessarily congruent, unless their sizes also match

30. Triangle ABC has angles $70°, 60°, 50°$ and sides $3, 4, 5$ cm. Triangle DEF has the SAME three angles but sides $6, 8, 10$ cm. A student calls them congruent because "the angles are identical." What is the real relationship?

- **A.** the triangles are congruent, since matching angles is what congruence actually checks
- **B.** the triangles are similar, at scale factor $2:1$, but not congruent
- **C.** the triangles are neither similar nor congruent, since their side lengths differ
- **D.** it cannot be decided, since the triangles' orientations were not given

2 more in this set online.

Part against part, not part against whole

The Basic Proportionality Theorem gives one ratio: $AD : DB = AE : EC$. Both sides compare a part with the other part of the same side. It is easy to compare a part with the whole side instead, by mistake. Take triangle ABC with D on AB, E on AC, and DE parallel to BC. If you write $AD : DB = AE : AC$, you have mixed a part with a whole. The right side of that ratio should compare AE with EC, not AE with the entire side AC. Let us build the habit that prevents this. Look at the divided side first. Name its two pieces. Then look at the other divided side, and name its two pieces the same way. Only then write the ratio, part against part. Your turn: one side is split into pieces of 2 cm and 5 cm. Which is the correct comparison: the two pieces, 2 and 5, or one piece against the full 7 cm side? (Answer: the two pieces, 2 and 5.)

BACK
The Basic Proportionality Theorem · p. 189

CAUTION
Write the theorem out as $AD : DB = AE : EC$ before a single number goes in: part against part on both sides.

KEY
Finish by reading the two ratios back. If $AD : DB$ and $AE : EC$ are not the same ratio, the line above them is wrong.

FIRST TRY

~~With $AD = 3$, $DB = 6$ and $AE = 4$, I wrote $AD/DB = AE/AC$.~~
~~That gives $3/6 = 4/AC$, so $AC = 8$ and $EC = 4$ cm.~~

SECOND LOOK

Both sides of a ratio must be built the same way. The right line is $AD/DB = AE/EC$, giving $3/6 = 4/EC$, so $EC = 8$ cm.

Match part to part on both sides of the ratio, never part to whole.

IN A TRIANGLE, ONE SIDE GIVES 3 CM AND 6 CM, THE OTHER GIVES 4 CM AND AN UNKNOWN PIECE. FIND THE UNKNOWN PIECE.

WEAK

The careless line writes $AD : DB = AE : AC$, comparing a part on one side with the whole of the other side.

Substituting, $3 : 6 = 4 : AC$, so $AC = 8$ cm, and $EC = AC - AE = 8 - 4 = 4$ cm.

Check the two ratios this gives: $AD : DB = 1 : 2$ and $AE : EC = 4 : 4 = 1 : 1$. These are not equal, so something has gone wrong, even though the theorem promised they would match.

STRONG

The right line writes $AD : DB = AE : EC$, part against part on both sides.

Substituting, $3 : 6 = 4 : EC$, so $EC = 8$ cm.

Check the two ratios now: $AD : DB = 1 : 2$ and $AE : EC = 4 : 8 = 1 : 2$. The two ratios agree, which is what the theorem promised.

Worked examples

WORKED EXAMPLE · **Finding EC from the ratio theorem**

1.	$AD = 1.5$ cm, $DB = 3$ cm, $AE = 1$ cm, with DE parallel to BC	*These are the given measurements. D is on AB and E is on AC.*
2.	$AD : DB = AE : EC$	*The Basic Proportionality Theorem applies here, since DE is parallel to BC.*
3.	$1.5 : 3 = 1 : EC$	*Substitute the given lengths into the ratio.*
4.	$1.5 \cdot EC = 3$	*Cross-multiply the two ratios.*
5.	$EC = 2$ cm	*Divide both sides by 1.5.*
6.	$AD : DB = 1.5 : 3 = 0.5$ and $AE : EC = 1 : 2 = 0.5$	CHECK ***Check the answer by confirming both ratios come out equal.*** *They do, so $EC = 2$ cm is correct.*

TRY

In triangle ABC, DE is parallel to BC; $AD = 2$ cm, $DB = 4$ cm, $AE = 3$ cm. Find EC.

Answer: $EC = 6$ cm, from $AD : DB = AE : EC$.

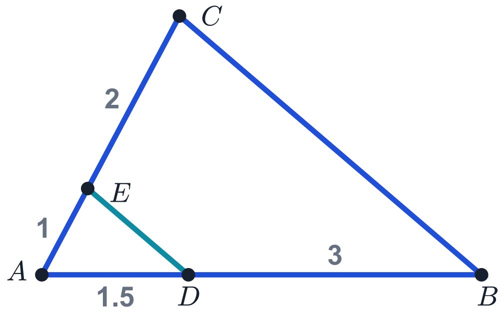
RULE

Pair each divided side into its two segments before comparing ratios, then cross-multiply to solve.

CAUTION

The ratio compares two PARTS of a side, $AD : DB$: never a part against the whole side AB.

The ratio, worked in numbers



A triangle with a line parallel to its base shows both ratios, 1.5 to 3 and 1 to 2, equal at 0.5.

FIND THE MISSING PIECE OF A SIDE, USING THE RATIO THEOREM.

1. **Name the two points** Mark D on one side and E on the other, with DE parallel to the third side.
2. **Write the ratio** $AD : DB = AE : EC$. Both sides of the ratio compare the same two pieces.
3. **Put in the numbers** Substitute the lengths you know into the ratio, leaving the missing piece as the unknown.
4. **Solve for the missing piece** Cross-multiply and divide to find the unknown length.

CHECK YOURSELF

31. In triangle ABC, D lies on AB, E lies on AC, and DE is parallel to BC. By BPT, which equation correctly relates the segments?

- **A.** $(AD)/(AB) = (AE)/(EC)$
- **B.** $(AD)/(DB) = (EC)/(AE)$
- **C.** $(AD)/(DB) = (AE)/(EC)$
- **D.** $(AD) \cdot (DB) = (AE) \cdot (EC)$

32. In triangle PQR, S lies on PQ and T lies on PR, with ST parallel to QR. Given $PS = 4$ cm, $SQ = 6$ cm, $PT = 3$ cm, find TR.

- **A.** 2 cm, from $PS - PT$
- **B.** 4.5 cm, from $6 : 4 = 3 : TR$
- **C.** 4.5 cm, from $4 : 6 = 3 : TR$
- **D.** 8 cm, from $6 + 3 - 4 + 3$

1 more in this set online.

WORKED EXAMPLE · **Checking whether EF is parallel to QR**

1.	$PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm, $FR = 2.4$ cm	*These are the given measurements. E is on PQ and F is on PR.*
2.	$PE : EQ = 3.9 : 3 = 1.3$	*Compute the first ratio.*
3.	$PF : FR = 3.6 : 2.4 = 1.5$	*Compute the second ratio.*
4.	$1.3 \neq 1.5$	*The two ratios disagree.*

WORKED EXAMPLE · **Checking whether EF is parallel to QR**

5. EF is not parallel to QR

The converse of the Basic Proportionality Theorem needs both ratios equal. They are not, so the line fails the test.

6. $PE \cdot FR = 3.9 \cdot 2.4 = 9.36$ and $EQ \cdot PF = 3 \cdot 3.6 = 10.8$

CHECK ***Check the same conclusion a second way, by cross-multiplying instead of dividing.*** *The two products disagree, confirming EF is not parallel to QR.*

BACK
Converse of the Basic Proportionality Theorem · p. 191

TRY
In triangle PQR, E on PQ, F on PR: $PE = 2$, $EQ = 4$, $PF = 3$, $FR = 6$. Is EF parallel to QR?

Answer: $PE : EQ = 0.5$ and $PF : FR = 0.5$. Equal, so EF is parallel to QR.

CAUTION
Both ratios need the same order: $PE : EQ$ compared with $PF : FR$, not one flipped over.

CHECK WHETHER A LINE IS PARALLEL TO THE THIRD SIDE.

1. **Compute the first ratio** Divide the first side's two pieces, in the same order every time.
2. **Compute the second ratio** Divide the second side's two pieces, in that same order.
3. **Compare the two ratios** Simplify both fully before you compare them.
4. **Decide parallel or not** Equal ratios mean the line is parallel. Unequal ratios mean it is not.

CHECK YOURSELF

33. In triangle PQR, E lies on PQ and F lies on PR, with $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm, $FR = 2.4$ cm. Which two values must be compared to test whether EF is parallel to QR?

- **A.** $PE : PF$ and $EQ : FR$
- **B.** $PQ : PR$ and $EQ : FR$
- **C.** $PE : EQ$ and $PF : FR$
- **D.** $PE + EQ$ and $PF + FR$

34. In triangle XYZ, M lies on XY with $XM = 5$ cm, $MY = 4$ cm, and N lies on XZ with $XN = 7.5$ cm, $NZ = 6$ cm. Is MN parallel to YZ?

- **A.** no, since XM does not equal XN
- **B.** no, since 5 and 4 do not divide evenly
- **C.** it cannot be decided without measuring YZ directly
- **D.** yes

1 more in this set online.

WORKED EXAMPLE · **A shadow and a lamp-post**

1. $BD = 1.2 \cdot 4 = 4.8$ m

This is the distance the girl walked in 4 seconds, at 1.2 m/s.

WORKED EXAMPLE · A shadow and a lamp-post

2.	angle E is shared; angle B = angle D = 90°	*Both are right angles, formed where the lamp-post and the girl meet the ground.*
3.	triangle ABE is similar to triangle CDE	*Two matching angles prove it, the AA criterion.*
4.	$AB : CD = BE : DE$	*Corresponding sides of similar triangles are in the same ratio.*
5.	$3.6 : 0.9 = (4.8 + x) : x$	*Substitute the lamp-post height, the girl's height, and* $BE = BD + DE = 4.8 + x$*, with the shadow length* $DE = x$*.*
6.	$3.6 : 0.9$ simplifies to $4 : 1$, so $4.8 + x = 4x$	*Cross-multiply the simplified ratio.*
7.	$3x = 4.8$, so $x = 1.6$ m	*Solve for* x*.*
8.	$(4.8 + 1.6) : 1.6 = 6.4 : 1.6 = 4$	CHECK ***Check by substituting*** $x = 1.6$ ***back into the ratio.*** *It matches* $3.6 : 0.9$*, which also simplifies to* 4*, so the shadow length is correct.*

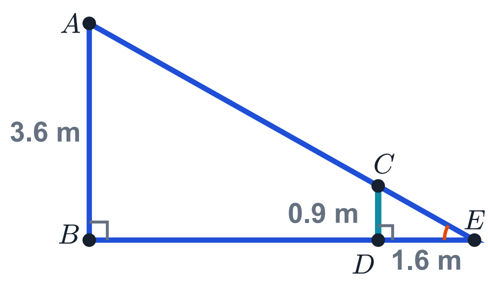
BACK
The AA similarity criterion · p. 192

TRY
If she had walked for 2 s instead of 4, what would BD be?

Answer: $BD = 1.2 \cdot 2 = 2.4$ m.

RULE
Find one shared angle and one equal right angle first: AA needs only those two matches.

A shadow and a lamp-post



A 3.6 m lamp-post and a 0.9 m girl share one ray of light, giving similar triangles with matching right angles.

Tara walks away from a lit lamp-post at dusk, her shadow falling ahead of her on the pavement.

USE TWO MATCHING ANGLES TO FIND A MISSING LENGTH.

1. **Find the shared angle** Look for one angle the two triangles have in common.
2. **Find the matching right angles** Two right angles, one in each triangle, give the second matching angle.
3. **Confirm similarity by AA** Two matching angles are enough. The triangles are similar.
4. **Write the ratio of sides** Match corresponding sides in the same order as the angles.
5. **Solve for the unknown** Cross-multiply the ratio and solve for the missing length.

CHECK YOURSELF

35. In the lamp-post and shadow problem, triangle ABE (lamp-post and shadow tip) and triangle CDE (girl and shadow tip) are shown similar by AA. Which two angle facts establish this?

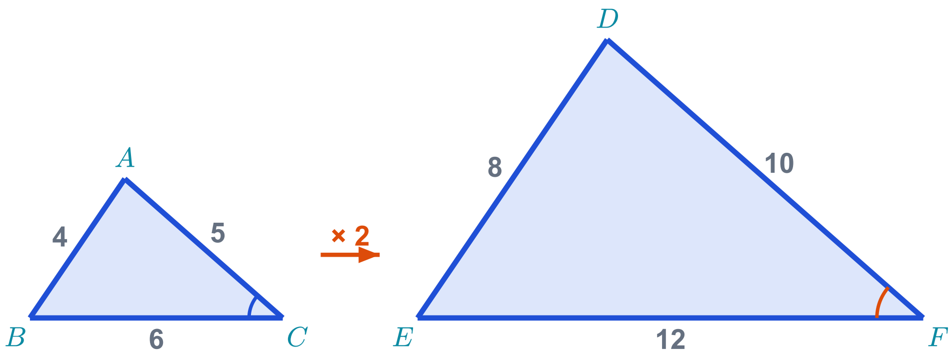
- **A.** the lamp-post and the girl are the same height
- **B.** the two shadows are the same length
- **C.** they share angle E, and angle B equals angle D, both right angles
- **D.** the lamp's angle changes noticeably between the lamp-post triangle and the girl's triangle

36. A pole 2 m tall casts a shadow 1.5 m long. At the same moment, a building casts a shadow 12 m long. Using AA similarity the same way as the lamp-post problem, find the building's height.

- **A.** 9 m, from $2 : 1.5 = 12 : h$
- **B.** 13.5 m, from $2 + 1.5 + 12 - 2$
- **C.** 16 m, from $2 : 1.5 = h : 12$
- **D.** 12 m, since the building's height should equal its shadow length

1 more in this set online.

Four, six, five doubles to eight, twelve, ten



every ratio is $1 : 2$, so $F = C = 180° - 80° - 60° = 40°$

Every side of DEF is exactly twice its partner in ABC. That one fact makes the triangles similar, so the angle at F is the angle at C, and the angle sum gives it as 40°.

Three matching ratios are enough on their own: once the sides say similar, the angles follow without measuring.

WORKED EXAMPLE · **SSS finds a missing angle**

1.	$AB : DE = 4 : 8,\ BC : EF = 6 : 12,\ CA : FD = 5 : 10$	*Form the three side ratios, in correspondence order.*
2.	all three ratios equal $1 : 2$	*Simplify each ratio.*
3.	triangle ABC is similar to triangle DEF	*All three side ratios match, the SSS criterion.*
4.	angle C = angle F	*Corresponding angles of similar triangles are equal.*
5.	angle $C = 180° - 80° - 60° = 40°$	*The angle sum property, using the given angle A and angle B.*
6.	angle $F = 40°$	*Equal to angle C, already found.*
7.	angle D + angle E + angle F = $80° + 60° + 40° = 180°$	CHECK ***Check the answer against the angle sum property.*** *The three angles of triangle DEF add to $180°$, confirming angle $F = 40°$ is correct.*

BACK
The SSS similarity criterion · p. 193

TRY
Triangle ABC has sides $3, 4, 5$; triangle DEF has sides $9, 12, 15$. Similar? By which criterion?

Answer: Yes. Every ratio is $1:3$, so similar by SSS.

CAUTION
Match sides by the STATED correspondence, AB with DE: not by pairing whichever lengths look close.

USE THREE SIDE RATIOS TO FIND A MISSING ANGLE.

1. **Form the three ratios** Match each side of one triangle to its corresponding side in the other, in order.
2. **Simplify each ratio** Check that all three come out to the same value.
3. **Confirm similarity by SSS** Equal ratios on all three sides prove the triangles are similar.
4. **Read off the angle** Use the matching correspondence and the angle sum property to find the missing angle.

CHECK YOURSELF

37. Triangle ABC has sides $4, 6, 5$; triangle DEF has sides $8, 12, 10$. What confirms $\triangle ABC \sim \triangle DEF$ by SSS, before angle F is found?

- **A.** the two triangles have the same perimeter ratio as their longest sides
- **B.** angle A is given as $80°$
- **C.** all three ratios, $AB : DE$, $BC : EF$, $CA : FD$, equal $1 : 2$
- **D.** DEF's sides are all even numbers

38. Triangle PQR has sides $PQ = 3$, $QR = 4$, $RP = 5$, with angle $Q = 90°$ (the largest angle here, opposite the longest side RP). Triangle XYZ has sides $XY = 6$, $YZ = 8$, $ZX = 10$. By SSS similarity, find angle Y.

- **A.** $45°$, half of angle Q since the sides are half as long
- **B.** it cannot be found without measuring angle Y directly
- **C.** $60°$, by the angle sum property applied to the other two angles
- **D.** $90°$

1 more in this set online.

WORKED EXAMPLE · **Vertical angles and SAS**

1.	$OA \cdot OB = OC \cdot OD$	*This is given.*
2.	$OA : OC = OD : OB$	*Rearrange the given product into a ratio.*
3.	angle AOD = angle COB	*Vertically opposite angles are always equal.*
4.	triangle AOD is similar to triangle COB	*Two sides in the same ratio, with the included angle equal: the SAS criterion.*
5.	angle A = angle C, angle D = angle B	*Corresponding angles of similar triangles are equal.*

WORKED EXAMPLE · **Vertical angles and SAS**

6. correspondence A to C, O to O, D to B gives angle A = angle C and angle D = angle B

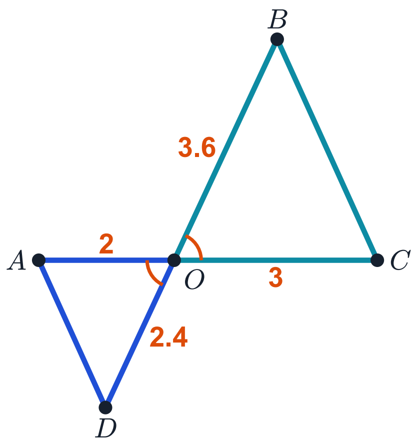
CHECK ***Check both conclusions against the same correspondence.*** *A was matched to C and D to B in the similarity above, so both angle equalities follow from that one match, not two separate arguments.*

BACK
The SAS similarity criterion · p. 194

CAUTION
Rearrange $OA \cdot OB = OC \cdot OD$ carefully: the wrong pairing gives a false ratio.

RULE
Confirm the equal angle sits BETWEEN the two matched sides before invoking SAS.

Vertical angles give SAS



Two triangles meet where two lines cross, sharing a vertically opposite angle and matching side ratios of 2 to 3.

CHECK YOURSELF

39. In the crossing-lines problem, $OA \cdot OB = OC \cdot OD$ rearranges to $OA : OC = OD : OB$, and angle AOD equals angle COB since they are vertically opposite. Why does this angle equality matter for SAS similarity here?

- **A.** it proves the two lines AC and BD are parallel
- **B.** it shows triangle AOD and triangle COB have equal area
- **C.** it lets OA be assumed equal to OD
- **D.** it supplies the included angle for the ratio-matched sides

40. Lines EG and FH cross at point O, with $OE \cdot OH = OF \cdot OG$. Using the same method as $OA \cdot OB = OC \cdot OD$, which triangle pair does SAS similarity establish?

- **A.** triangle EOF similar to triangle HOG, using the vertically opposite angles at O
- **B.** triangle EOH similar to triangle FOG, using the same vertically opposite angles at O
- **C.** no similarity can be established without also knowing the four lengths individually
- **D.** triangle EOF is congruent, not merely similar, to triangle HOG

1 more in this set online.

WORKED EXAMPLE · **Similar but not congruent, then a congruent pair**

1.	triangle ABC has angles 60°, 60°, 60° and sides $AB = BC = CA = 4$ cm	*This is the first triangle, given.*
2.	triangle DEF has the same three angles, with sides $DE = EF = FD = 8$ cm	*Every side is exactly double the first triangle's.*
3.	triangle ABC is similar to triangle DEF	*The angles match and the sides are in the ratio $1 : 2$, so both similarity conditions hold.*
4.	triangle ABC is not congruent to triangle DEF	*The actual side lengths do not match, 4 cm against 8 cm, and no rotation or flip can change that.*
5.	triangle PQR has sides $PQ = QR = RP = 4$ cm	*This is a third triangle, built to match triangle ABC's sides exactly.*
6.	triangle ABC is congruent to triangle PQR	*The two share the same shape and the same size, at scale $1 : 1$.*
7.	$AB : DE = 4 : 8, \;\; BC : EF = 4 : 8, \;\; CA : FD = 4 : 8$, all $= 1 : 2$	CHECK ***Check the similar pair by confirming all three side ratios agree.*** *They do, and the ratio for the congruent pair, triangle ABC to triangle PQR, comes out $4 : 4 = 1 : 1$ the same way.*

BACK
Similar figures, defined · p. 181

TRY
If triangle DEF's sides were 4 cm instead of 8 cm, would it be congruent to triangle ABC?

Answer: Yes. Equal angles and equal side lengths, scale factor 1.

RULE
Equal angles alone never prove congruence: check whether the actual side lengths also match.

CHECK YOURSELF

41. Triangle ABC has angles 60° each and sides 4 cm each; triangle DEF has the same three angles but sides 8 cm each; triangle PQR has angles 60° each and sides 4 cm each, matching ABC exactly. How should the three triangles be classified in pairs?

- **A.** all three of the triangles are fully congruent to one another, with no exceptions anywhere at all
- **B.** ABC and DEF are similar only; ABC and PQR are congruent (and also similar)
- **C.** ABC and DEF are congruent; ABC and PQR are only similar
- **D.** none of the three triangles are similar to each other, since only two match exactly

42. Triangle JKL has angles $45°, 45°, 90°$ and sides $5, 5, 5\sqrt{2}$ cm. Triangle MNO has the same three angles and sides $10, 10, 10\sqrt{2}$ cm. Triangle RST has angles $45°, 45°, 90°$ and sides $5, 5, 5\sqrt{2}$ cm, matching JKL exactly. Classify the relationship between each pair.

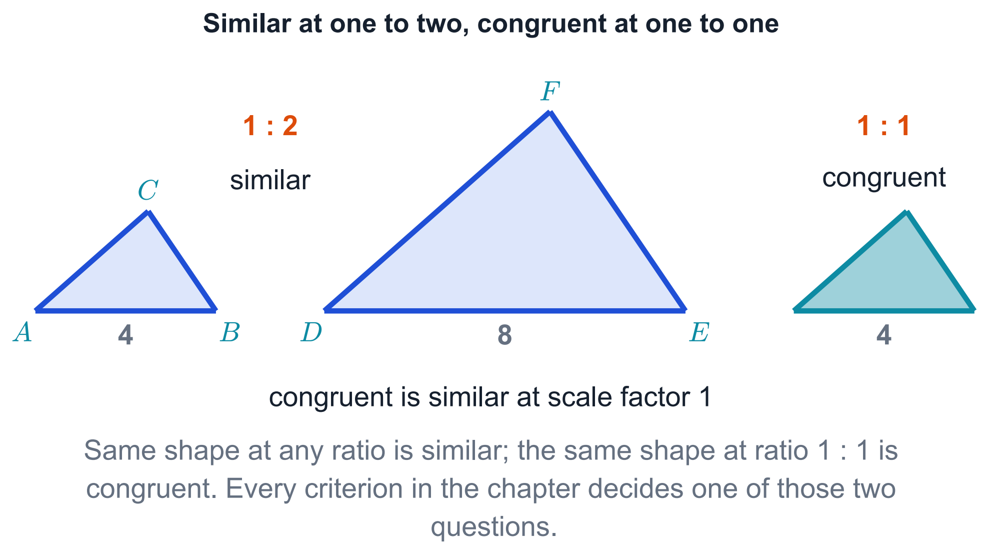
- **A.** JKL and MNO are similar only; JKL and RST are congruent
- **B.** JKL and MNO are congruent, since both are right-angled isosceles triangles
- **C.** JKL and RST are only similar, not congruent, since they are two different triangles
- **D.** none of the three are related, since irrational side lengths cannot be compared

1 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap



Congruence is the scale-one case of similarity, so a similarity test with the ratio fixed at 1 turns into a congruence test.

Let us gather everything into five lines.

- SIMILAR: equal angles, proportional sides, any ratio. Triangle ABC and triangle DEF with sides 4 cm and 8 cm are similar at ratio $1 : 2$.
- CONGRUENT: similar at ratio $1 : 1$. Two triangles with sides 4 cm and 4 cm, and the same angles, are congruent.
- BASIC PROPORTIONALITY: a line parallel to one side divides the other two in the same ratio. $AD : DB = 1.5 : 3$ gives $EC = 2$ cm.
- AA: two matching angles prove similarity. Angle A = angle D = 80° is one of the two checked, with no side measured.
- SSS AND SAS: matching side ratios, or one angle plus two side ratios, prove similarity. A $1 : 2$ side ratio found triangle DEF's missing angle at 40°.

Every criterion here answers the same question. Do two triangles have the same shape, and the same size? AA, SSS and SAS decide similarity. SSS, SAS, ASA and RHS decide congruence. Similarity asks only about shape: equal corresponding angles and proportional corresponding sides. Congruence adds size to the requirement, which is the same as asking for the ratio to be locked at exactly $1 : 1$. The Basic Proportionality Theorem sits underneath the similarity criteria. A line parallel to one side of a triangle divides the other two sides in that same fixed ratio. Its converse runs the test the other way. Same shape, any size, is the one question every line above answers a different way.

KEY
One theorem (Basic Proportionality), three shortcuts built on it (AA, SSS, SAS), and one boundary case (congruence, at scale factor 1).

BACK
The Basic Proportionality Theorem · p. 189

CHECK YOURSELF

43. Which statement best summarises the central question every result in this chapter answers?

- **A.** when do two triangles have the same area, whatever their shape?
- **B.** when do two triangles have the same shape, at any size?
- **C.** when do two triangles have the same perimeter?
- **D.** when can a triangle be drawn inside another triangle?

44. The chapter builds AA, SSS and SAS similarity on top of the Basic Proportionality Theorem, letting a check on just two or three matching parts stand in for all six. What does this buy, practically?

- **A.** it means every triangle is automatically similar to every other triangle
- **B.** similarity can be confirmed without measuring every one of a triangle's six parts
- **C.** it removes the need to check any angles at all, since only the sides are ever compared
- **D.** it proves BPT no longer needs its own proof once the criteria exist

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

45. Triangle PQR has angles $50°, 60°, 70°$. Triangle XYZ has the same three angles. Every side of XYZ is exactly double the corresponding side of PQR. Are the two triangles similar?

- **A.** No — two triangles of different sizes can never be similar, whatever their angles. A fixed doubling like this already rules similarity out.
- **B.** No — similar triangles need exactly equal side lengths, not merely a fixed ratio between them.
- **C.** Cannot be decided without measuring the actual side lengths, since equal angles alone say nothing about similarity.
- **D.** Yes — equal corresponding angles and sides in a fixed ratio (1 : 2 here) are exactly what similarity requires.

46. Triangle ABC is similar to triangle DEF, matched in that correspondence order — A with D, B with E, C with F. Which of these must be true?

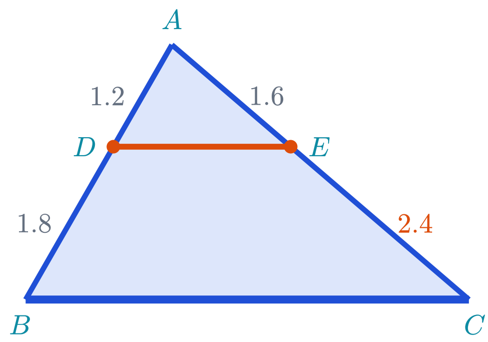
- **A.** angle B equals angle F
- **B.** angle A equals angle E
- **C.** angle B equals angle E
- **D.** side AB has no fixed relationship to any side of triangle DEF, since the correspondence only fixes matching angles

3 more in this set online.

IN EVERY CHAPTER

Where you will meet this

The shortcut splits both trails alike



DE is parallel to BC, so $AD : DB = AE : EC$. $1.2 : 1.8 = 1.6 : 2.4$,
and $EC = 2.4$ km is the distance wanted.

A shortcut D E parallel to the road cuts the two trails in the same ratio, 1.2 to 1.8 and 1.6 to 2.4.

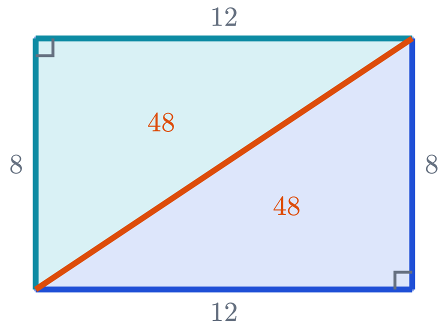
You will meet similar and congruent triangles often. A shape gets scaled to a new size. Two pieces must match exactly. Here are seven of those places.

- **Sewing a matching pair of pennant flags.** A flag maker sews a small triangular pennant with sides 30, 40, 50 cm. A large pennant for the same client must be similar, at scale factor 2.
  The maths: equal angles and sides in the same ratio make two triangles similar, whatever the size.
  Large sides 60, 80, 100 cm: $60/30 = 2$, $80/40 = 2$, $100/50 = 2$, one ratio throughout.
- **Scaling up a triangular rug pattern.** A paper pattern piece has sides 5, 6, 9 cm. The rug maker cuts a piece at scale factor 2 and wants to check it was cut correctly.
  The maths: all three pairs of sides are in the same ratio. That alone makes the two triangles similar, with no angle measured.
  Cut sides 10, 12, 18 cm: $10/5 = 2$, $12/6 = 2$, $18/9 = 2$. The cut is correct.
- **Checking two garden beds need equal edging.** One triangular flower bed has sides 3, 4, 5 m. Another has sides 5, 4, 3 m, measured in a different order.
  The maths: all three sides of one triangle equal the three sides of another. That makes the triangles congruent, so they need the same edging length.
  Both beds: $3 + 4 + 5 = 12$ m of edging, since the two lists of sides are the same three numbers.
- **Finding a distance along a shortcut trail.** Two trails leave a gate A and reach the main road at B and C. A shortcut parallel to the road crosses them at D and E.
  The maths: a line parallel to one side of a triangle divides the other two sides in the same ratio.
  From the start, $AD = 1.2$ km and $DB = 1.8$ km. Also $AE = 1.6$ km, so $AD : DB = AE : EC$ gives $EC = 2.4$ km.
- **Cutting a sandwich exactly in half.** A rectangular sandwich, 12 cm by 8 cm, is cut once along the diagonal into two triangles.
  The maths: each half has sides 12 and 8 cm with a right angle between them. So SAS makes

the two halves congruent.
Each triangle has legs 12 and 8 cm and area 48 square cm. Together, $48 + 48 = 96$, the whole sandwich.

BACK
Similarity, applied to triangles · p. 183

One cut, two equal halves



Each half has sides 12 and 8 with the right angle between them, so
SAS makes the halves congruent: 48 + 48 = 96, the whole sandwich.

A diagonal always splits a rectangle into two congruent triangles, so each half carries exactly half the area.

- **Checking two sailboat sails are the same shape.** A sailing club's junior sail and senior sail are both triangular. Both have base angles 70° and 60°.
  The maths: two matching angles are enough to prove similarity. That is AA. The larger sail behaves the same way, just bigger.
  The third angle is fixed too: $180 - 70 - 60 = 50°$, in both sails.
- **Buying a set square in two sizes.** A stationery shop sells a 30°-60°-90° set square in two sizes, junior and senior.
  The maths: the two set squares share the same three angles, so they are similar. Their sides are not equal, so they are not congruent.
  The junior size has hypotenuse 10 cm. The senior size has hypotenuse 20 cm. Both keep angles 30°, 60°, 90°.

Your turn. Two triangular kite frames have sides 6, 8, 10 cm and 9, 12, 15 cm. Are they similar? What is the scale factor? *Answer:* Check the ratios: $9/6 = 1.5$, $12/8 = 1.5$, $15/10 = 1.5$. All equal, so the kites are similar, at scale factor 1.5.

Practice: Exercise 6.1

PRACTICE SET

1. PRACTICE Are any two regular hexagons similar? Give a reason. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1. every angle of a regular hexagon is 120°, in both figures

similarity asks two separate questions, and the angles are the first — settle them on their own before looking at any side

SOLUTION — Q1 · **Worked in full**

2.	all six sides of each hexagon are equal, so corresponding sides are in one fixed ratio	*the second question is whether corresponding sides are in the SAME ratio — which ratio it is does not matter*
3.	both conditions hold, so any two regular hexagons are similar	*neither condition alone is enough, which is why both were checked and not just the one that looked obvious*

2. PRACTICE Two right triangles each have an angle of 30°. Are they similar? Give a reason. (Start the same way — write down all three angles of each triangle before deciding.)

3. PRACTICE One isosceles triangle has a vertex angle of 40° and another has a vertex angle of 80°. Are they similar? Give a reason. (Work out the two base angles of each first.)

4. PRACTICE For each pair below, state whether they are always similar, never similar, or only sometimes similar, and give a reason. (a) Any two circles. (b) Any two squares. (c) Any two equilateral triangles. (d) Any two rectangles.

5. PRACTICE Give one example of two figures that are similar, and one example of two figures that are not similar. Give a reason for each.

6. PRACTICE A quadrilateral has all four angles equal to 90°. A second quadrilateral also has all four angles equal to 90°, but its sides are not in the same ratio as the first. Are the two quadrilaterals similar? Give a reason.

7. PRACTICE Two rhombuses have all four sides in the ratio $1 : 1 : 1 : 1$ to each other, but one has an angle of 70° and the other has an angle of 100°. Are the two rhombuses similar? Give a reason.

8. PRACTICE State whether every pair of squares is similar, and why.

ANSWERS

1. Yes, always. Every angle is 120° in both, and all six sides of a regular hexagon are equal, so corresponding sides are in the same ratio.
2. Yes. Each has angles 90°, 30° and 60°, so all three angles match.
3. No. The first has angles 40°, 70°, 70° and the second has 80°, 50°, 50°, so the angles do not match.
4. (a) Always similar — every circle is the same shape, whatever its radius. (b) Always similar — every square has four 90° angles and all sides equal. (c) Always similar — every angle is fixed at 60°. (d) Only sometimes similar — angles always match at 90°, but sides need not be in the same ratio.
5. Sample answer: any two circles are similar (equal angles are not needed; every circle has the same shape at some ratio). A square and a non-square rectangle are not similar — equal angles, but sides not in the same ratio.
6. Not similar — the angles match, but the sides are not in the same ratio; both conditions are needed together.
7. Not similar — the sides are in the same ratio, but the angles differ.

8. Yes — every square has four 90° angles and all sides equal, so any two squares always satisfy both similarity conditions.
Full solutions: **manthika.com/learn/math10-ch06**

Further practice

PRACTICE SET

9. PRACTICE Are all isosceles triangles always similar to each other? Give a reason.

10. PRACTICE An equilateral triangle has side 3 cm and a second equilateral triangle has side 9 cm. Are the two triangles similar? Give a reason.

11. PRACTICE A photograph of size 10 cm by 15 cm is enlarged to 20 cm by 25 cm. Are the original and the enlarged photograph similar rectangles? Give a reason.

12. PRACTICE Which one of the following pairs is NOT always similar?
A. Two squares
B. Two equilateral triangles
C. Two isosceles triangles
D. Two circles

13. PRACTICE One quadrilateral has all four angles equal to 90°. A second quadrilateral has angles 100°, 80°, 100°, 80°, in order. Are the two quadrilaterals similar? Give a reason.

ANSWERS

9. No — an isosceles triangle with angles 50°, 50°, 80° and one with angles 40°, 40°, 100° are both isosceles, but their angles differ, so they are not similar.
10. Yes — every equilateral triangle has all three angles equal to 60°, whatever its side length, so the angle condition always holds.
11. No — the ratio of the widths is $10 : 20 = 1 : 2$ and the ratio of the heights is $15 : 25 = 3 : 5$. The two ratios are not equal, so the sides are not in the same ratio, and the rectangles are not similar.
12. C — Two isosceles triangles.
13. No — the corresponding angles are not equal, so the first condition for similarity already fails; the sides need not even be compared.
Full solutions: **manthika.com/learn/math10-ch06**

Practice: Exercise 6.2

PRACTICE SET

1. PRACTICE In triangle ABC, D lies on AB and E lies on AC, with DE parallel to BC. If $AD = 2$ cm, $DB = 5$ cm and $AE = 3$ cm, find EC. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1. $AD/DB = AE/EC$ — *write the theorem out before a single number goes in — part over part on BOTH sides*

SOLUTION — Q1 · Worked in full

2.	$2/5 = 3/EC$	*substitute the three known lengths, leaving the unknown one on its own*
3.	$2 \cdot EC = 15$, so $EC = 7.5$ cm	*cross-multiply, then divide*
4.	$AD : DB = 2 : 5$ and $AE : EC = 3 : 7.5 = 2 : 5$	*the check — read both ratios back, and if they are not equal the first line was written the wrong way round*

2. PRACTICE In triangle PQR, S lies on PQ and T lies on PR, with $PS = 2$ cm, $SQ = 3$ cm, $PT = 3$ cm and $TR = 4.5$ cm. Is ST parallel to QR? (Start the same way, but here both ratios are known — work them out and compare.)

3. PRACTICE In triangle ABC, a line parallel to BC meets AB at D and AC at E, with $AD : DB = 2 : 3$. If $AE = 4$ cm, find EC. (The ratio is handed to you this time — it goes straight into the left-hand side.)

4. PRACTICE In triangle ABC, D lies on AB and E lies on AC, with DE parallel to BC. If $AD = 4$ cm, $DB = 6$ cm and $AE = 6$ cm, find EC.

5. PRACTICE In triangle PQR, S lies on PQ and T lies on PR, with $PS = 4$ cm, $SQ = 4.5$ cm, $PT = 8$ cm, $TR = 9$ cm. Is ST parallel to QR?

6. PRACTICE In triangle ABC, D and E are the midpoints of AB and AC. Prove that DE is parallel to BC.

7. PRACTICE $ABCD$ is a trapezium with AB parallel to DC. The diagonals AC and BD meet at O. Prove that $AO : OC = BO : OD$.

8. PRACTICE In triangle ABC, a line parallel to BC meets AB at D and AC at E, with $AD = 2DB$. If $AE = 6$ cm, find EC.

ANSWERS **1.** $EC = 7.5$ cm.
2. Yes. $PS : SQ = 2 : 3$ and $PT : TR = 3 : 4.5 = 2 : 3$. The two ratios are equal, so by the converse of the theorem ST is parallel to QR.
3. $EC = 6$ cm. **4.** $AD : DB = AE : EC$, so $4 : 6 = 6 : EC$, giving $EC = 9$ cm.
5. $PS : SQ = 4 : 4.5 = 8 : 9$ and $PT : TR = 8 : 9$. The two ratios are equal, so by the converse of the Basic Proportionality Theorem, ST is parallel to QR.
6. D and E are midpoints, so $AD : DB = 1 : 1 = AE : EC$. Equal ratios on both sides means, by the converse of the Basic Proportionality Theorem, DE is parallel to BC.
7. Since AB is parallel to DC, angle OAB = angle OCD and angle OBA = angle ODC (alternate angles). So triangle AOB is similar to triangle COD by AA, giving $AO : OC = BO : OD$.
8. $AD = 2DB$ means $AD : DB = 2 : 1$. By the Basic Proportionality Theorem, $AE : EC = 2 : 1$ too, so $EC = 3$ cm.
Full solutions: **manthika.com/learn/math10-ch06**

Further practice

PRACTICE SET

9. PRACTICE In triangle ABC, D lies on AB and E lies on AC, with DE parallel to BC. If $AD = 2$ cm, $DB = 3$ cm and $AE = 2.4$ cm, find EC.

SOLUTION — Q9 · **Worked in full**

1.	$AD : DB = AE : EC$	*The Basic Proportionality Theorem applies, since DE is parallel to BC.*
2.	$2 : 3 = 2.4 : EC$	*Substitute the given lengths into the ratio.*
3.	$2 \cdot EC = 3 \cdot 2.4 = 7.2$	*Cross-multiply the two ratios.*
4.	$EC = 7.2/2 = 3.6$	*Divide to find EC, in cm.*

10. PRACTICE In triangle PQR, S lies on PQ and T lies on PR, with ST parallel to QR. If $PS = 3$ cm, $SQ = 5$ cm and $PT = 6$ cm, find TR.

11. PRACTICE In triangle PQR, E lies on PQ and F lies on PR, with $PE = 3$ cm, $EQ = 4.5$ cm, $PF = 4$ cm and $FR = 5$ cm. Which statement is correct?

A. EF is parallel to QR

B. EF is not parallel to QR

C. EF and QR cannot be compared without more information

D. EF is perpendicular to QR

12. PRACTICE A surveyor marks points D on side AB and E on side AC of a triangular plot ABC, to check whether a proposed fence DE can run parallel to BC. She measures $AD = 6$ m, $DB = 9$ m, $AE = 8$ m and $EC = 12$ m. Can the fence DE be drawn parallel to BC? Give a reason.

13. PRACTICE In triangle ABC, D lies on AB and E lies on AC, with DE parallel to BC. If $AD : DB = 3 : 4$ and $AE = 6$ cm, find EC.

14. PRACTICE A carpenter fixes a beam DE parallel to the base BC inside a triangular truss ABC, with D on AB and E on AC. If $AD = 2.5$ m, $DB = 5$ m and $AE = 3$ m, find EC.

15. PRACTICE $ABCD$ is a trapezium with AB parallel to DC. The diagonals AC and BD meet at O, so that $AO : OC = BO : OD$. If $AO = 3$ cm, $BO = 4.5$ cm and $OD = 6$ cm, find OC.

16. PRACTICE In triangle ABC, D, E and F are the midpoints of sides BC, CA and AB. Show that triangle DEF is similar to triangle ABC, and state the scale factor.

17. PRACTICE $ABCD$ is a trapezium with AB parallel to DC. E lies on AD and F lies on BC, with EF parallel to AB. Prove that $AE : ED = BF : FC$.

ANSWERS **9.** $EC = 3.6$ cm **10.** $TR = 10$ cm **11.** B — EF is not parallel to QR.
12. Yes — $AD : DB = 6 : 9 = 2 : 3$ and $AE : EC = 8 : 12 = 2 : 3$. The two ratios are equal, so by the converse of the Basic Proportionality Theorem, DE can be drawn parallel to BC.
13. $EC = 8$ cm **14.** $EC = 6$ m **15.** $OC = 4$ cm

16. Since E and F are midpoints of CA and AB, segment EF is parallel to BC and $EF = 1/2BC$ by the midpoint theorem. Likewise $DE = 1/2AB$ and $FD = 1/2CA$. All three sides of triangle DEF are exactly half the corresponding sides of triangle ABC (D opposite A, E opposite B, F opposite C), so triangle DEF is similar to triangle ABC by SSS, with scale factor $1 : 2$.

17. Join the diagonal AC, meeting EF at G. In triangle ADC, EG is parallel to DC (both parallel to AB), so by the Basic Proportionality Theorem $AE : ED = AG : GC$. In triangle ABC, GF is parallel to AB, so by the Basic Proportionality Theorem $CG : GA = CF : FB$, which gives $AG : GC = BF : FC$. Combining the two, $AE : ED = BF : FC$.

Full solutions: **manthika.com/learn/math10-ch06**

Practice: Exercise 6.3

PRACTICE SET

1. PRACTICE Triangle ABC has angle $A = 40°$ and angle $B = 75°$. Triangle PQR has angle $P = 40°$ and angle $Q = 75°$. State whether the two triangles are similar and name the criterion. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	angle A = angle P = 40°	*match the angles in corresponding order — A with P, B with Q, C with R*
2.	angle B = angle Q = 75°	*that is already two pairs, which is the whole of what the AA criterion asks for*
3.	angle C = angle R = 180° − 40° − 75° = 65°	*the third pair comes free, because the angles of any triangle add to 180° — so two pairs really is enough*
4.	triangle ABC is similar to triangle PQR, by AA	*name the criterion, and write the correspondence in the order the angles matched*

2. PRACTICE Triangle LMN has sides $LM = 4$ cm, $MN = 6$ cm and $NL = 8$ cm. Triangle XYZ has sides $XY = 6$ cm, $YZ = 9$ cm and $ZX = 12$ cm. Show that the two triangles are similar and name the criterion. (Start the same way, but here it is three side ratios that must be compared.)

5. PRACTICE Triangle PQR has sides $PQ = 3$ cm, $QR = 4$ cm, $RP = 5$ cm. Triangle XYZ has sides $XY = 6$ cm, $YZ = 8$ cm, $ZX = 10$ cm. Show that the two triangles are similar, and name the criterion used.

6. PRACTICE In triangle ABC, angle $A = 40°$; in triangle DEF, angle $D = 40°$. If $AB : DE = AC : DF = 2 : 3$, are the two triangles similar? Name the criterion.

3. PRACTICE In triangle ABC, $AB = 4$ cm, $AC = 6$ cm and angle $A = 60°$. In triangle PQR, $PQ = 6$ cm, $PR = 9$ cm and angle $P = 60°$. Are the two triangles similar? Name the criterion. (Two sides and the angle between them — check the two ratios, then the angle.)

4. PRACTICE Triangle ABC has angle $A = 50°$ and angle $B = 70°$. Triangle DEF has angle $D = 50°$ and angle $E = 70°$. State, with reason, whether the two triangles are similar.

7. PRACTICE Triangle ABC is similar to triangle DEF, with angle $A = 65°$ and angle $B = 55°$. Find angle F.

8. PRACTICE AD is the altitude from A to BC in a right triangle ABC, right-angled at A. Show that triangle ABD is similar to triangle CBA.

ANSWERS

1. Yes, by AA. Two pairs of angles are equal, and the third pair follows, since the angles of a triangle add to 180°.
2. $LM : XY = 4 : 6$, $MN : YZ = 6 : 9$ and $NL : ZX = 8 : 12$ are all $2 : 3$, so the two triangles are similar by SSS.
3. Yes, by SAS. $AB : PQ = 4 : 6$ and $AC : PR = 6 : 9$ are both $2 : 3$, and the included angles A and P are both 60°.
4. Yes, similar by AA — angle A = angle D = 50° and angle B = angle E = 70°, matching in correspondence order.
5. $PQ : XY = 3 : 6$, $QR : YZ = 4 : 8$, $RP : ZX = 5 : 10$ all equal $1 : 2$, so the triangles are similar by SSS.
6. Yes, similar by SAS — one matching angle, with the two including sides in the same ratio.
7. Angle $C = 180° - 65° - 55° = 60°$; equal to angle F, so angle $F = 60°$.
8. Triangle ABD and triangle CBA share angle B, and angle ADB = angle BAC = 90° (both right angles). Two matching angles are enough — the two triangles are similar by AA.

Full solutions: **manthika.com/learn/math10-ch06**

BACK
The AA similarity criterion · p. 192

Further practice

PRACTICE SET

9. PRACTICE Triangle ABC has angle $A = 55°$ and angle $B = 50°$. Triangle DEF has angle $D = 55°$ and angle $F = 75°$. Are the two triangles similar? Name the criterion, and state the correspondence.

15. PRACTICE Triangle ABC is right-angled at B. M is a point on ray AB and P is a point on ray AC, so that triangle AMP is right-angled at M. Show that triangle ABC is similar to triangle AMP, and that $CA : PA = BC : MP$.

10. PRACTICE Triangle ABC has sides 4 cm, 5 cm and 6 cm. Triangle PQR has sides 8 cm, 10 cm and 12 cm. Triangle XYZ has sides 8 cm, 9 cm and 10 cm. Which one of triangle PQR and triangle XYZ is similar to triangle ABC? Name the criterion.

11. PRACTICE Triangle ABC has $AB = 5$ cm, $AC = 7$ cm and angle $A = 40°$. Triangle DEF has $DE = 10$ cm, $DF = 14$ cm and angle $D = 40°$. Are the two triangles similar? Name the criterion.

12. PRACTICE Triangle ABC is similar to triangle PQR, with angle $A = 80°$ and angle $C = 45°$. Find angle Q.

13. PRACTICE Triangle ABC is similar to triangle DEF, with $AB = 6$ cm, $DE = 9$ cm and $BC = 8$ cm. Find EF.

14. PRACTICE Triangle ABC is similar to triangle DEF, with correspondence A to D, B to E, C to F. Which of these ratios must equal $AB : DE$?

A. $BC : EF$

B. $BC : DE$

C. $AC : DE$

D. $DE : AB$

16. PRACTICE In triangle ABC, D is the midpoint of BC, and E lies on AC so that DE is parallel to AB. Show that triangle CDE is similar to triangle CBA, and find the ratio $CD : CB$.

17. PRACTICE In triangle ABC, D lies on AB and E lies on AC, with angle ADE = angle ACB. Show that triangle ADE is similar to triangle ACB.

18. PRACTICE In triangle ABC, AD is the median to BC. In triangle PQR, PM is the median to QR. Given that $AB : PQ = BC : QR = AD : PM$, show that triangle ABC is similar to triangle PQR.

19. PRACTICE A vertical pole 4 m tall casts a shadow 3 m long on the ground. At the same time, a nearby tower casts a shadow 15 m long. Find the height of the tower.

ANSWERS

9. Yes — angle $C = 180° - 55° - 50° = 75°$, and angle $E = 180° - 55° - 75° = 50°$. So angle A = angle D, angle B = angle E and angle C = angle F, and the two triangles are similar by AA, with correspondence A to D, B to E, C to F.
10. Triangle PQR — its sides are each exactly twice the matching side of triangle ABC ($4 : 8 = 5 : 10 = 6 : 12 = 1 : 2$), so the two are similar by SSS. Triangle XYZ is not similar to triangle ABC: the ratios $4 : 8$, $5 : 9$ and $6 : 10$ are not all equal.
11. Yes — $AB : DE = 5 : 10 = 1 : 2$ and $AC : DF = 7 : 14 = 1 : 2$, with the included angle A = angle D = 40°. The two triangles are similar by SAS.
12. angle $Q = 55°$ **13.** $EF = 12$ cm **14.** A — $BC : EF$.
15. Angle A is common to both triangles, since M lies on ray AB and P lies on ray AC. Angle B = angle M = 90°. Two matching angles are enough, so triangle ABC is similar to triangle AMP by AA, with correspondence A to A, B to M, C to P. Corresponding sides are then in the same ratio, so $CA : PA = BC : MP$.
16. Angle C is common to both triangles. Since DE is parallel to AB, angle CDE = angle CBA (corresponding angles). Two matching angles are enough, so triangle CDE is similar to triangle CBA by AA. Since D is the midpoint of BC, $CD : CB = 1 : 2$.
17. Angle A is common to both triangles, since angle DAE and angle CAB are the same angle. Angle ADE = angle ACB is given. Two matching angles are enough, so triangle ADE is similar to triangle ACB by AA, with correspondence A to A, D to C, E to B.
18. Since AD and PM are medians, $BD = 1/2BC$ and $QM = 1/2QR$, so $BD : QM = BC : QR$ too. Triangle ABD has all three sides (AB, BD, AD) in the same ratio as the matching sides (PQ, QM, PM) of triangle PQM, so triangle ABD is similar to triangle PQM by SSS. This gives angle B = angle Q. In triangles ABC and PQR, $AB : PQ = BC : QR$ is given, and the included angle B = angle Q has just been shown, so triangle ABC is similar to triangle PQR by SAS.
19. $h = 20$ m — the tower is 20 m tall.
Full solutions: **manthika.com/learn/math10-ch06**