CHAPTER 9

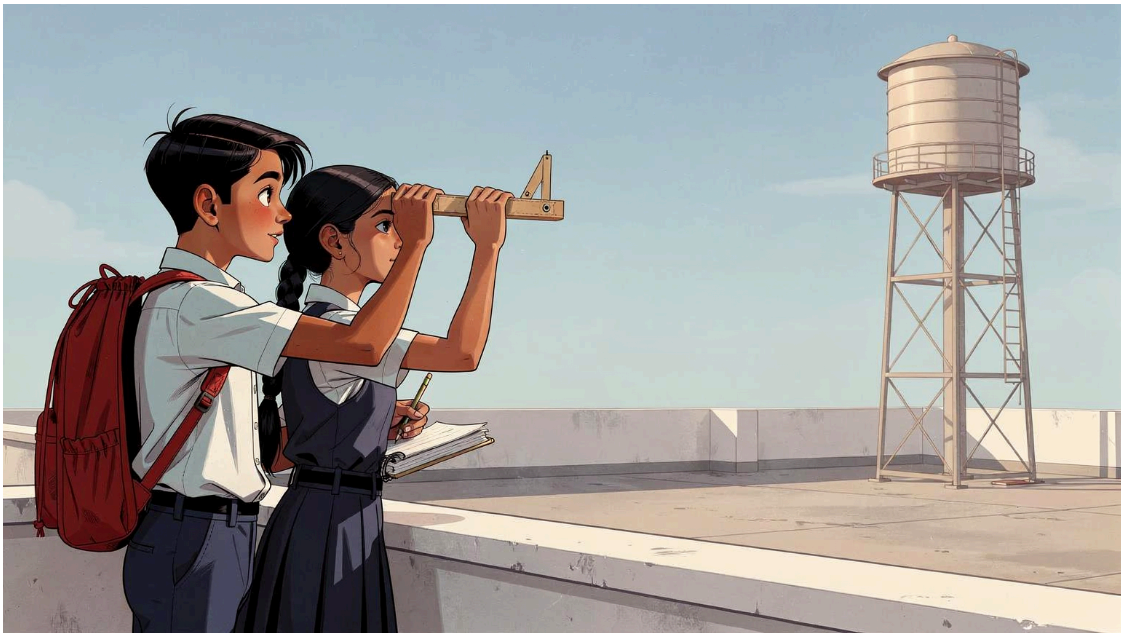
9 Some Applications of Trigonometry

WHAT THIS CHAPTER BUILDS

You will learn to find a height or a distance you cannot reach by measuring one angle and one length and applying a trigonometric ratio.

TRIGONOMETRY UNIT, WORD PROBLEMS

IN THIS CHAPTER



Nila sights the top of the roof's water tank with Dhruv's help, and one angle with one distance gives its height without a climb.

Getting started

Measuring what cannot be reached

A STORY

The tank on the roof

The water tank on the school roof has a leak. The plumber has to bring a ladder, so he asks how tall the tank is. Nobody knows.

After class, Nila and Dhruv go up to the roof. The tank stands at the far end, high on four tall legs. It is too high to climb, and no tape reaches up there.

Dhruv has made a sighting tool from two strips of wood. He holds one end steady. Nila looks along its top edge at the top of the tank. The tool gives the angle: $30°$ above the level of her eye.

“That is one number,” Dhruv says. “We still need the distance to the tank.”

They count their steps across the roof to the tank’s legs. It is about 20 m.

“One angle and one distance,” Nila says. “Can that really give the height?”

It can. How would you get the height from just those two numbers?

A tower too tall to climb. A river too wide to cross. A kite string too far to reach. Each one still yields to arithmetic. We never touch the object itself.

Stand at a known distance from it. Look up or down at a measured angle. The tower or the river becomes one right triangle. You already know how to solve it. The tool is the trigonometric ratio from the last chapter: $\sin$, $\cos$ and $\tan$. Now it aims at a real angle and a real distance, not an abstract one.

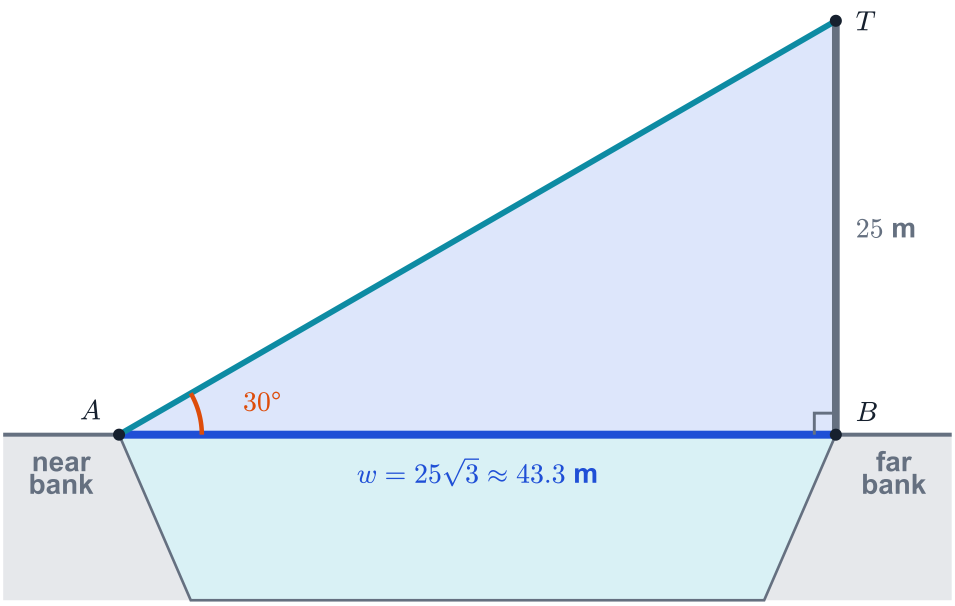
Nothing new is learned about $\sin$, $\cos$ or $\tan$ here. Only the aim changes.

Your turn: stand 14 m from a chimney’s foot and look up at $30°$ to its top. Name the triangle’s three sides. (Answer: the ground distance, the chimney’s height, and the line of sight joining them.)

RULE
Draw and label the right triangle first. The angle of elevation or depression only means something once it exists.

KEY
A tower too tall to climb and a river too wide to cross both become one right triangle, once the angle is named.

A river's width, from the near bank



The unknown lies flat across the water, not standing in the air: the side next to the one measurable angle.

A SCHOLAR INDIA REMEMBERS

Brahmagupta worked in a school of astronomy that predicted eclipses with shadow geometry. Nobody could climb up to the sun or the moon. The measuring was done from the ground, with lengths and angles. You will work the same way with a tower. Measure a length and an angle from the ground. Then a right triangle gives you the height.

Brahmagupta · the brahminy kite

IN THE WILD

Stand fifty paces from a building you cannot climb. Point a phone's angle app at its top and read the angle of elevation. That angle and the paced distance give the building's height, with no tape near the roof. A river too wide to wade is measured the same way, from one bank, without crossing it.

CHECK YOURSELF

1. This chapter finds the height of a tower or the width of a river without touching either. What two things does the method actually need?

- **A.** two distances, never an angle
- **B.** two angles, never a distance
- **C.** the answer itself, checked afterward
- **D.** an angle and a distance

2. Why does measuring an angle and a distance give the height of something you cannot reach?

- **A.** they are two parts of one right triangle
- **B.** the angle alone fixes the height, regardless of distance
- **C.** the distance alone fixes the height, regardless of angle
- **D.** a second, unmeasured distance is secretly compared

1 more in this set online.

IN EVERY CHAPTER

Before you start

DO THIS FIRST

You cannot climb the tower or cross the river. Stand where you are, and measure one angle. Try each check below.

- **Alternate angles** (Class 8, Ganita Prakash Part 1, page 99).
  Here: the top's depression angle equals the bottom's elevation angle.
  Check: Angles on a line: $180° - 125° = 55°$, its alternate angle too.
- **Angle sum of a triangle** (Class 8, Ganita Prakash Part 1, page 94).
  Here: a right triangle's other two angles add to $90°$, so elevation is acute.
  Check: A right triangle with angles $90°$ and $60°$ has a third angle of $30°$.
- **Multiplying both sides** (Class 8, Ganita Prakash Part 1, page 170).
  Here: isolating l in $\sin 30° = 8/l$ multiplies both sides by l.
  Check: $x/4 = 3$ gives $x = 12$, multiplying both sides by 4.
- **Solving a linear equation** (Class 9, Ganita Manjari, page 20).
  Here: the flagpole's height is the x found by solving its equation.
  Check: $5x = 20$ gives $x = 4$.
- **Irrational numbers** (Class 9, Ganita Manjari, page 53).
  Here: the tower's height, $12 \cdot \sqrt{3}$ m, is exactly such a number.
  Check: $12\sqrt{3}$ is about 20.8 m, since $\sqrt{3}$ is about 1.732.

If any of these felt new, read the page named before going on.

The line of sight

Every heights-and-distances problem starts the same way. Name one line precisely. The line of sight runs from the observer's eye to the exact point you are looking at.

Nothing else counts as the line of sight. Not a line to the object's base. Not a line to some other point on it. Look at the top of a tower. You see the line of sight end there, nowhere else on it.

Every angle of elevation or depression is measured from this line. Get its endpoint wrong, and your whole triangle goes wrong with it.

Your turn: someone looks at a bird resting on a wire above them. Name the two points the line of sight joins. (Answer: the observer's eye and the bird, since a line of sight runs from the eye to the object seen.)

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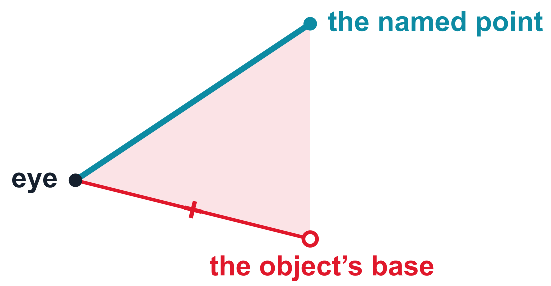
RULE
The line of sight always runs to the specific point named in the problem, never to the object's base or any other point on it.

KEY
One line, eye to the exact point being looked at, nothing more.

The line of sight is ONE line



One ray, in teal, reaches the named point; the other, crossed out in red, wrongly points at the object's base.

CHECK YOURSELF

3. The line of sight, in this chapter's diagrams, is

- **A.** the line from the eye to the point viewed
- **B.** the horizontal line at the observer's eye
- **C.** the vertical line up the tower or pole
- **D.** the ground between the observer and the object

4. In the triangle drawn for a heights problem, why is the line of sight usually the slanted (hypotenuse) side, not one of the two legs?

- **A.** it is always the shortest of the three sides
- **B.** it lies flat along the horizontal, like the ground
- **C.** it runs straight from eye to object
- **D.** it always equals the vertical side in length

1 more in this set online.

The angle of elevation

Look up at a point above eye level. The angle between your line of sight and the horizontal is the angle of elevation.

That angle is always measured from the horizontal. It is never measured from the vertical pole, tower or cliff face nearby. Look straight ahead, and you trace the horizontal. Tilt your head up to the object, and you trace the line of sight instead. The angle of elevation is the gap between the two.

Picture a kite above the horizontal, a plane overhead, the top of a building seen from the street. Each one gives an angle of elevation. Each is read off the same horizontal.

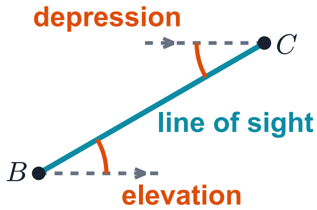
Your turn: someone on the ground looks up at a lit window near a building's top. Name the angle this makes with the horizontal. (Answer: an angle of elevation, since the window sits above eye level.)

TRY
Looking up at a kite, is the angle from the horizontal an angle of elevation or depression?

Answer: Elevation. The kite is above the horizontal.

KEY
Looking up at something above eye level, measured from the horizontal.

One line of sight, two names for its angle



A line of sight joins two observers; each measures from their own horizontal, so elevation at the bottom equals depression at the top.

A SCHOLAR INDIA REMEMBERS

Every angle here is measured from something fixed. What is it? It is the level line through your eye. Look straight ahead. That line is 0°. Now raise your eyes to the top of the tree. The turn from the level line to the top is the angle of elevation.

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CHECK YOURSELF

5. The angle of elevation is the angle between the line of sight and

- **A.** the vertical, when looking upward
- **B.** the horizontal, when looking upward
- **C.** the horizontal, when looking downward
- **D.** the ground, at the foot of the object

6. An observer looks up at the top of a tower. Which angle does this line of sight make?

- **A.** an angle of depression
- **B.** an angle of elevation
- **C.** a right angle
- **D.** an alternate angle

1 more in this set online.

The angle of depression

Look down at a point below eye level. The angle between your line of sight and the horizontal is the angle of depression.

Like elevation, this angle is measured from the horizontal at the observer's eye. It is never measured from the cliff, tower or wall the observer stands on. Picture a boat seen from a cliff top. Picture a car seen from a tall building's roof. Both give an angle of depression, off that same horizontal.

The only difference between depression and elevation is which way you look, up or down, from that same horizontal.

Your turn: a person on a balcony looks down at a car parked below. Name the angle this makes with the horizontal. (Answer: an angle of depression, since the car sits below eye level.)

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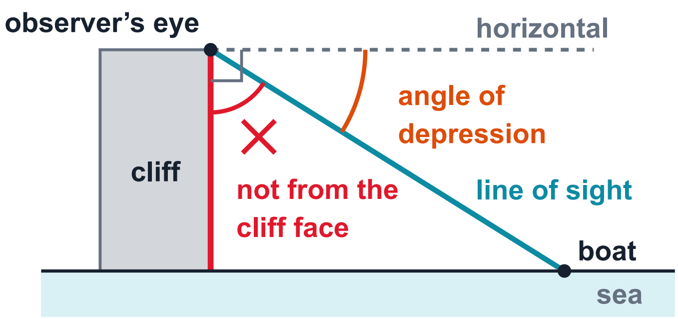
TRY
Looking down from a cliff at a boat, is the angle measured from the horizontal or the cliff's vertical face?

Answer: The horizontal. Never the cliff's vertical face.

KEY
Looking down at something below eye level, measured from the horizontal.

Depression is not read from the cliff face



One line of sight, two lines it could be read from: depression is the angle off the horizontal at the eye, never off the cliff face.

A line of sight from a cliff-top observer down to a boat gives the angle of depression, measured from the horizontal.

CHECK YOURSELF

7. The angle of depression is the angle between the line of sight and

A. the vertical, when looking downward

B. the horizontal, when looking downward

C. the horizontal, when looking upward

D. the ground, at the object being viewed

8. From the top of a lighthouse, a lookout spots a boat on the sea below. The angle this line of sight makes with the horizontal at the lookout's eye is

A. an angle of elevation

B. an angle of depression

C. the boat's own angle of tilt

D. a right angle, since the lighthouse is vertical

1 more in this set online.

The ideas

One angle, two directions

Elevation and depression are not two different kinds of angle. They are the same kind. We read both off the same reference line: the horizontal at the observer's eye.

Look upward to a point above that line, and you call it an angle of elevation. Look downward to a point below it, and you call it depression instead. The horizontal itself never changes. Only the direction of the line of sight does.

Two different observers looking at two different objects still each use their own horizontal. The reference line sits at each observer's own eye. It is never borrowed from the other observer's line of sight.

Your turn: one person on the ground looks up at a plane. A passenger inside the plane looks down at that person. Name the angle each one measures. (Answer: elevation for the one on the ground, depression for the one in the plane, both from their own horizontal.)

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TRY
Two different observers, two different objects. Do elevation and depression use two different reference lines?

Answer: No. Both use the same horizontal, each at its own observer's eye.

RULE
Never measure from the vertical. The reference is always the horizontal, no matter which way the observer looks.

CHECK YOURSELF

9. Elevation and depression are the same kind of angle because both are measured from

- **A.** the vertical pole or tower
- **B.** the ground at the object's foot
- **C.** the horizontal at the observer's eye
- **D.** whichever line makes the angle come out smaller

10. If the only difference between elevation and depression is the direction of looking, what stays exactly the same between them?

- **A.** the horizontal reference line itself
- **B.** the size of the angle
- **C.** which object is being viewed
- **D.** the observer's own height above the ground

1 more in this set online.

Depression up top, elevation down below

One equality follows directly from how elevation and depression are defined. The angle of depression at a high point equals the angle of elevation at the low point below it.

The two horizontals are parallel. One sits at the high point, one at the low point. Both are simply horizontal. The line of sight joins the two points. It cuts across both horizontals, so it acts as a transversal. **A transversal crossing two parallel lines makes equal alternate angles.** The angle of depression at the top and the angle of elevation at the bottom are the same angle, named from opposite ends.

You use this equality to move a given angle down. It goes from the top of a tall figure to the base of the right triangle we are actually solving. Stand at the top of a cliff and look down at a boat: the angle of depression is $30°$. The boat's own angle of elevation, looking up at the cliff top, is $30°$ too.

Your turn: from a watchtower, the angle of depression to a hiker below is 45°. Name the angle of elevation from the hiker up to the watchtower. (Answer: 45°, since the two horizontals are parallel, so the two angles are alternate angles and equal.)

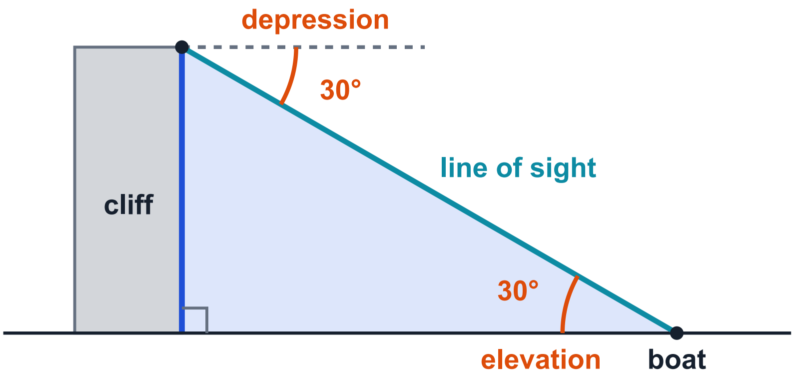
TRY
From a cliff top, the depression to a boat is 30°. What is the boat's own elevation to the cliff top?

Answer: 30°, the same angle, by alternate angles.

RULE
Use this to move the given angle from the top of a tall figure down to the base of the triangle actually being solved.

KEY
Depression from above equals elevation from below, by alternate angles.

The angle comes down to the boat



Two parallel horizontals, one line of sight across them: the 30° of depression at the top is the 30° of elevation at the boat, so the given angle sits at the base of the right triangle.

Alternate angles carry the given angle from the observer to the far end of the base, where the working needs it.

CHECK YOURSELF

11. The angle of depression measured from a tall building's top, looking at a point on the ground, equals

- **A.** half the angle of elevation measured from that ground point
- **B.** the angle of elevation measured from that ground point
- **C.** the complement of the elevation angle from that ground point
- **D.** unrelated to the elevation angle from that ground point

12. Why are the depression angle at the top and the elevation angle at the ground equal, rather than just related somehow?

- **A.** both angles are measured against the same vertical tower
- **B.** the triangle happens to be isosceles
- **C.** parallel horizontals cut by one transversal
- **D.** the two points happen to be the same distance from the tower

2 more in this set online.

Turning words into a triangle

Every heights-and-distances problem starts with one right triangle. It has three sides. A vertical side. A horizontal side. The line of sight joins them as the third side.

Let us draw the triangle together. We mark the right angle first, where the vertical side meets the horizontal side. That is where the tower, pole or building meets level ground. Next we mark the given angle, elevation or depression, at the observer. Then we label the side the problem gives, and the side it asks for.

Label opposite, adjacent and hypotenuse relative to the marked angle, never to the shape of the triangle. The vertical side is opposite the angle in some problems. It is adjacent to it in others. The marked angle decides the labels every time.

Your turn: stand 9 m from a pole's foot and look up at $60°$ to its top. Find the pole's height. (Answer: $9 \cdot \sqrt{3}$ m, since $\tan 60° = h/9$ gives $h = 9 \cdot \sqrt{3}$.)

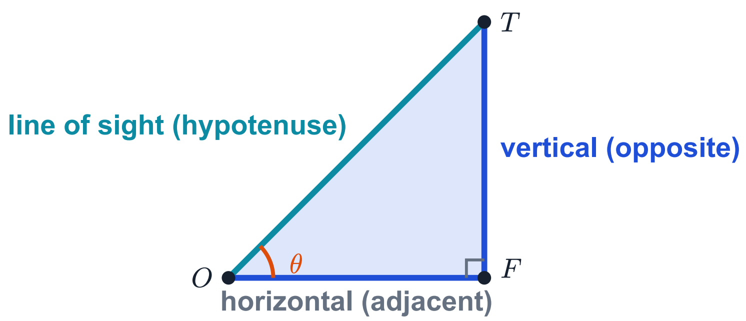
TRY
In the triangle, where does the right angle go?

Answer: Where the vertical side meets the horizontal side.

RULE
Label opposite, adjacent and hypotenuse relative to the MARKED angle, never to the shape of the triangle.

KEY
Vertical side, horizontal side, line of sight, the same three sides every time.

Name the three sides, mark the two angles



The right angle sits where the vertical meets the horizontal, and the marked angle at the observer names opposite, adjacent and hypotenuse.

CHECK YOURSELF

13. Every heights-and-distances triangle in this chapter has three sides: a vertical side, a horizontal side, and

- **A.** a second vertical side
- **B.** the angle of elevation itself
- **C.** the line of sight
- **D.** a line joining the two given distances

14. In this triangle, where is the right angle marked?

- **A.** where the line of sight meets the horizontal side
- **B.** where the vertical side meets the horizontal side
- **C.** where the line of sight meets the vertical side
- **D.** wherever makes the two legs come out equal

1 more in this set online.

The observer's own height counts

An observer rarely stands with an eye exactly at ground level. Say the observer has some height of their own. Then the triangle's vertical side runs from EYE level, not ground level, up to the point being viewed.

Solve that triangle yourself, and we get only the part of the height above the observer's eye. We still have to add the observer's own height back afterward. Only then do we reach the object's full height above the ground.

Folding the observer's height in too early changes what the triangle's vertical side measures. Take an observer 1.5 m tall who finds a triangle side of 18.5 m up to a chimney top. The chimney's height is $18.5 + 1.5 = 20$ m, not 18.5 m.

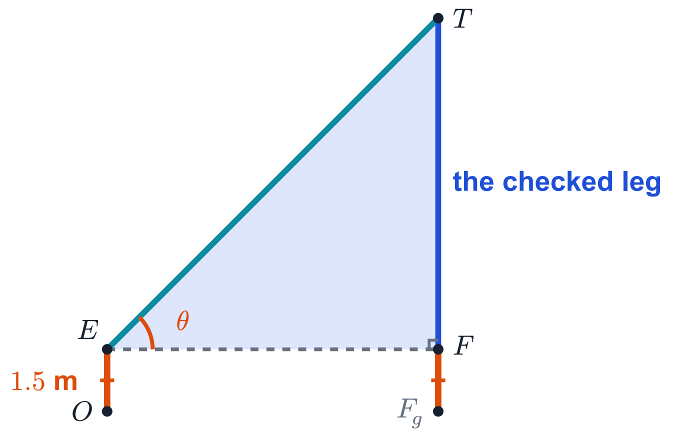
Your turn: an observer 1.4 m tall stands 10 m from a pole. The angle of elevation to its top is $45°$. Find the pole's full height. (Answer: 11.4 m, since $\tan 45° = 1$ gives a 10 m rise above eye level, plus the observer's 1.4 m.)

TRY
An observer 1.5 m tall computes a triangle side of 18.5 m up to a chimney top. What is the chimney's height?

Answer: $18.5 + 1.5 = 20$ m.

KEY
The triangle starts at the eye, not the feet. Add the observer's height back at the end.

The triangle starts at the eye



The triangle runs only from eye level to the top; the observer's own height, tied to it, is added back afterward.

A SCHOLAR INDIA REMEMBERS

Your answer says the tower is 30 m tall. The builder says 31.5 m. Who is wrong? Check your own working first. Did you measure the angle from your eye, 1.5 m above the ground? Then your triangle starts 1.5 m up. Add it on: $30 + 1.5 = 31.5$. Now your answer matches the tower.

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CHECK YOURSELF

15. When an observer standing on top of something looks at a distant object, the triangle's vertical side should be measured from

- **A.** the ground the observer stands on
- **B.** the midpoint between the observer's eye and feet
- **C.** the observer's eye level, not the ground
- **D.** the top of whatever the observer is standing on

16. After solving the triangle for the vertical distance from eye level up to the object, why must the observer's own height still be added?

- **A.** to correct for a rounding error in the trig ratio
- **B.** because the angle was measured incorrectly
- **C.** to make the final answer come out to a whole number
- **D.** the triangle stops short, at eye level

2 more in this set online.

Which ratio connects the two sides

Once we draw and label the right triangle, one question decides the ratio to use. Which two sides does the problem actually connect, relative to the marked angle?

Look at the triangle. If both the known and the wanted side are legs, opposite and adjacent, we use $\tan$ or $\cot$ to connect them. If one of them is the hypotenuse instead, we use $\sin$, $\cos$, or their reciprocals.

If the hypotenuse is unknown and one leg is known, use $\sin$ or $\cos$. $\tan$ never involves the hypotenuse, so it can never be the answer there.

Your turn: a slide is 5 m long and meets the ground at $30°$. Find the horizontal distance it covers. (Answer: 4.33 m, since the length is the hypotenuse and the ground distance is adjacent, so $\cos 30° = d/5$.)

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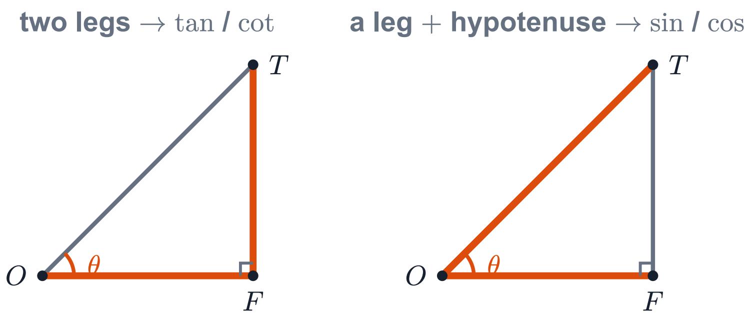
TRY
The hypotenuse is unknown and one leg is known. Which ratio applies?

Answer: $\sin$ or $\cos$. Never $\tan$, since $\tan$ never involves the hypotenuse.

RULE
Pick the ratio only after naming the two sides. Guessing the ratio first wastes the whole set-up.

Two legs, or a leg and the hypotenuse



The same triangle appears twice: once with its two legs highlighted, once with a leg and the hypotenuse, showing which pair sets the ratio.

CHECK YOURSELF

17. If both the known side and the wanted side are legs of the right triangle (not the hypotenuse), which ratio connects them?

- **A.** sin or cos
- **B.** only sin
- **C.** any of the six ratios equally
- **D.** tan or cot

18. Why does a problem giving the hypotenuse and one leg call for sin or cos, rather than tan?

- **A.** tan only works for angles under 45°
- **B.** tan gives a less accurate answer than sin or cos
- **C.** the hypotenuse can always be swapped for a leg
- **D.** tan never involves the hypotenuse

2 more in this set online.

Two triangles, one shared side

Some problems hand over two angles instead of one. These need two right triangles to answer. The syllabus allows at most two, usually sharing one side: the same base point, or the same vertical line.

We write one trigonometric equation from each triangle. Then we solve the two equations together for the unknown height or distance. Neither equation alone pins the answer down. Only both together do.

A third right triangle is outside this syllabus's cap. If a problem seems to need one, the set-up has been misread.

Your turn: a 6 m pole stands between two people on level ground, in line with its foot. One sees its top at 45°, the other at 60°. How far apart are they? (Answer: $6 + 2\sqrt{3}$ m, about 9.46 m, since the two distances are 6 m and $6/\sqrt{3}$ m.)

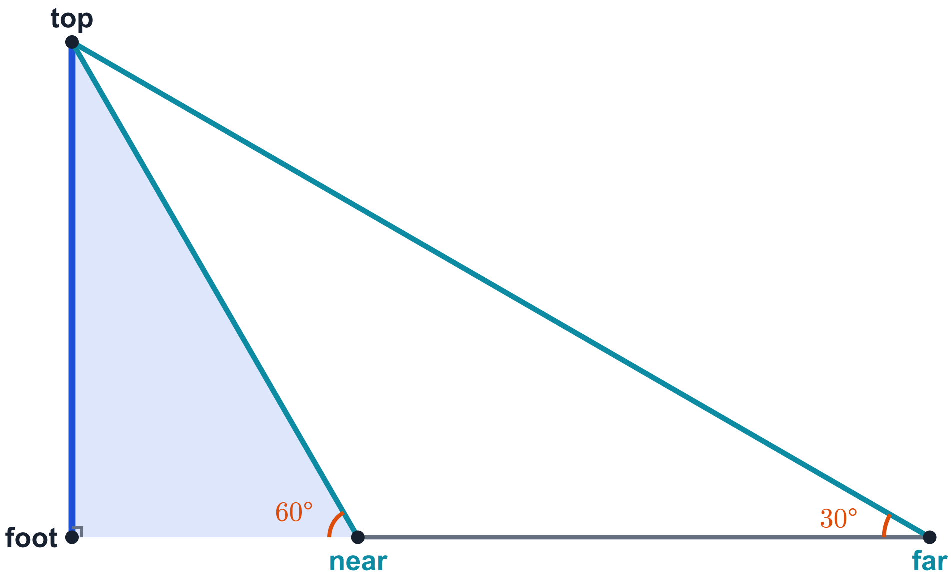
CAUTION
A third triangle is outside the syllabus cap. If a problem seems to need one, re-read the set-up.

TRY
A problem seems to need a third right triangle. What does that mean?

Answer: The set-up has been misread. At most two triangles are ever needed here.

KEY
One equation per triangle, then combine them. Never solve one triangle alone and stop.

One tower, two vantage points



One tower is seen from two ground points, sixty degrees from the near one and thirty from the far, sharing one vertical side.

A SCHOLAR INDIA REMEMBERS

One angle gave you one triangle, with two unknowns in it. What now? Walk closer and measure again. The second angle gives a second triangle. Both triangles share the same height. Each triangle gives one equation. Two equations find both unknowns.

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CHECK YOURSELF

19. This chapter allows at most two right triangles in one problem, usually sharing

- **A.** the same angle at every vertex
- **B.** the same base point or vertical line
- **C.** the same hypotenuse
- **D.** no side at all — they are entirely separate

20. Why does a two-triangle problem need one trigonometric equation from each triangle, rather than one equation overall?

- **A.** the problem always has two unknown angles
- **B.** one triangle's answer is always wrong
- **C.** each triangle has its own angle
- **D.** to check the same answer twice, for accuracy

1 more in this set online.

Common mistakes

Elevation is not measured from the vertical

A common mistake treats the angle as measured from the vertical pole or tower itself, not the horizontal. It is an easy slip. The pole stands right there in the picture, and it feels like the natural reference.

But the reference line is always the horizontal at the observer's eye. It is never the vertical structure beside it. The two references differ by a full $90°$. Confusing them does not give a slightly wrong answer. It gives a completely different triangle.

For a tower 15 m away with elevation $60°$, the correct height is $15 \cdot \sqrt{3}$ m. Mistake the vertical for the reference, and it secretly swaps in $30°$ for $60°$, giving only $5 \cdot \sqrt{3}$ m: a different, wrong height.

Your turn: before you solve any elevation or depression problem, name the reference line out loud. (Answer: the horizontal at the observer's eye, never the pole, tower or cliff face beside it.)

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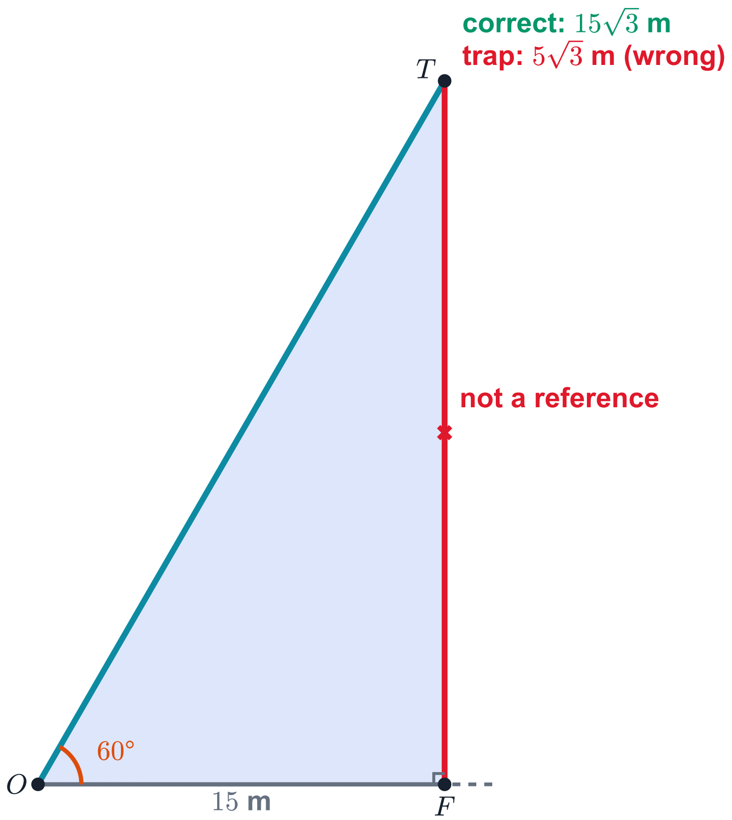
One angle, two directions · p. 296

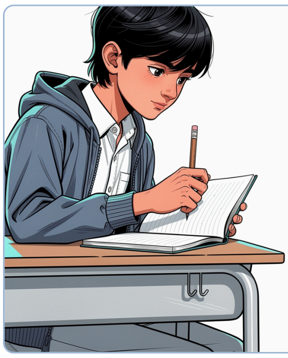
CAUTION
The reference line is always horizontal, never the pole, tower, or cliff face itself.

KEY
The same trap catches elevation and depression both, always horizontal, never vertical.

Elevation is not measured from the vertical



Only the horizontal angle is real; the crossed-out tower face gives the wrong height, five root three metres instead of fifteen.



FIRST TRY

~~Tower 15 m away, angle of elevation 60°. I put the 60° at the top, against the tower, and got $15 \cdot \tan 30° = 5\sqrt{3}$ m.~~

SECOND LOOK

An angle of elevation opens from the horizontal at my eye, never from the tower. So the 60° sits at my eye, and the height is $15 \cdot \tan 60° = 15\sqrt{3}$ m.

Find the horizontal first. Every angle of elevation opens from it.

A BOAT'S DISTANCE FROM A CLIFF-TOP DEPRESSION

WEAK

A boat is seen from the top of a 60 m cliff at an angle of depression of 30°. Measure that angle from the cliff face instead of the horizontal, and the cliff's height sits adjacent to it: $60 \cdot \tan 30° = 20 \cdot \sqrt{3}$ m. But a 30° depression is a shallow look down. The boat should sit farther out than the cliff is tall, not closer. An answer shorter than the cliff's own height means the angle was taken from the wrong line.

STRONG

The angle of depression, 30°, is always measured from the horizontal at the observer's eye, never from the cliff face. In the true triangle, the cliff's 60 m height is opposite that angle, and the distance is adjacent. $\tan 30° = 60/x$, so $x = 60 \cdot \sqrt{3}$ m. A shallow 30° look down does put the boat well beyond the cliff's own height, so the size now makes sense.

CHECK YOURSELF

21. A student says: "the angle of elevation is measured between the line of sight and the tower itself, not the horizontal." For a tower $15m$ away with an elevation of $60°$, what does this mistake actually give?

- **A.** the same height, $15\sqrt{3}$ m, since the ratio still works out
- **B.** an undefined result, since the vertical has no angle to the tower
- **C.** a taller tower than the true height, since the angle looks bigger
- **D.** $5\sqrt{3}$ m, instead of the true $15\sqrt{3}$ m

22. A tower is $18m$ from an observer, and the true angle of elevation of its top, measured from the horizontal, is $30°$. A student instead measures from the tower's own vertical line and plugs the same $30°$ into $\tan$ from that wrong reference. What wrong height results?

- **A.** $6\sqrt{3}$ m — the same as the true height
- **B.** 18 m exactly, using the distance with no ratio applied
- **C.** a smaller height than the true one, since 30° is shallow
- **D.** $18\sqrt{3}$ m, far more than the true $6\sqrt{3}$ m

3 more in this set online.

Worked examples

WORKED EXAMPLE · **A tower's height from one triangle**

1. a tower on level ground has its top at an angle of elevation of 60°, seen from a point 12 m from its foot

the given information, one known distance and one known angle

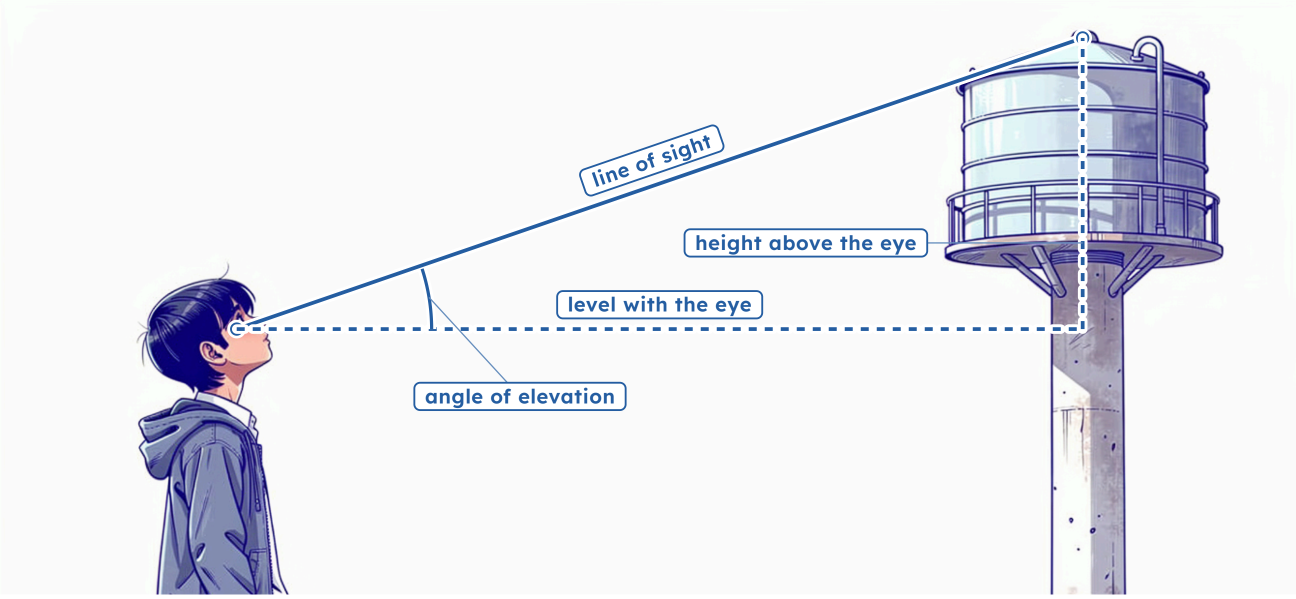
WORKED EXAMPLE · A tower's height from one triangle

2.	the 12 m distance is adjacent to the marked angle, and the tower's height is opposite it	*both are legs of the right triangle, so* tan *connects them*
3.	$\tan 60° = h/12$, so the height is $12 \cdot \tan 60° = 12 \cdot \sqrt{3}$ m	$\tan 60° = \sqrt{3}$, *from the standard-angle table in Chapter 8*
4.	$(12 \cdot \sqrt{3})/12 = \sqrt{3}$, exactly the value the tangent table gives for 60°	CHECK ***Check the height by dividing it back by the known base.*** *The* 12*s cancel and leave* $\sqrt{3}$*, exactly the tangent table's value for* 60°.

TRY
A tower stands 9 m from an observer, elevation 60°. Find the height.

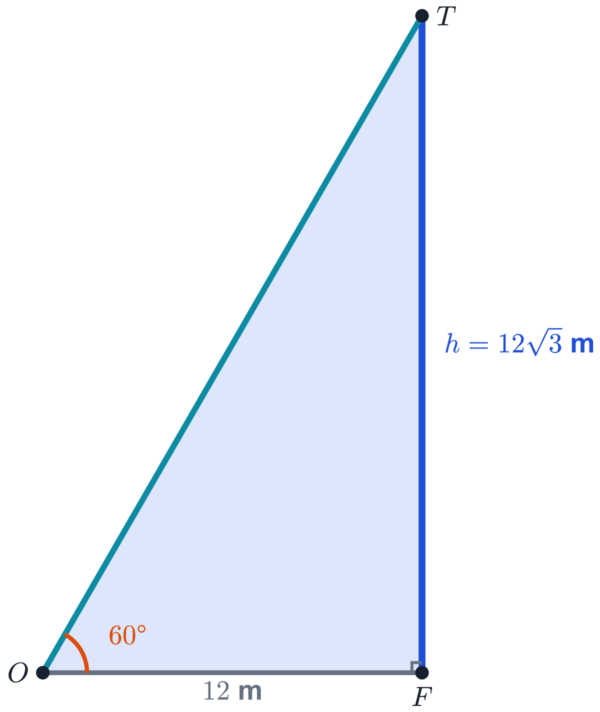
Answer: Since $\tan 60° = \sqrt{3}$, the height is $9 \cdot \sqrt{3}$ m.

RULE
Name the two known quantities first, distance 12 m, angle 60°, then pick tan because both are legs.



Siddharth's eye, the tank's top and a point on the tower level with his eye form one right triangle.

A tower's height from one triangle



Twelve metres and sixty degrees are both parts of one triangle, and the tangent of sixty degrees turns one into the other.

A HEIGHT FROM ONE DISTANCE AND ONE ANGLE

1. **Draw the triangle** Draw a right triangle. The height is the vertical side, the known distance is the horizontal side, and the right angle sits at the foot.
2. **Mark the angle** Mark the given angle at the observer's point. For a tower 12 m away, that angle is 60°.
3. **Name the sides** Both the known distance and the wanted height are legs of the triangle, not the hypotenuse.
4. **Pick the ratio** Two legs need tan. $\tan 60° = h/12$, so $h = 12 \cdot \sqrt{3}$ m.
5. **Check the size** An angle over 45° means the height is more than the distance. $12 \cdot \sqrt{3}$ is about 20.8 m, more than 12 m, so the size is right.

CHECK YOURSELF

23. A tower stands $12m$ from an observer, with an elevation angle of $60°$ to its top. Which single equation gives the tower's height h?

- **A.** $\sin 60° = h/12$
- **B.** $\tan 60° = 12/h$
- **C.** $\cos 60° = 12/h$
- **D.** $\tan 60° = h/12$

24. Using the same method, a flagpole stands $9m$ from an observer whose angle of elevation to its top is $60°$. What is the flagpole's height?

- **A.** $9/\sqrt{3}$ m
- **B.** 9 m, the same as the given distance
- **C.** 18 m, double the given distance
- **D.** $9\sqrt{3}$ m

1 more in this set online.

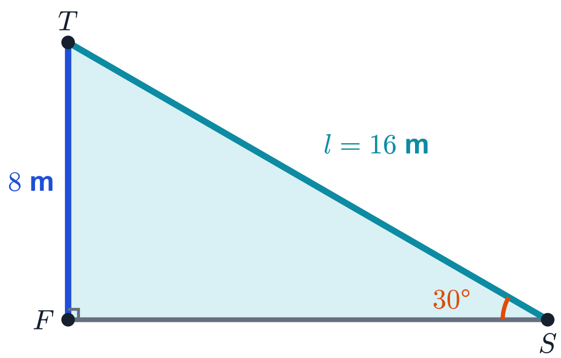
WORKED EXAMPLE · A rope's length using sine

1.	a rope ties the top of a pole 8 m tall to a point on the ground, making an angle of 30° with the ground	*the given information. Let the rope's length be l*
2.	the pole's 8 m height is opposite the marked angle, and the rope is the hypotenuse	*the rope runs from the ground to the pole's top, so it is the longest side, the hypotenuse*
3.	$\sin 30° = 8/l$, and $\sin 30° = 1/2$, so $l = 16$ m	*opposite over hypotenuse is* sin. *Solving $1/2 = 8/l$ for l gives 16*
4.	$8/16 = 1/2$, exactly $\sin 30°$ again	CHECK ***Check the length by substituting it back into the ratio.*** *Eight over sixteen simplifies to one half, the same value* $\sin 30°$ *has.*

BACK
Which ratio connects the two sides · p. 301

RULE
Hypotenuse unknown, one leg known. That is exactly when sin or cos applies, not tan.

A rope's length, found by sine



The rope is the hypotenuse here, so sine, not tangent, connects the eight metre pole to the rope's sixteen metre length.

CHECK YOURSELF

25. A rope ties the top of an $8m$ pole to a ground point, making $30°$ with the ground. Which ratio finds the rope's length l?

- **A.** $\sin 30° = 8/l$
- **B.** $\tan 30° = 8/l$
- **C.** $\cos 30° = 8/l$
- **D.** $\sin 30° = l/8$

26. Using the same method, a rope ties the top of a $10m$ pole to the ground, making $30°$ with the ground. What is the rope's length?

- **A.** 5 m
- **B.** $10\sqrt{3}$ m
- **C.** 10 m, the same as the pole
- **D.** 20 m

1 more in this set online.

WORKED EXAMPLE · Depression with the observer's own height

	Step	Reason
1.	a person 1.6 m tall stands on a tower 18.4 m high, so the eyes sit 20 m above the ground	$18.4 + 1.6 = 20$. *We add the observer's own height to the tower before drawing the triangle*
2.	a car is seen at an angle of depression of 45°	*the given angle, measured from the observer's horizontal at eye level*
3.	the 20 m eye height is opposite the marked angle at the car's own elevation angle, which equals 45° by alternate angles	*the angle of depression from the eye equals the angle of elevation from the car*
4.	$\tan 45° = 20/d$, and $\tan 45° = 1$, so $d = 20$ m	*the horizontal distance equals the eye height exactly when the ratio is* 1
5.	$20/20 = 1$, exactly $\tan 45°$	CHECK ***Check the distance by dividing it back by the eye height.*** *Twenty over twenty is one. The car sits exactly as far out as the eyes are high, exactly what a 45° angle means.*

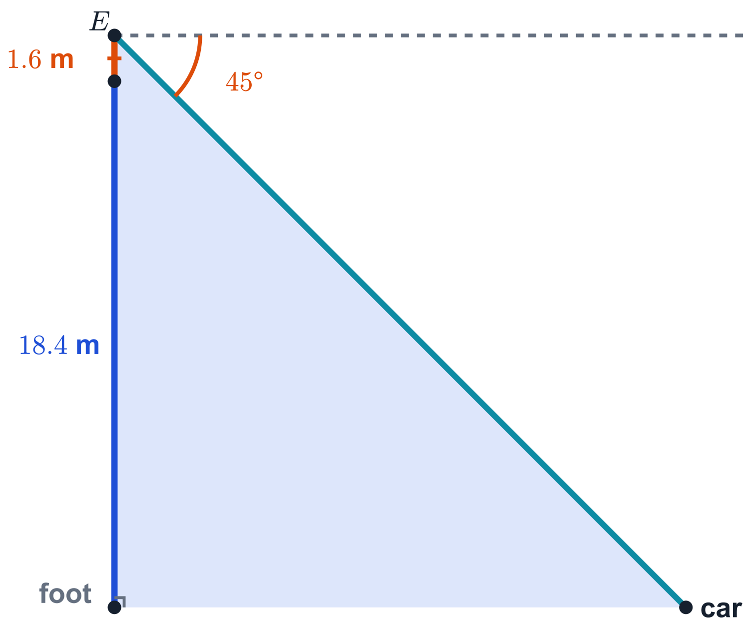
BACK
The observer's own height counts · p. 300

TRY
A 1.5 m person stands on a 16.5 m tower. A car is at 45° depression. Find its distance.

Answer: Eyes are $16.5 + 1.5 = 18$ m up. The distance is also 18 m, since $\tan 45° = 1$.

KEY
The 20 m eye height is built from $18.4 + 1.6$ before the triangle is even drawn.

Depression, with the eye height built in



The tower and the observer's own height stack to twenty metres of eye level, matching the twenty metre distance to the car.

A DISTANCE FROM AN EYE HEIGHT AND ONE ANGLE

1. **Add the heights** $18.4 + 1.6 = 20$ m. The tower's height and the observer's own height together give the eye's height above the ground.
2. **Draw from the eye** Draw the right triangle from the 20 m eye level down to the car, not from the tower's foot.
3. **Mark the angle** Mark the given angle of depression, 45°, at the eye.
4. **Use alternate angles** The 45° depression at the eye equals the 45° elevation at the car.
5. **Pick the ratio and solve** $\tan 45° = 1$, so the horizontal distance equals the 20 m eye height.

CHECK YOURSELF

27. A person $1.6m$ tall stands on an $18.4m$ tower, eyes at $20m$. Looking at a car with a $45°$ angle of depression, which equation finds the horizontal distance d to the car?

- **A.** $\tan 45° = d/20$
- **B.** $\tan 45° = 18.4/d$
- **C.** $\sin 45° = 20/d$
- **D.** $\tan 45° = 20/d$

28. Using the same setup — eyes at $20m$ — but with the angle of depression now $30°$, what is the distance to the car?

- **A.** $20/\sqrt{3}$ m
- **B.** 20 m, the same as at 45°
- **C.** $20\sqrt{3}$ m
- **D.** $10\sqrt{3}$ m

2 more in this set online.

WORKED EXAMPLE · **A building and its flagpole**

1.	a building 20 m tall carries a flagpole on its roof, seen from a ground point P at elevation 30° to the roof and 45° to the flag's top	*the given information. Let d be the distance from P to the foot of the building, and x the flagpole's length*
2.	in the smaller triangle, $\tan 30° = 20/d$, so $d = 20 \cdot \sqrt{3}$ m	*the building's own height and distance form one right triangle, solved first*
3.	in the larger triangle, $\tan 45° = (20 + x)/d = 1$, so $20 + x = d$	*the flagpole's top adds x to the building's height, over the same distance d*
4.	$x = 20 \cdot \sqrt{3} - 20 = 20 \cdot \left(\sqrt{3} - 1\right) \approx 14.6$ m	*substitute the value of d found from the smaller triangle*
5.	$20 + x = 20 + 20 \cdot \left(\sqrt{3} - 1\right) = 20 \cdot \sqrt{3} = d$	CHECK ***Check by adding the flagpole's length back onto the building's height.*** *The 20s cancel, leaving $20\sqrt{3}$, exactly the distance d found from the smaller triangle.*

BACK
Two triangles, one shared side · p. 302

TRY
A 15 m building has a flagpole. Elevation from P is 45° to the roof, 60° to the flag. Find its length.

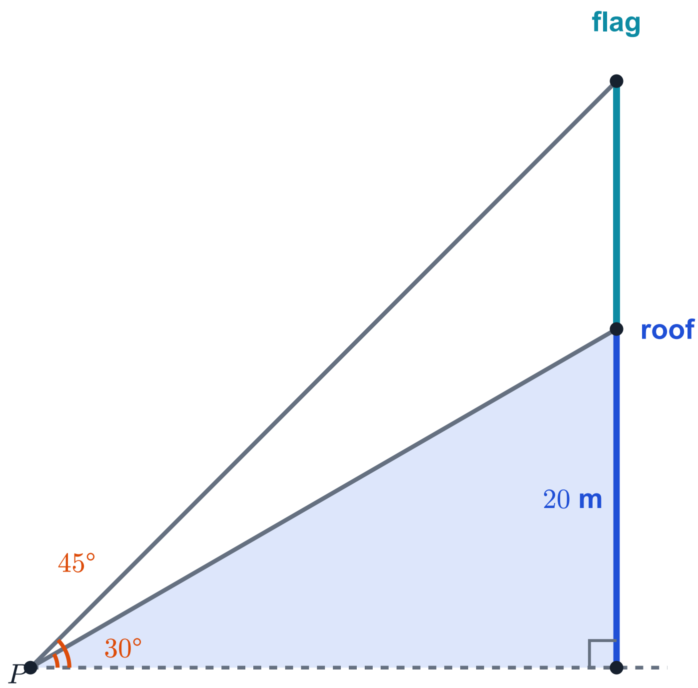
Answer: $d = 15$ m, since $\tan 45° = 1$. Then $x = 15 \cdot \left(\sqrt{3} - 1\right) \approx 11.0$ m.

RULE
Two triangles, same base point P. Solve the smaller one first, then reuse its distance in the larger one.

TWO TRIANGLES SHARING ONE SIDE

1. **Draw both triangles** Draw one triangle to the building's roof and a taller one to the flagpole's top, both from the same ground point P.
2. **Mark both angles** Mark 30° to the roof and 45° to the flag, both at P.
3. **Name the shared side** Both triangles share the same base, the distance d from P to the building's foot.
4. **Solve the smaller triangle** $\tan 30° = 20/d$, so $d = 20 \cdot \sqrt{3}$ m.
5. **Solve the larger triangle** $\tan 45° = (20 + x)/d$, using the same d. Solve for x.
6. **Check the size** A bigger angle to the flag means x must be positive. Here $x \approx 14.6$ m, which checks out.

A building and its flagpole



Two angles from one ground point, thirty degrees to the roof and forty-five to the flag, are enough to find the flagpole's length.

CHECK YOURSELF

29. A $20m$ building carries a flagpole on its roof. From point P, the building's top has elevation $30°$ and the flagpole's top has elevation $45°$. Which equation finds d, the distance from P to the building's foot?

- **A.** $\tan 45° = 20/d$
- **B.** $\tan 30° = 20/d$
- **C.** $\tan 30° = d/20$
- **D.** $\sin 30° = 20/d$

30. Using the same method, a $10m$ building carries a flagpole. From point P, the building's top has elevation $30°$ and the flagpole's top has elevation $60°$. What is the flagpole's length?

- **A.** $10\sqrt{3} - 10$ m
- **B.** 10 m, the same as the building
- **C.** 30 m, the total height including the building
- **D.** 20 m

1 more in this set online.

WORKED EXAMPLE · **Two buildings, by angle of depression**

1. a shorter building 8 m tall stands near a taller one, depression 30° to its top and 60° to its foot, both measured from the taller building's top

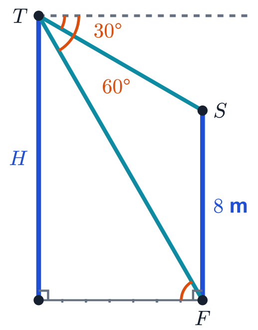
the given information. Let x be the distance between the buildings, and H the taller building's height

WORKED EXAMPLE · Two buildings, by angle of depression

2.	by alternate angles, these depression angles equal the elevation angles measured from the shorter building's top and foot	*the alternate-angle equality proved earlier, applied twice*
3.	from the foot: $\tan 60° = H/x$	*the full height H and the shared distance x form one right triangle*
4.	from the top: the remaining vertical drop is $H - 8$, so $\tan 30° = (H-8)/x$	*the shorter building's own height, 8 m, is subtracted from H first*
5.	solving the two equations together gives $x = 4\cdot\sqrt{3}$ m and $H = 12$ m	*substitute $H = x\cdot\sqrt{3}$ from the first equation into the second, then solve for x*
6.	$H/x = 12/(4\cdot\sqrt{3}) = \sqrt{3}$, matching $\tan 60°$, and $(H-8)/x = 4/(4\cdot\sqrt{3}) = 1/\sqrt{3}$, matching $\tan 30°$	**CHECK** ***Check both original equations by substituting $x = 4\sqrt{3}$ and $H = 12$ back in.*** *Both ratios land exactly on the tangents the problem started from.*

RULE
Alternate angles move each depression angle from the top of the taller building to the base of its own right triangle.

Two buildings, angles of depression



The depression angles from the taller roof match the elevation angles at the shorter building, which is how both triangles get solved.

CHECK YOURSELF

31. A shorter $8m$ building stands near a taller one. From the taller building's top, the depression angle to the shorter one's top is $30°$, and to its foot is $60°$. By alternate angles, what does the $60°$ depression angle equal, seen from the shorter building's foot?

- **A.** an elevation angle of $30°$
- **B.** an elevation angle of $60°$
- **C.** the complement of $60°$, which is $30°$
- **D.** no fixed relationship — it depends on the buildings' heights

32. Using the same method — a shorter $6m$ building near a taller one, with depression angles of $30°$ to its top and $60°$ to its foot — what is the height of the taller building?

- **A.** 12 m
- **B.** 6 m, the same as the shorter building
- **C.** 9 m
- **D.** $3\sqrt{3}$ m

2 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap

- TRIANGLE: vertical side, horizontal side and line of sight, angle marked at the observer. The tower problem marked its $60°$ angle there, 12 m from the foot.
- RATIO: two legs need $\tan$ or $\cot$; a leg with the hypotenuse needs $\sin$ or $\cos$. The rope problem used $\sin 30° = 8/l$ for its hypotenuse.
- EYE LEVEL: add the observer's own height back after solving the triangle. The tower's 18.4 m plus the person's 1.6 m made 20 m of eye height.
- ALTERNATE ANGLES: depression from a high point equals elevation from the low point below it. A $30°$ depression from a cliff top gave a $30°$ elevation from the boat.
- TWO TRIANGLES: at most two triangles, sharing one side, solved as two equations together. The flagpole came out $20 \cdot \left(\sqrt{3} - 1\right) \approx 14.6$ m from both.

Step back. We have run the same three steps every time. Draw and label one right triangle, at most two if the problem needs a second. Mark the given angle of elevation or depression at the observer. Then choose the ratio that connects the known side to the wanted one.

A tower, a rope, a flagpole and two buildings: different pictures, the same triangle underneath each one.

Your turn: an observer 1.2 m tall stands 5 m from a pole and measures $60°$ of elevation to its top. Find the pole's full height. (Answer: 9.86 m, since $\tan 60°$ gives a rise of $5 \cdot \sqrt{3} \approx 8.66$ m, plus the observer's 1.2 m.)

BACK
Turning words into a triangle · p. 298

KEY
Draw, mark the angle, pick the ratio. Three steps, every problem, at most two triangles.

CHECK YOURSELF

33. This chapter's method, every time, is: draw and label one right triangle, mark the given angle, and then

- **A.** measure the object directly, to double-check
- **B.** add a second, unrelated triangle for safety
- **C.** choose the connecting ratio
- **D.** guess the answer and adjust it afterward

34. This chapter never introduces a new trigonometric ratio or identity. What does it add to what Chapter 8 already built?

- **A.** a fourth trigonometric ratio, for slanted lines specifically
- **B.** a rule for when $\sin\theta$ and $\cos\theta$ are equal
- **C.** a correction to how elevation and depression are defined
- **D.** a method for aiming those ratios at real problems

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

35. A student wants to find the height of a mobile tower without climbing it. Which two measurements does the method in this chapter actually need?

- **A.** the tower's shadow length and the time of day
- **B.** two different distances measured from the tower's foot, with no angle taken at all
- **C.** one angle of elevation to the top of the tower, and the distance from the point of measurement to the tower's foot
- **D.** two different angles of elevation taken from the same spot, with no distance measured at all, relying on the two angles alone to fix the height

36. An observer, from one spot, looks up at a plane and down at a boat on a lake. What line is each of the two angles measured from?

- **A.** the angle to the plane is measured from the horizontal. The angle to the boat is measured from the vertical drop down to the lake below
- **B.** each angle is measured from the observer's line of sight to the OTHER object. Neither is based on any horizontal line
- **C.** the plane and the boat each need their own separate horizontal line. They point in different directions
- **D.** both angles are measured from the same horizontal line at the observer's eye. The plane's angle is read upward, the boat's downward

3 more in this set online.

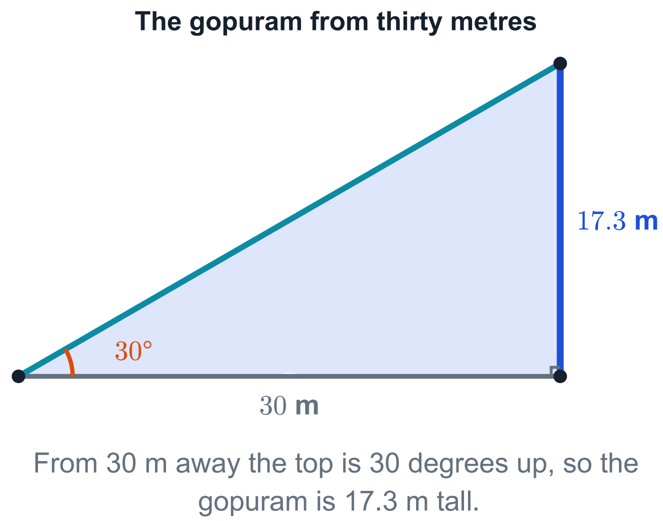
IN EVERY CHAPTER

Where you will meet this

You judge a height or a distance you cannot reach more often than you think. Here are seven places one angle and one triangle give you that number.

- **Measuring a temple gopuram's height.** You stand 30 m from the foot of a temple gopuram. Its top has an angle of elevation of 30°.
  The maths: the height comes from the horizontal distance and the tangent of the elevation angle.
  Since $\tan 30° = 1/\sqrt{3}$, the height is $30/\sqrt{3} = 10\sqrt{3} \approx 17.3$ m.

BACK
The angle of elevation · p. 294



One angle and one distance, and the tangent turns them into the height you could not climb up to measure.

- **Estimating a kite's height from the string.** A kite string is let out to 50 m, taut, at an angle of elevation of $60°$ from the flyer's hand.
  The maths: the kite's height uses the sine ratio, since the taut string is the hypotenuse.
  Since $\sin 60° = \sqrt{3}/2$, the height is $50 \cdot \sqrt{3}/2 = 25\sqrt{3} \approx 43.3$ m.
- **Judging a parked car's distance from your terrace.** You stand on a terrace 12 m high. You see a car parked on the road at an angle of depression of $30°$.
  The maths: the angle of depression from the terrace equals the angle of elevation measured from the car looking up.
  Since $\tan 30° = 1/\sqrt{3}$, the car's distance is $12\sqrt{3} \approx 20.8$ m.
- **Measuring a tree's height for a school project.** A student's eyes are 1.5 m above the ground. From 20 m away, the tree's top has an elevation of $30°$.
  The maths: the tree's full height adds the observer's own eye level to the triangle's vertical side.
  Since $\tan 30° = 1/\sqrt{3}$, the triangle gives $20/\sqrt{3} \approx 11.5$ m. Adding the eyes' 1.5 m, the tree is about 13.0 m tall.
- **Finding a statue's pedestal height.** A 1.6 m statue stands on a pedestal in a town square. From one point on the ground, the pedestal's top is at $45°$ and the statue's top at $60°$.
  The maths: two triangles share the same base distance, and each gives one equation for it.
  With the pedestal h m tall, the base distance is h. So $h\sqrt{3} = h + 1.6$, and $h = 1.6/(\sqrt{3} - 1) \approx 2.19$ m.
- **Estimating a Ferris wheel's height at a fair.** You walk back from a Ferris wheel until its top is at an angle of elevation of $45°$. You are then 40 m from its base.
  The maths: a $45°$ elevation makes the height equal the horizontal distance, since $\tan 45° = 1$.
  So the wheel is 40 m tall, read off your own paces with no table needed.
- **Checking a builder's claimed apartment height.** A builder claims an apartment tower is 52 m tall. From 30 m away, you measure the top's angle of elevation as $60°$.
  The maths: the tangent ratio turns your distance and angle into the tower's real height, which you can check against the claim.
  Since $\tan 60° = \sqrt{3}$, the measured height is $30\sqrt{3} \approx 51.96$ m, close to the claimed 52 m.

Your turn. You stand 15 m from the foot of your school's flagpole. Its top has an angle of elevation of 30°. How tall is the pole? *Answer:* Since $\tan 30° = 1/\sqrt{3}$, the height is $15/\sqrt{3} = 5\sqrt{3} \approx 8.66$ m.

Practice: Exercise 9.1

PRACTICE SET

1. PRACTICE A tower stands on level ground. From a point 18 m from its foot, the angle of elevation of the top of the tower is 60°. Find the height of the tower. (Worked in full below — read it, then do the next three the same way.)

SOLUTION — Q1 · **Worked in full**

1.	the tower is the vertical side, the 18 m is the horizontal side, and the 60° sits on the ground between the line of sight and the horizontal	*draw and label before calculating — the angle of elevation is measured from the HORIZONTAL, never from the tower, and marking it in the right place is what stops the whole question going wrong*
2.	$\tan 60° = h/18$	*the known side is next to the angle and the unknown is opposite it, and tangent is the ratio that joins those two — choose the ratio before any value goes in*
3.	$h = 18 \cdot \tan 60° = 18 \cdot \sqrt{3}$	*substitute the table value $\tan 60° = \sqrt{3}$ and multiply*
4.	$h = 18 \cdot \sqrt{3} \approx 31.2$ m	*the check — the angle is bigger than 45°, so the height has to come out bigger than the 18 m distance, and 31.2 m is bigger than 18 m*

2. PRACTICE A ladder 12 m long leans against a vertical wall and makes an angle of 30° with the WALL. Find how high up the wall the ladder reaches. (Mark the angle first — this 30° is at the top, between the ladder and the wall, so the wall is the side next to it and the ladder is the hypotenuse. That makes the ratio cosine, not tangent.)

3. PRACTICE A girl 1.5 m tall stands 12 m from the foot of a lamp post. From her eyes, the angle of elevation of the top of the post is 30°. Find the height of the post. (Two steps — the triangle gives only the rise ABOVE her eyes, so work that out first and then add her own height.)

11. PRACTICE A building 15 m tall carries a flagpole on its roof. From a ground point, the angle of elevation of the building's top is 30° and of the flagpole's top is 45°. Find the length of the flagpole.

12. PRACTICE A shorter building 10 m tall stands near a taller one. From the taller building's top, the angle of depression of the shorter building's top is 30° and of its foot is 60°. Find the taller building's height and the distance between them.

13. PRACTICE From the top of a lighthouse 60 m tall, the angle of depression of a boat is 60°. Find the boat's distance from the foot of the lighthouse.

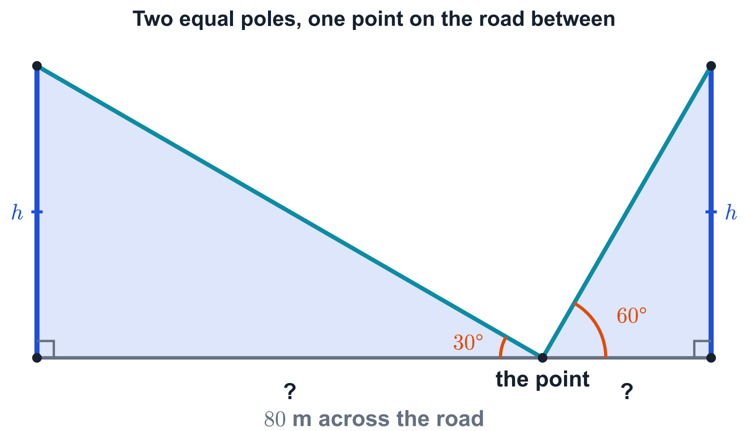
4. PRACTICE A statue stands on top of a pedestal 12 m tall. From a point on the ground, the angle of elevation of the top of the pedestal is 45° and of the top of the statue is 60°. Find the height of the statue. (Two triangles share the same ground distance — use the smaller one to find that distance first, then put it into the larger one.)
5. PRACTICE A ladder leans against a wall so that its foot is 2.5 m from the wall and it makes an angle of 60° with the ground. Find the length of the ladder.
6. PRACTICE A 10 m ladder rests against a vertical wall, making an angle of 30° with the wall itself. Find how far the foot of the ladder is from the wall.
7. PRACTICE A tower stands on level ground. From a point 20 m from its foot, the angle of elevation of the top is 45°. Find the height of the tower.
8. PRACTICE A tower stands on level ground. From a point 30 m from its foot, the angle of elevation of the top is 30°. Find the height of the tower.
9. PRACTICE A kite's taut string is 40 m long and makes an angle of 60° with the ground. Find the height of the kite above the ground.
10. PRACTICE Two points lie on the same side of a tower, in line with its foot, 20 m apart. From the nearer point, the angle of elevation of the tower's top is 60°. From the farther point, the angle is 30°. Find the height of the tower.
14. PRACTICE From the top of a building 30 m tall, the angles of depression of two cars on the same road, both on the same side, are 60° (nearer car) and 30° (farther car). Find the distance between the two cars.
15. PRACTICE An observer 1.6 m tall stands 30 m from a building. The angle of elevation of the building's top, measured from the observer's eye, is 30°. Find the building's height.
16. PRACTICE A person's eyes are 10 m above the ground, standing on a tower. The angle of depression of a car on the ground is 30°. Find the car's distance from the foot of the tower.
17. PRACTICE A tower stands on one bank of a river. From its top, the angle of depression of a point directly opposite, on the other bank, is 60°. The angle of depression of a second point, 24 m further away on the line joining the first point to the foot of the tower, is 30°. Find the width of the river and the height of the tower.
18. PRACTICE A pole stands on top of a pedestal 5 m tall. From a ground point, the angle of elevation of the pedestal's top is 30° and of the pole's top is 60°. Find the length of the pole.
19. PRACTICE Two poles of equal height stand on either side of a road 80 m wide. From a point on the road between them, the angles of elevation of the tops are 30° and 60°. Find the height of the poles and the distances of the point from each pole.

ANSWERS **1.** $h = 18 \cdot \sqrt{3}$ m, about 31.2 m.
2. $\cos 30° = x/12 = \sqrt{3}/2$, so $x = 6 \cdot \sqrt{3}$ m, about 10.4 m.
3. The rise above eye level is $12 \cdot \tan 30° = 12/\sqrt{3} = 4 \cdot \sqrt{3}$ m. Adding her own height, the post is $1.5 + 4 \cdot \sqrt{3}$ m tall, about 8.4 m.

4. $\tan 45° = 12/d = 1$, so $d = 12$ m. The full height is $d \cdot \tan 60° = 12 \cdot \sqrt{3}$ m, so the statue is $12 \cdot \sqrt{3} - 12 = 12 \cdot \left(\sqrt{3} - 1\right)$ m, about 8.8 m.
5. $\cos 60° = 2.5/l$, and $\cos 60° = 1/2$, so $l = 5$ m.
6. The angle at the top, between the ladder and the wall, is 30°; the horizontal distance is opposite it. $\sin 30° = d/10 = 1/2$, so $d = 5$ m.
7. $\tan 45° = h/20 = 1$, so $h = 20$ m.
8. $\tan 30° = h/30 = 1/\sqrt{3}$, so $h = 30/\sqrt{3} = 10 \cdot \sqrt{3}$ m, about 17.3 m.
9. $\sin 60° = h/40 = \sqrt{3}/2$, so $h = 20 \cdot \sqrt{3}$ m, about 34.6 m.
10. Let the nearer distance be a. $\tan 60° = h/a$ and $\tan 30° = h/(a + 20)$. Solving together gives $a = 10$ m and $h = 10 \cdot \sqrt{3}$ m, about 17.3 m.
11. $d = 15/(\tan 30°) = 15 \cdot \sqrt{3}$ m. The flagpole's own length is $d \cdot \tan 45° - 15 = 15 \cdot \sqrt{3} - 15 = 15 \cdot \left(\sqrt{3} - 1\right)$ m, about 11.0 m.
12. Let x be the distance and H the taller building's height. $H = x \cdot \sqrt{3}$ and $H - 10 = x/\sqrt{3}$. Solving together gives $x = 5 \cdot \sqrt{3}$ m, about 8.7 m, and $H = 15$ m.
13. $\tan 60° = 60/d = \sqrt{3}$, so $d = 60/\sqrt{3} = 20 \cdot \sqrt{3}$ m, about 34.6 m.
14. Nearer car: $\tan 60° = 30/a$, so $a = 10 \cdot \sqrt{3}$ m. Farther car: $\tan 30° = 30/b$, so $b = 30 \cdot \sqrt{3}$ m. The distance between them is $b - a = 20 \cdot \sqrt{3}$ m, about 34.6 m.
15. The rise above eye level is $30 \cdot \tan 30° = 10 \cdot \sqrt{3}$ m. Adding the observer's own height, the building is $10 \cdot \sqrt{3} + 1.6$ m tall, about 18.9 m.
16. $\tan 30° = 10/d = 1/\sqrt{3}$, so $d = 10 \cdot \sqrt{3}$ m, about 17.3 m.
17. Let the width be a. $\tan 60° = h/a$ and $\tan 30° = h/(a + 24)$. Solving together gives $a = 12$ m and $h = 12 \cdot \sqrt{3}$ m, about 20.8 m.
18. $d = 5/(\tan 30°) = 5 \cdot \sqrt{3}$ m. The full height is $d \cdot \tan 60° = 5 \cdot \sqrt{3} \cdot \sqrt{3} = 15$ m, so the pole itself is $15 - 5 = 10$ m.
19. Let the distance from the nearer pole be a. $a \cdot \tan 60° = (80 - a) \cdot \tan 30°$ gives $a = 20$ m. The height is $20 \cdot \tan 60° = 20 \cdot \sqrt{3}$ m, about 34.6 m; the two distances are 20 m and 60 m.

Full solutions: **manthika.com/learn/math10-ch09**

Further practice



Two equal poles stand at each end of an 80 metre road, sighted from one point between them at thirty and sixty degrees.

PRACTICE SET

20. PRACTICE In a right triangle formed for a heights-and-distances problem, the hypotenuse is unknown and one leg is known. Which trigonometric ratio should be used to find the hypotenuse?

A. tan only
B. sin or cos
C. cot only
D. Any of sin, cos, tan

SOLUTION — Q20 · Worked in full

1.	the two legs are opposite and adjacent to the marked angle; the hypotenuse is neither	*naming the sides relative to the marked angle decides which ratio connects them*
2.	tan and cot relate the two legs to each other	*neither one ever touches the hypotenuse, so neither can be used to find it*
3.	sin or cos is the ratio needed	sin *connects the opposite leg to the hypotenuse and* cos *connects the adjacent leg to the hypotenuse — whichever leg is known decides which of the two to use*

21. PRACTICE An escalator in a shopping mall rises at an angle of 30° to the horizontal floor. A shopper travels 8 m along the escalator, from its bottom to its top. Find the vertical height gained.

22. PRACTICE A guy wire is stretched from the top of a mobile-network tower to a peg fixed in the ground, making an angle of 60° with the ground. If the wire is 16 m long, find the distance of the peg from the foot of the tower.

23. PRACTICE A ladder 8 m long is placed so that it reaches a window 4 m above the ground, at the foot of a vertical wall. What angle does the ladder make with the ground?

A. 30°
B. 45°
C. 60°
D. 90°

28. PRACTICE A fire engine's ladder extends at an angle of 60° to the ground to reach a window $15\sqrt{3}$ m above the ground. Find the length the ladder must be extended to.

29. PRACTICE A helicopter is hovering at a height of 250 m above a straight road. The angle of depression of a car on the road, measured from the helicopter, is 30°. Find the line-of-sight distance between the helicopter and the car.

30. PRACTICE A tree breaks in a storm partway up its trunk. The broken upper part bends over so that its tip touches the ground, making an angle of 30° with the ground. The tip touches the ground 9 m from the foot of the tree. Find the height of the tree before it broke.

31. PRACTICE A boy standing on level ground finds that the angle of elevation of the top of a building 45 m tall increases from 30° to 60° as he walks in a straight line towards it. Find the distance he walked.

24. PRACTICE A person 1.6 m tall stands 9 m from the foot of a pole. The angle of elevation of the top of the pole, measured from the person's eyes, is 60°. Find the straight-line distance from the person's eyes to the top of the pole.

25. PRACTICE A person stands at some distance from a tower and works out, from the triangle formed by the angle of elevation, that the rise above eye level is 22 m. If the person's eyes are 1.5 m above the ground, what is the height of the tower above the ground?

A. 20.5 m
B. 22 m
C. 23.5 m
D. 33 m

26. PRACTICE A flagstaff $7\sqrt{3}$ m tall stands on level ground. From a point on the ground, the angle of elevation of its top is 60°. Find the distance of the point from the foot of the flagstaff.

27. PRACTICE From the top of a tower, the angle of depression of a car on the ground is 60°. What is the angle of elevation of the top of the tower, as seen from the car?

A. 30°
B. 45°
C. 60°
D. 90°

32. PRACTICE A tower and a building stand directly opposite each other on level ground. The angle of elevation of the top of the tower, measured from the foot of the building, is 60°. The angle of elevation of the top of the building, measured from the foot of the tower, is 30°. The tower is 30 m tall. Find the height of the building.

33. PRACTICE From the top of a building 12 m high, the angle of elevation of the top of a nearby cable tower is 60°, and the angle of depression of the tower's foot is 45°. Find the height of the cable tower.

34. PRACTICE A girl 1.6 m tall spots a balloon moving horizontally at a constant height of 61.6 m above the ground. At one instant, the angle of elevation of the balloon from her eyes is 60°; a little later, it is 30°. Find the distance travelled by the balloon in that interval.

ANSWERS **20.** B — sin or cos. **21.** 4 m. **22.** 8 m. **23.** A — 30°. **24.** 18 m. **25.** C — 23.5 m. **26.** 7 m. **27.** C — 60°. **28.** 30 m. **29.** 500 m. **30.** $9\sqrt{3}$ m, about 15.6 m. **31.** $30\sqrt{3}$ m, about 52.0 m. **32.** 10 m. **33.** $12 + 12\sqrt{3}$ m, about 32.8 m. **34.** $40\sqrt{3}$ m, about 69.3 m.
Full solutions: **manthika.com/learn/math10-ch09**