10 CHAPTER 10 Circles

WHAT THIS CHAPTER BUILDS
You will learn that a tangent meets the radius at a right angle, that the two tangents from a point are equal, and how those two facts settle every circle proof.

GEOMETRY UNIT, PROOFS

IN THIS CHAPTER

Arjun holds the bicycle steady while Tara crouches by the front wheel, which touches the flat road at just one spot.

Getting started

A line that touches

A STORY

Where the wheel meets the road

Arjun holds his bicycle steady by the handlebar. Tara crouches by the front wheel, pointing low at the tyre.

“Only one point of this wheel is on the road,” Tara says. “Right here, where I am pointing.”

“And the hub sits straight above that point,” Arjun says. “The line from the hub down to it meets the road at a right angle.”

“Every wheel on flat ground does that,” Tara says. “A small wheel or a big one.”

“Now suppose I stand at one spot on the road, away from the wheel,” Arjun says. “Could I draw two straight lines from my spot, each just touching the wheel?”

“Exactly two,” Tara says. “The road itself is one of them. And the two lines turn out equal in length, from your spot to where each one touches.”

Arjun looks from the wheel to the road and back.

Why should those two lengths be equal?

Draw a circle. Then draw a straight line that just brushes past it. Slide the line a little. It now either misses the circle, or cuts through it at two points. At one exact position only, the line touches the circle at a single point and nowhere else. That one line is what this chapter studies.

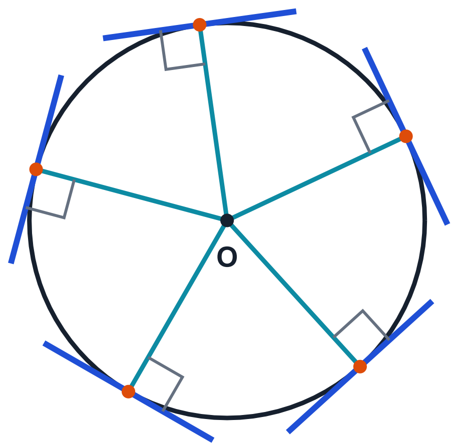
A line meeting a circle at exactly one point is a TANGENT. The single shared point is the POINT OF CONTACT. **A tangent is always perpendicular to the radius drawn to its point of contact.** Hold onto that one right angle. We will use it in every proof and every worked example ahead.

Now pick any point outside the circle. From that point, we can always draw exactly two tangents back to the circle. They touch the circle at two different places. Draw both tangents, then measure their lengths with a ruler. The two lengths always come out equal.

RULE
Mark the radius to the point of contact first. That right angle is where every proof here starts.

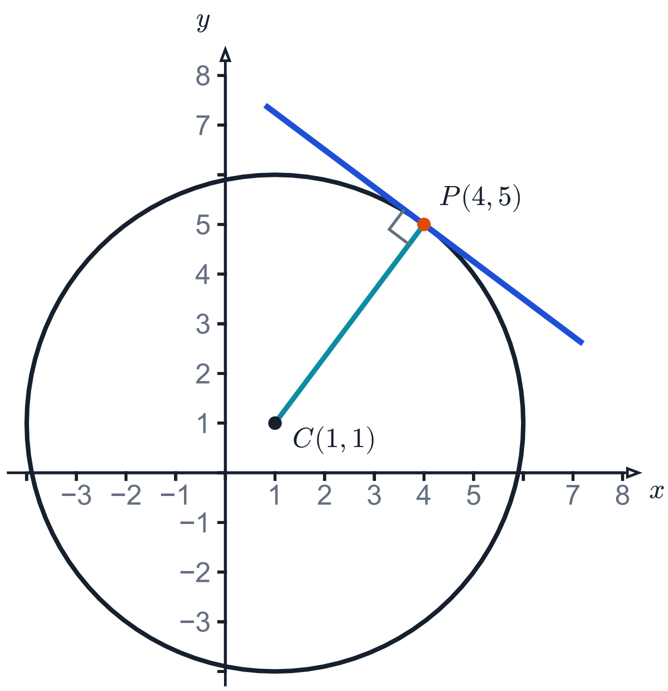
KEY
One right angle drives every tangent fact here: the radius meets the tangent at the point of contact.

The same right angle, all the way round



Five points of contact spaced unevenly around one circle, each with the same right angle between its radius and its tangent line.

The circle and its tangent, on axes



A circle with centre C and a tangent at P, on a coordinate grid, with a small square marking the right angle at P.

IN THE WILD

A bicycle wheel touches the road at exactly one point. The hub sits straight above that point, so the radius from the hub to the road meets it at a right angle, and the road is a tangent to the wheel's circle. That one right angle lets you find a tangent's length from any point outside the circle, and tells you, without drawing, that a second tangent of equal length exists.

CHECK YOURSELF

1. A tangent to a circle is best described as a line that

- **A.** meets the circle at exactly two points
- **B.** always passes through the centre
- **C.** never touches the circle at all
- **D.** meets the circle at exactly one point

2. At the point where a tangent touches a circle, the tangent and the radius drawn to that point are

- **A.** perpendicular to each other
- **B.** parallel to each other
- **C.** equal in length
- **D.** at $45°$ to each other

1 more in this set online.

IN EVERY CHAPTER

Before you start

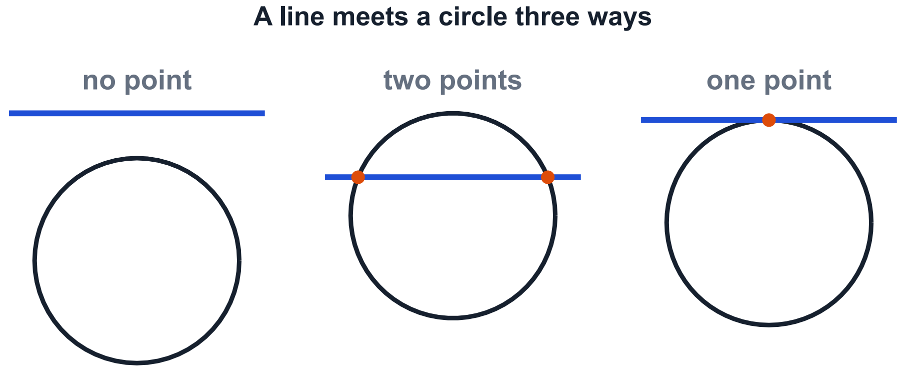
DO THIS FIRST

You cannot always draw a triangle to prove a circle fact. Two straight lines from outside do the same job. Try each check below.

- **RHS congruence** (Class 9, Ganita Manjari, page 104).
  Here: right triangles OPT, OQT share OT and have equal radii.
  Check: Hypotenuse 13 and one leg 5 leave the other leg 12.
- **Perpendicular bisects a chord** (Class 9, Ganita Manjari, page 102).
  Here: point P splits the chord AB into two equal halves this same way.
  Check: A chord 3 cm from radius 5 has half-length 4: $3^2 + 4^2 = 5^2$.
- **Opposite sides equal** (Class 8, Ganita Prakash Part 1, page 98).
  Here: the proof starts from $AB = CD$ and $AD = BC$, already true here.
  Check: A parallelogram with sides 7 and 9 has perimeter $2(7 + 9) = 32$.
- **Isosceles base angles** (Class 8, Ganita Prakash Part 1, page 87).
  Here: triangle TPQ is isosceles, so its base angles are equal.
  Check: Sides $6, 6$ cm and top angle $40°$ give base angles $70°$ each.
- **Pythagoras' theorem** (Class 8, Ganita Prakash Part 2, page 44).
  Here: the tangent length PQ comes from $OQ^2 = OP^2 + PQ^2$.
  Check: $5^2 + 12^2 = 13^2$, since $25 + 144 = 169$.
- **Quadrilateral angle sum** (Class 8, Ganita Prakash Part 1, page 95).
  Here: quadrilateral $OPTQ$ uses this to find $\angle PTQ$.
  Check: Angles $90°, 90°, 130°$ leave a fourth of $50°$, adding to $360°$.

If any of these felt new, read the page named before going on.

Tangent, secant and point of contact



Three panels show the same circle with a line missing it, crossing it at two points, and resting on top at one point.

Take any straight line, and any circle drawn in the same plane. Between them, only three things can happen. The line can miss the circle completely. It can cross the circle at two separate points. Or it can touch the circle at exactly one point.

A line meeting the circle at two points is a SECANT. A line meeting it at exactly one point is a TANGENT. That single meeting point is the POINT OF CONTACT.

Try it yourself. Draw a line through two points that sit close together. You will find it is still a secant, however small the gap. It only becomes a tangent once those two points are exactly the same point.

TRY
How many points does a secant share with the circle?

Answer: Exactly two.

BACK
A line that touches · p. 323

RULE
A tangent meets the circle at exactly ONE point, never two points close together.

CHECK YOURSELF

3. A line meeting a circle at exactly two points is called a

- **A.** tangent
- **B.** chord
- **C.** secant
- **D.** diameter

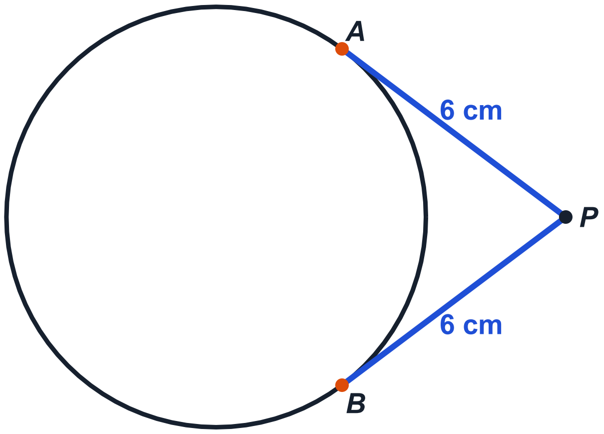
4. A student says: "A line and a circle can meet at two points, touch at one point, or not meet at all — those are the only three possibilities." Is this correct?

- **A.** Yes — those are the only three cases
- **B.** No — a line can also meet a circle at three points
- **C.** No — every line must meet the circle at least once

1 more in this set online.

The length of a tangent from a point

Two tangents, two lengths, one number



from P to its own point of contact, and no further

The length of a tangent is measured from the outside point to that tangent's own point of contact. P has two such lengths, one per tangent, and they always agree, so 'the' tangent length from P is a fair phrase.

Because the two lengths always agree, a single number is enough to describe how far a point is from the circle along a tangent.

Take a point P outside a circle, with one tangent drawn from it. You can measure exactly one distance here: from P to the point where that tangent touches the circle. That distance is the LENGTH OF THE TANGENT from P.

Two tangents can be drawn from P, so the term applies separately to each one. P has two tangent lengths: one for the tangent touching the circle at one point, and one for the tangent touching it at the other.

The two tangent lengths from P always come out equal. So you can safely say "the" tangent length from P, without naming which of the two tangents you mean.

Your turn: a tangent from a point P touches a circle at a point 6 cm away. What is the length of that tangent? (Answer: 6 cm, the distance from P to the point of contact.)

TRY
From how many points can "the length of the tangent" be measured, for one external point?

Answer: Two, one for each of the two tangents drawn from it.

KEY
Length of tangent = distance from the external point to where THAT tangent touches the circle.

CHECK YOURSELF

5. The "length of the tangent" from an external point P means the distance from P to

- **A.** the centre of the circle
- **B.** the nearest point on the circle
- **C.** the tangent's point of contact
- **D.** the far side of the circle

6. From an external point P, two tangents touch a circle at Q and R. How many different "tangent lengths" does P have, and what are they?

- **A.** two — PQ and PR, one for each tangent from P
- **B.** one — both tangents from the same point must share a single length
- **C.** three — PQ, PR, and the distance PO to the centre
- **D.** none — length only makes sense for a chord, not a tangent

1 more in this set online.

The ideas

A tangent is a secant's limit

Picture a secant sliding across a circle. Its two crossing points drift toward each other. Watch closely as they move closer. The line still crosses the circle at two separate places. It is still a secant.

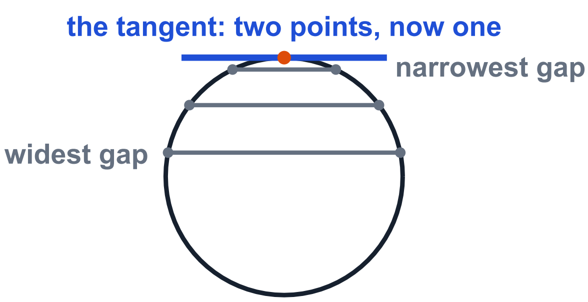
A tangent is what a secant becomes only at the exact moment its two points merge into one. Not one instant before. However close the two points remain, while still distinct, the line stays a secant.

This is why a tangent is called the LIMITING POSITION of a secant. The secant approaches the tangent as its two points approach each other. You only get a tangent at that single moment of exact coincidence.

CAUTION
Two points arbitrarily close together are still two points, not yet a tangent.

KEY
A tangent is what a secant becomes when its two points merge into one.

A secant closing into a tangent



Three secants cross the circle twice each, closer together as the line rises, and a fourth line touches it once, at the tangent.

A SCHOLAR INDIA REMEMBERS

You say a tangent touches the circle at one point. What is your reason? Watch a secant. It cuts the circle at two points. Slide one point towards the other. The two points come closer, and at the end they meet. The line through that one point is the tangent.

Gautama · the sarus crane

CHECK YOURSELF

7. As a secant is moved so its two intersection points move closer together, at what stage does the line become a tangent?

- **A.** only when the two points coincide exactly into one point
- **B.** as soon as the two points are too close to tell apart
- **C.** once the gap is smaller than the circle's radius
- **D.** the line stays a secant forever and never becomes a tangent

8. Why is "the two points are very close together" not an acceptable description of a tangent?

- **A.** distinct points, however close, still make a secant
- **B.** because a tangent must always be drawn outside the circle
- **C.** because closeness between two points on a circle cannot be measured
- **D.** because a secant always has fewer points than a tangent

1 more in this set online.

How many tangents from a point

How many tangents can you draw to a circle from a chosen point? The answer depends on where the point sits.

- **Inside the circle:** no tangent exists at all. Every line through an interior point crosses the circle twice, so it is always a secant.
- **On the circle:** exactly one tangent can be drawn, at that point itself.
- **Outside the circle:** exactly two tangents can be drawn, touching the circle at two different points.

Your turn: a point sits exactly on the circle. How many tangents pass through it? (Answer: exactly one, at that point.)

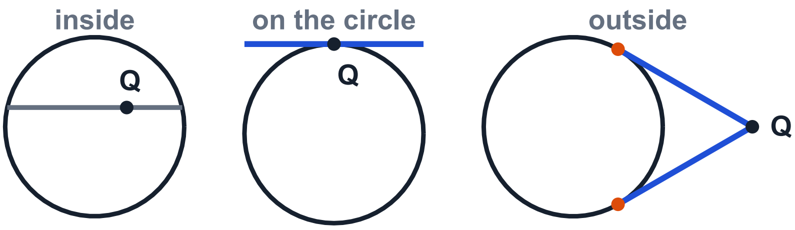
BACK
Tangent, secant and point of contact · p. 326

TRY
How many tangents from a point ON the circle?

Answer: Exactly one, at that point.

RULE
Locate the point first, inside, on, or outside, before asking how many tangents it has.

Inside, on, outside: how many tangents



Three panels place a point Q inside, on, and outside the same circle, showing how many tangents can be drawn from each position.

CHECK YOURSELF

9. From a point lying INSIDE a circle, how many tangents can be drawn to the circle?

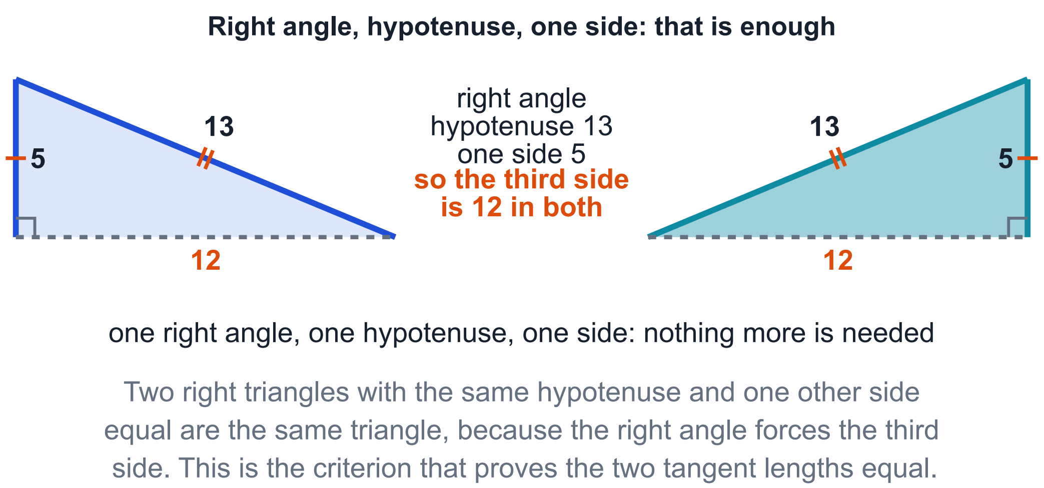
- **A.** none
- **B.** exactly one
- **C.** exactly two
- **D.** infinitely many

10. From a point lying ON a circle, how many tangents can be drawn at that point?

- **A.** none
- **B.** exactly one
- **C.** exactly two
- **D.** it depends on the radius

2 more in this set online.

RHS: a shortcut for right triangles



Two right triangles share a hypotenuse of 13 and a leg of 5, forcing the third side to be 12 in both.

RHS congruence is a shortcut for right triangles only. The name gives the checklist: Right angle, Hypotenuse, Side.

Let us check the right angle first. Both triangles must carry one. Next, we match the HYPOTENUSE, the side opposite the right angle, between the two triangles. Then we match one more side, any one we like.

SSS, SAS, ASA and RHS between them cover every pair of congruent triangles. Only RHS is used here, to show that the two tangent lengths from an outside point are equal.

BACK
A line that touches · p. 323

RULE
RHS only applies to RIGHT triangles: check for the right angle first.

KEY
Hypotenuse + one side match → the two right triangles are congruent (RHS).

CHECK YOURSELF

11. The RHS congruence criterion proves two triangles congruent when

- **A.** an equal hypotenuse plus one equal side, in right triangles
- **B.** all three corresponding sides equal, with no angle needed at all
- **C.** two sides and the angle between them equal, with no right angle needed
- **D.** two angles and the side between them equal, with no right angle needed

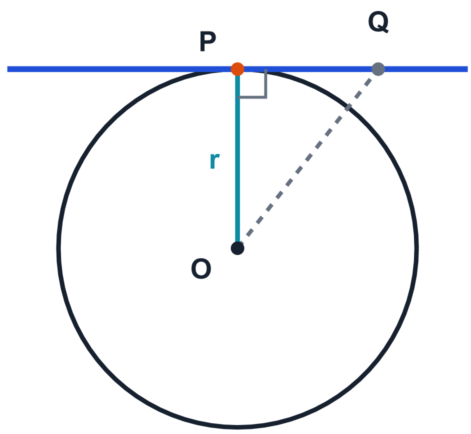
12. Why does RHS need the two triangles to be right triangles, unlike SSS?

- **A.** it does not really need a right angle; the name is just a label
- **B.** right triangles are always larger than other triangles
- **C.** only right triangles have a hypotenuse worth measuring
- **D.** the right angle fixes the third side from the other two

1 more in this set online.

Tangent is perpendicular to the radius

The radius meets the tangent square



A circle with centre O and radius r to the point P, where a tangent line rests, with a square marking the right angle there.

THEOREM. **The tangent to a circle, at the point of contact, is perpendicular to the radius drawn to that point.**

Let O be the centre of a circle. Let XY be a tangent to the circle, touching it at the point P. We must show that OP is perpendicular to XY.

Before the proof, let us see what it compares. It compares the length of OP against the distance from O to every other point on XY. It shows OP is the SHORTEST such distance, and only a perpendicular can be the shortest one. Follow the numbered proof below, one justification per step, and check each line against the figure as we go.

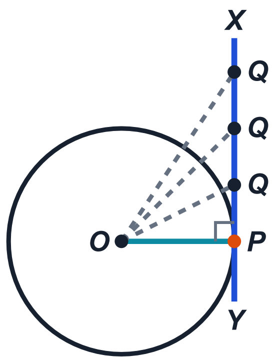
TRY
Why does Q lie outside the circle?

Answer: A tangent meets the circle at exactly one point, P, so any other point on the line cannot be on the circle.

RULE
Every point on the tangent except P lies OUTSIDE the circle. That is what makes OQ longer than OP.

KEY
The shortest segment from the centre to the tangent is the perpendicular: that segment is the radius OP.

Every other OQ is longer than OP



every OQ reaches outside the circle; only OP stops on it

P is the only point of the tangent that lies on the circle, so every other point Q lies outside it and is further from O. The shortest segment from O to the line is OP, and the shortest is always the perpendicular.

The proof never measures an angle: it shows OP is the shortest segment, and the shortest is the perpendicular.

Given: A circle has centre O. Its tangent XY touches the circle at the point P. OP is the radius drawn to that point of contact.

To Prove: The tangent to a circle at any point is perpendicular to the radius through the point of contact: OP is perpendicular to XY.

Proof:

Step	Statement	Reason
1.	let O be the centre of the circle and XY a tangent touching it at the point P; draw the radius OP	*This is the construction the whole proof turns on.*
2.	let Q be any point on the tangent XY other than P, and join OQ	*We draw this auxiliary line so we can compare the two distances.*
3.	Q lies outside the circle	*A tangent meets the circle at exactly one point, P. So any other point of XY cannot lie on the circle.*
4.	OQ is longer than OP	*OP reaches a point ON the circle, while OQ reaches a point OUTSIDE it, by the step before.*
5.	OQ is longer than OP for every choice of Q on XY other than P	*Step 4 did not depend on which point Q we chose.*
6.	the shortest segment from a point to a line is the perpendicular from that point to the line	*We already know this fact from Class 9.*
7.	OP is perpendicular to XY	*By step 5, OP is shorter than the segment to every other point of XY, so it is the shortest such segment. By step 6, the shortest segment is the perpendicular.*

■

A SCHOLAR INDIA REMEMBERS

Gautama's way of arguing, from around the 2nd century BCE, was used by schools that agreed on almost nothing else. They disagreed about many big questions. Still, they could all check the same steps of an argument. A proof in geometry works like that. You may doubt the result at first. You can still check each step, one at a time, and agree when every step holds.

Gautama · the sarus crane

CHECK YOURSELF

13. In the proof that a tangent is perpendicular to the radius, why must Q, any other point on the tangent besides P, lie outside the circle?

- **A.** Q was deliberately chosen to sit far away from the point P
- **B.** the radius OP happens to be shorter than the circle's diameter
- **C.** the tangent meets the circle only at P
- **D.** tangents to a circle are always constructed to lie outside it

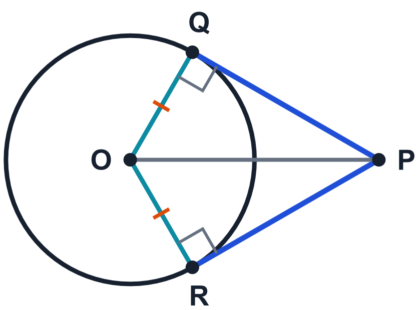
14. Suppose the line XY through P were a secant instead of a tangent. Which step of the perpendicularity proof breaks down first?

- **A.** the step joining OQ, since OQ cannot be drawn for a secant
- **B.** the final step invoking the shortest-segment fact, since it only applies to tangents
- **C.** the step claiming every other point Q on the line lies outside the circle
- **D.** no step breaks down — the same proof works unchanged for a secant

3 more in this set online.

The two tangent lengths are equal

Two right triangles share a hypotenuse



Radii OQ and OR carry matching ticks, tangents from P touch Q and R sharing segment OP, with right angles at Q and R.

THEOREM. **The lengths of the two tangents drawn from an external point to a circle are equal.**

Let O be the centre of a circle. Let P be a point outside it, from which two tangents touch the circle at Q and R. We must show that PQ equals PR.

Let us watch for the shape the proof builds: two right triangles sharing one common hypotenuse. We compare them using the RHS congruence criterion we just met. The equal tangent lengths then follow as corresponding sides of congruent triangles. Read the numbered proof below, one justification per step, and see the same shape appear there.

TRY
Which congruence criterion proves $\triangle OQP \cong \triangle ORP$?

Answer: RHS: common hypotenuse OP, equal radii $OQ = OR$, right angles at Q and R.

RULE
Both right angles come from Theorem 10.1: the radius is always perpendicular to its own tangent.

KEY
Same radius, same hypotenuse, right angle at the point of contact: RHS does the rest.

Pull the two triangles apart

same radius, same right angle, same hypotenuse, so PQ = PR

Step 5 says the two triangles are the same triangle. Lay them side by side and the matched parts line up: radius to radius, right angle to right angle, the shared OP to itself. What is left over, PQ and PR, must match too.

The two right triangles OQP and ORP share a radius, a right angle and the hypotenuse OP, so PQ equals PR.

Given: A point P sits outside a circle with centre O. Two tangents PQ and PR touch the circle at Q and R. The radii OQ and OR go to the two points of contact, and OP is joined.

To Prove: The two tangents drawn to a circle from an external point are equal in length: $PQ = PR$.

Proof:

	Statement	Reason
1.	let P be a point outside the circle, with tangents PQ and PR touching it at Q and R. Draw the radii OQ and OR, and join OP	*This is the construction the proof compares.*
2.	$OQ = OR$	*Both are radii of the same circle.*
3.	$\angle OQP = \angle ORP = 90°$	*Each is the angle between a radius and the tangent at its own point of contact. The theorem before this one makes every such angle a right angle.*
4.	$OP = OP$	*This is the same segment, common to both triangles.*
5.	$\triangle OQP \cong \triangle ORP$	*By the RHS criterion: the hypotenuse OP is common, and the radii $OQ = OR$ are equal.*
6.	$PQ = PR$	*Corresponding sides of congruent triangles are equal. This is CPCT.*

■

A SCHOLAR INDIA REMEMBERS

Walk me through the steps again, slowly. Join the centre to the outside point. Join the centre to each point of contact. Each radius meets its tangent at $90°$*. The two right triangles share their hypotenuse, and their radii are equal. So the triangles are congruent, and the two tangents are equal.*

Gautama · the sarus crane

CHECK YOURSELF

15. In the proof that $PQ = PR$, why are OQ and OR equal?

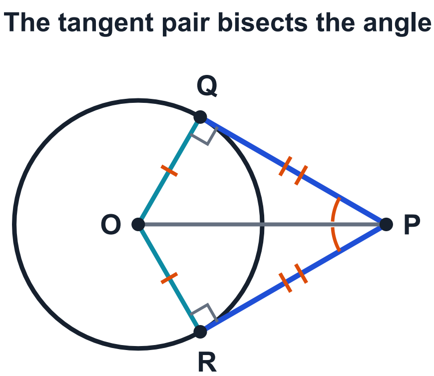
- **A.** because both are tangents from the same point
- **B.** because both are radii of the same circle
- **C.** because the two triangles are given as congruent from the start
- **D.** because Q and R are the same distance from P

16. Which earlier theorem supplies the two right angles the RHS step needs?

- **A.** the theorem that opposite angles of a cyclic quadrilateral are supplementary
- **B.** the fact that all radii of a circle are equal
- **C.** the tangent-perpendicular-to-radius theorem
- **D.** the definition of a secant

3 more in this set online.

The tangent pair bisects the angle



The same circle and tangents, now with equal tangent lengths marked, and two equal arcs at P splitting the angle into two equal halves.

Let us look back at the congruence we just used to prove the two tangent lengths equal. It gives us one more fact for free.

Triangle OQP is congruent to triangle ORP. So every corresponding pair of angles is equal too, not only the sides. In particular, $\angle OPQ = \angle OPR$.

So the line OP, from the centre to the external point, bisects the angle between the two tangents. We get this from CPCT, applied a second time to the same congruence. No new construction is needed.

Your turn: if $\angle OPQ = 35°$, what is $\angle OPR$? (Answer: $35°$, since OP bisects the angle between the tangents.)

RULE
OP is the angle bisector of the angle between the two tangents from P.

KEY
Equal tangent lengths come with equal angles too: CPCT gives both for free.

CHECK YOURSELF

17. For two tangents PQ and PR drawn from an external point P, the line OP to the centre

- **A.** is perpendicular to QR but does not bisect the angle
- **B.** bisects the tangent lengths PQ and PR instead
- **C.** has no fixed relationship to $\angle QPR$
- **D.** bisects $\angle QPR$, the angle between the tangents

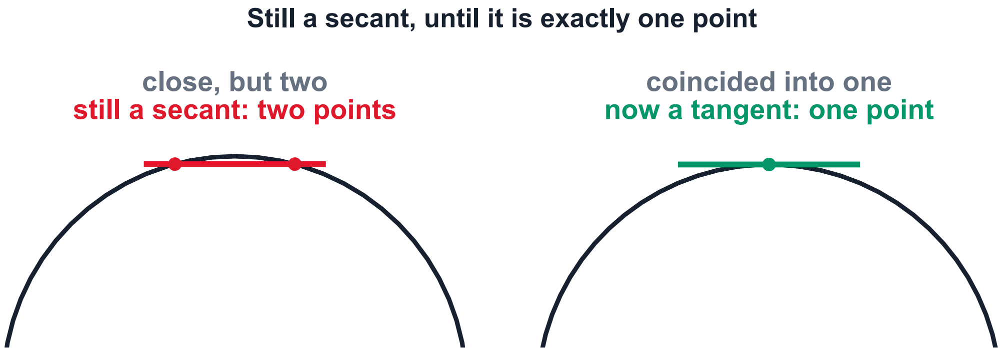
18. Why does the same congruence, $\triangle OQP \cong \triangle ORP$, give both equal tangent lengths AND equal angles at P?

- **A.** tangent lengths and angles are always numerically equal to each other
- **B.** it is a coincidence specific to this one theorem
- **C.** OP is drawn twice, once for each fact
- **D.** congruent triangles match in every corresponding part

1 more in this set online.

Common mistakes

A tangent is not two very close points



Two panels show the same arc, with two close but separate points on the left and one coincided point on the right.

You may already be carrying this mix-up.

You might picture a tangent as touching a circle at two points placed very close together, rather than at exactly one point. That picture is wrong, however close the two points seem.

Only when the two points coincide EXACTLY, becoming one single point, does the line become a tangent.

Check your own picture against that rule. The distinction is not about closeness. It is about whether there are two points, or one.

IS A LINE THROUGH TWO POINTS THAT SIT VERY CLOSE TOGETHER ON A CIRCLE A TANGENT?

WEAK
A line passes through P and a second point Q that sits very near P on the circle. It looks like it touches at one place, so it gets called a tangent. But count the points where the line meets the circle: P and Q are two separate points, however close together, so the line is a secant, not a tangent.

STRONG
The tangent at P meets the circle at P and at no other point at all, however closely you look. One point of contact, not two close ones, is what makes a line a tangent.

CAUTION
“Close together” is not “the same point”: a tangent needs exact coincidence, not proximity.

BACK
A tangent is a secant’s limit · p. 328

KEY
A secant stays a secant no matter how close its two points get. Only exact coincidence makes a tangent.

A SCHOLAR INDIA REMEMBERS

Two very close points are still two points. So a line through them still cuts the circle twice. What is the reason a tangent is different? A tangent meets the circle at exactly one point. Even when the two points are very close, the line is a secant. It becomes a tangent only when the two points meet.
Gautama · the sarus crane

WATCH OUT

The trap: Zooming into the circle until the two crossing points look like one, the student calls the closeness a tangent.

The correction: Count the points where the line meets the circle: two makes it a secant no matter how small the gap, and only one makes it a tangent.

CHECK YOURSELF

19. A student says: “A tangent touches a circle at two points that are extremely close together.” Is this correct?

- **A.** Yes — when two points are close enough, the line counts as a tangent
- **B.** Yes, but only for very small circles
- **C.** No — however close two points are, the line is still a secant
- **D.** No — a tangent actually touches the circle at zero points

20. What exactly has to happen to a secant’s two intersection points for the line to become a tangent?

- **A.** the two points must move to within a hundredth of the radius
- **B.** the two points must coincide exactly into a single point
- **C.** the two points must both move to the centre of the circle
- **D.** the line must be rotated by exactly 90°

3 more in this set online.

The distance to the centre is the hypotenuse

A right triangle sits at the point of contact. It often gets remembered only as "use Pythagoras", so the two given lengths get squared and added, no matter which two they are. That is not always the right move.

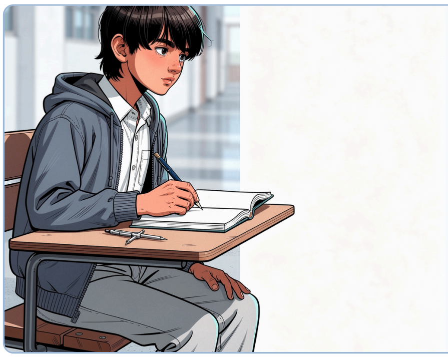
The right angle sits at the point of contact. The side facing it, from the centre to the outside point, is the hypotenuse. The hypotenuse is the square you subtract from, never a square you add.

Your turn: a radius is 7 cm, and the distance from the centre to an outside point is 25 cm. What is the tangent length? (Answer: 24 cm, since $25^2 - 7^2 = 625 - 49 = 576$.)

BACK
Tangent is perpendicular to the radius · p. 331

CAUTION
Mark the right angle at the point of contact first. The side facing it, OQ, is the one you subtract from.

KEY
The tangent always comes out shorter than the distance to the centre. Longer means you added instead of subtracting.



FIRST TRY
~~Radius 12 cm, distance to the centre 20 cm. I squared both and added: $12^2 + 20^2 = 544$, so the tangent is $\sqrt{544} \approx 23.3$ cm.~~

SECOND LOOK
The right angle sits at the point of contact, so the distance to the centre is the hypotenuse, the longest side. I should subtract: $20^2 - 12^2 = 256$, so the tangent is $\sqrt{256} = 16$ cm.

The line to the centre is the hypotenuse. Take the square of the radius from its square, never add.

FIND THE TANGENT LENGTH WHEN THE RADIUS IS 12 CENTIMETRES AND THE DISTANCE TO THE CENTRE IS 20 CENTIMETRES.

WEAK
Square and add: $PQ^2 = 20^2 + 12^2 = 544$, so $PQ \approx 23.3$ cm. But 23.3 cm is longer than the 20 cm from the outside point to the centre, and a leg of a right triangle is never longer than its hypotenuse.

STRONG
The right angle sits at P, so OQ is the hypotenuse: $PQ^2 = OQ^2 - OP^2 = 20^2 - 12^2 = 256$, so $PQ = 16$ cm.

Worked examples

WORKED EXAMPLE · **A tangent length from the radius and the distance to the centre**

1.	a tangent touches a circle of radius 5 cm at the point P; the distance from the centre O to an external point Q is $OQ = 13$ cm	*These are the given measurements.*
2.	$\angle OPQ = 90°$	*The radius to the point of contact is always perpendicular to the tangent there.*
3.	$OQ^2 = OP^2 + PQ^2$	*Triangle OPQ is right-angled at P, so Pythagoras' theorem applies.*
4.	$13^2 = 5^2 + PQ^2$	*Substitute the given lengths.*

WORKED EXAMPLE · A tangent length from the radius and the distance to the centre

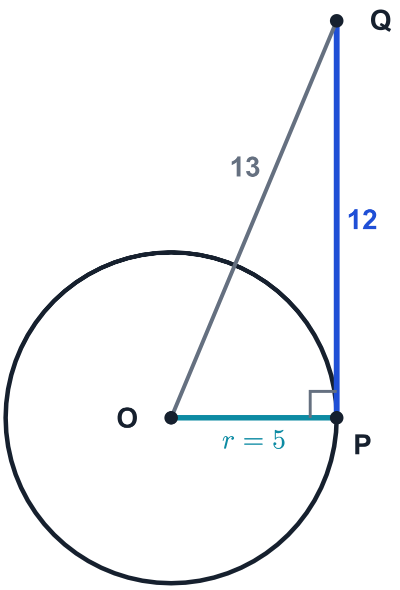
5.	$PQ^2 = 169 - 25 = 144$	*Evaluate the squares, then subtract.*
6.	$PQ = 12$ cm	*Take the square root of* 144.
7.	$5^2 + 12^2 = 25 + 144 = 169 = 13^2$	CHECK ***Check the answer by substituting back.*** *5, 12 and 13 satisfy Pythagoras exactly, so $PQ = 12$ cm is correct.*

TRY
A tangent touches a circle of radius 8 cm at P. From an external point Q, $OQ = 17$ cm. Find the tangent length PQ.

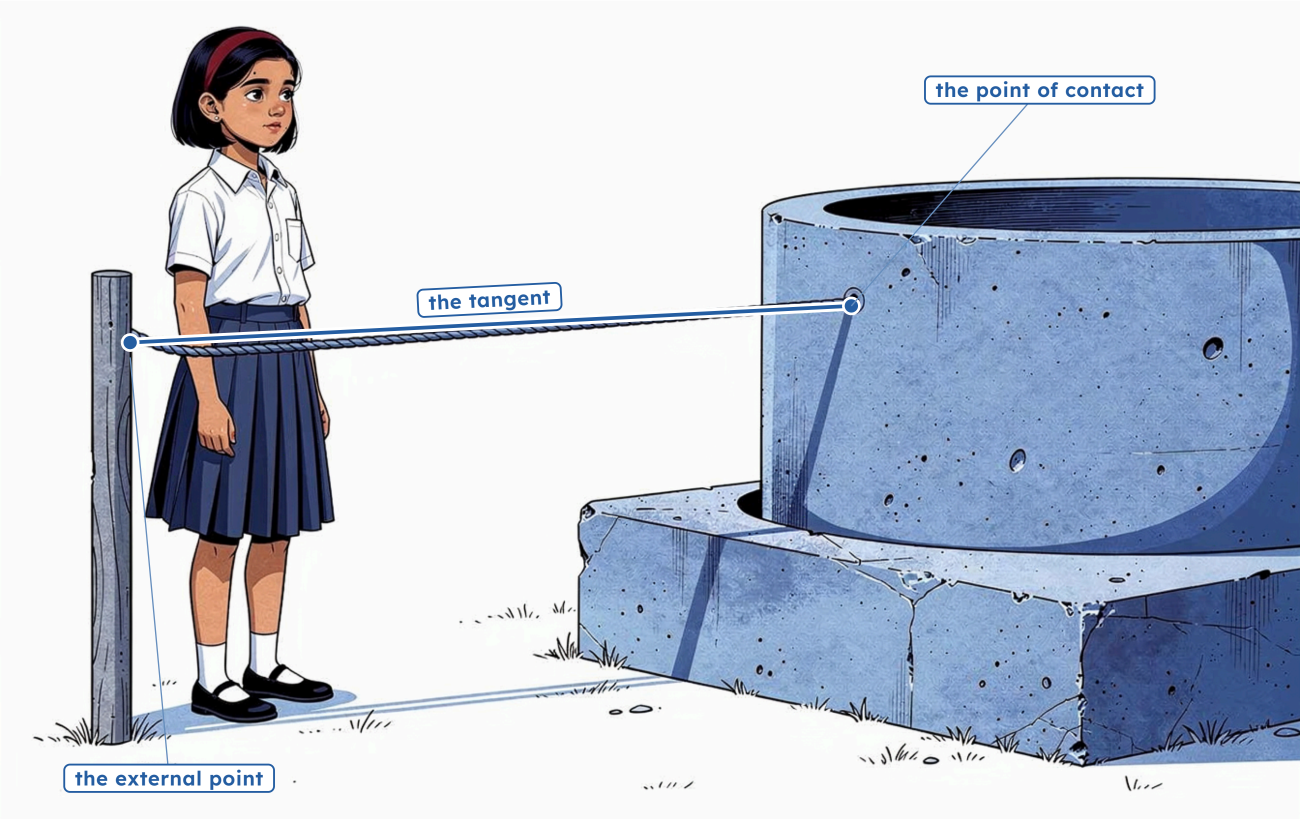
Answer: Since OP is perpendicular to the tangent, $PQ^2 = 17^2 - 8^2 = 225$, so $PQ = 15$ cm.

KEY
The radius to the point of contact is always perpendicular to the tangent, which gives the right angle Pythagoras needs.

Tangent length from radius and distance



A right triangle with radius 5 at OP, tangent 12 at PQ, and OQ equal to 13, with a right angle at P.



The taut rope runs from the stake and meets the round tank at one point. It lies along a tangent.

FINDING A TANGENT LENGTH FROM THE RADIUS AND THE DISTANCE TO THE CENTRE

1. **Mark the right angle** The radius meets the tangent at 90°, at the point of contact P. Here the radius is $OP = 5$ cm and $\angle OPQ = 90°$.
2. **Name the hypotenuse** The side from the centre to the outside point is the hypotenuse. Here that is $OQ = 13$ cm.
3. **Subtract the squares** $PQ^2 = OQ^2 - OP^2 = 13^2 - 5^2 = 144$, so $PQ = 12$ cm.
4. **Check the tangent is shorter** The tangent must come out shorter than OQ. 12 cm is shorter than 13 cm, so the answer checks out.

CHECK YOURSELF

21. In finding a tangent length from the radius and the distance to the centre, which theorem makes the triangle right-angled in the first place?

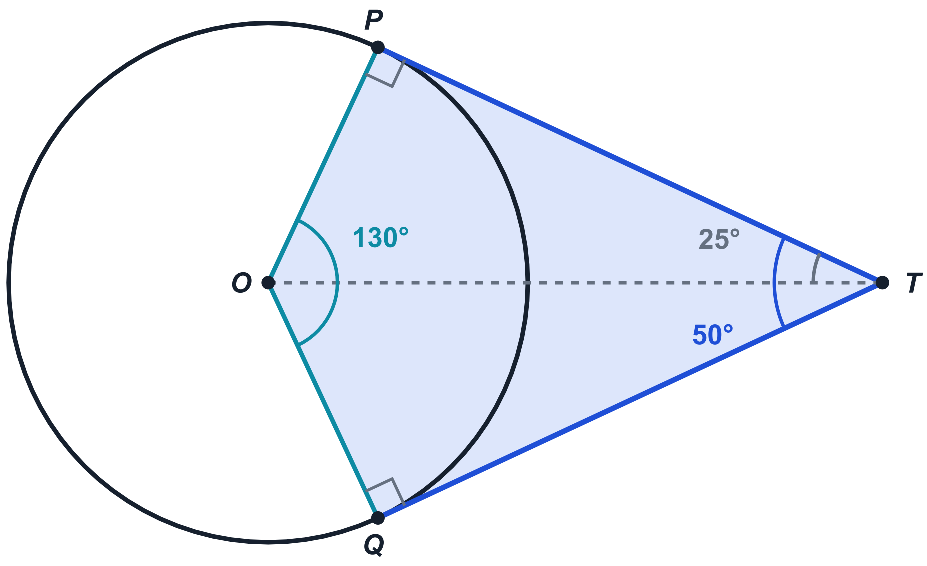
- **A.** the Pythagorean theorem itself supplies the right angle
- **B.** the tangent-perpendicular-to-radius theorem
- **C.** the equal-tangent-lengths theorem
- **D.** no theorem is needed — the triangle is automatically right-angled

22. A tangent touches a circle of radius 9 cm at P. From an external point Q, $OQ = 15$ cm. What is the tangent length PQ?

- **A.** 24 cm, from adding the two given lengths
- **B.** 6 cm, from subtracting the two given lengths
- **C.** 18 cm, from doubling the radius
- **D.** 12 cm, from $15^2 = 9^2 + PQ^2$

2 more in this set online.

The centre angle and the tangent angle add to 180



130 + 50 = 180: the two right angles take the other 180

The four angles of OPTQ add to 360°, and two of them are right angles. Whatever is left, 180°, is shared between the angle at the centre and the angle at T, so knowing one gives the other.

In the kite OPTQ, the right angles at P and Q leave the angle at O, 130 degrees, and at T, 50 degrees.

WORKED EXAMPLE · The angle between two tangents, given the angle at the centre

1.	two tangents TP and TQ are drawn to a circle with centre O from an external point T, touching it at P and Q; $\angle POQ = 130°$	*These are the given measurements.*
2.	$\angle OPT = \angle OQT = 90°$	*Each radius meets its own tangent at a right angle.*
3.	$\angle POQ + \angle OPT + \angle PTQ + \angle TQO = 360°$	*The four angles of the quadrilateral $OPTQ$ add to $360°$.*
4.	$130° + 90° + 90° + \angle PTQ = 360°$	*Substitute $\angle POQ = 130°$ and the two right angles from step 2.*
5.	$\angle PTQ = 50°$	*Solve for $\angle PTQ$.*
6.	$\angle OTP = 25°$	*The line OT bisects the angle between the two tangents, so it splits $\angle PTQ$ into two equal halves.*
7.	$130 + 50 = 180$	CHECK ***Check it without redoing the work.*** *The two right angles use up $180°$ of the quadrilateral, so the angle at the centre and the angle between the tangents must always add to $180°$.*

RULE
The angle at the centre and the angle between the two tangents always add to 180°.

TRY
Two tangents TP and TQ touch a circle with centre O at P and Q. If $\angle POQ = 140°$, find $\angle PTQ$.

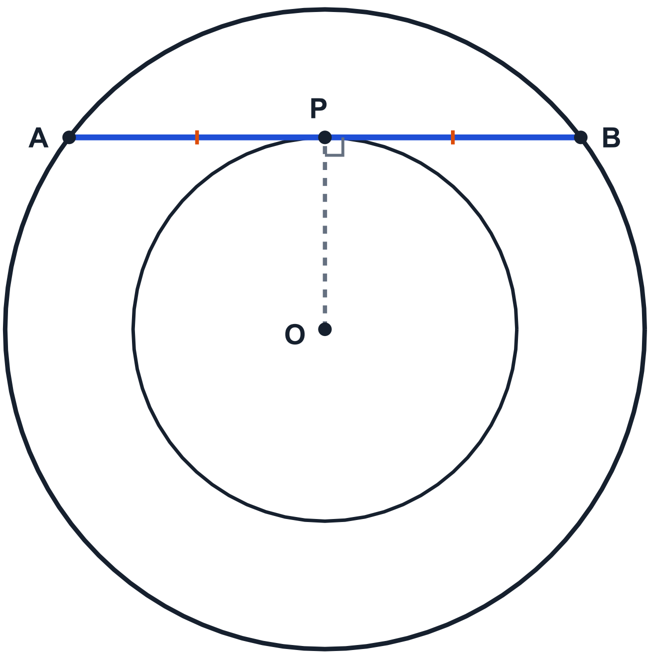
Answer: The two right angles use up 180°, so $\angle PTQ = 360° - 140° - 180° = 40°$.

KEY
Two right angles at the points of contact turn the tangent pair into a quadrilateral, so the four angles simply add.

FINDING THE ANGLE BETWEEN TWO TANGENTS FROM THE ANGLE AT THE CENTRE

1. **Mark the two right angles** At each point of contact, the radius meets the tangent at 90°. Here $\angle OPT = \angle OQT = 90°$.
2. **Add the quadrilateral's angles** The angle at the centre, the two right angles, and the angle at the outside point add to 360°. Here $130° + 90° + 90° + \angle PTQ = 360°$.
3. **Solve for the angle at T** $\angle PTQ = 360° - 130° - 90° - 90° = 50°$.
4. **Halve it for the bisected angle** The line from the centre to the outside point bisects that angle, so $\angle OTP = 50°/2 = 25°$.

The smaller circle bisects the chord



Two circles share centre O, with a chord AB touching the smaller one at P and matching ticks on AP and PB.

WORKED EXAMPLE · **A chord of the larger circle, bisected where it touches the smaller one**

1.	two circles share the same centre O; a chord AB of the larger circle touches the smaller circle at the point P	*This is the given configuration.*
2.	join OP	*This is the auxiliary line the proof turns on.*
3.	OP is perpendicular to AB	*AB is a tangent to the smaller circle at P. A radius is always perpendicular to the tangent at its point of contact.*

WORKED EXAMPLE · **A chord of the larger circle, bisected where it touches the smaller one**

4.	OP is also a line from the centre of the LARGER circle to a point on its own chord AB	*O is the centre of both circles, and AB is a chord of the larger circle.*
5.	a perpendicular from a circle's centre to a chord always bisects that chord	*We already know this fact from Class 9.*
6.	$AP = BP$	*By steps 3 and 4, OP is a perpendicular from the larger circle's centre to its own chord AB. So, by step 5, it bisects that chord.*
7.	reflecting the whole figure across the line OP swaps A and B but leaves the picture unchanged	CHECK ***Check the result by symmetry.*** *The reflection sends AP to BP and back again, so the two lengths have to be equal.*

RULE
A perpendicular from a circle's centre to any chord always bisects that chord.

KEY
The tangent gives the right angle; the chord-bisection rule (Class 9) finishes the proof.

CHECK YOURSELF

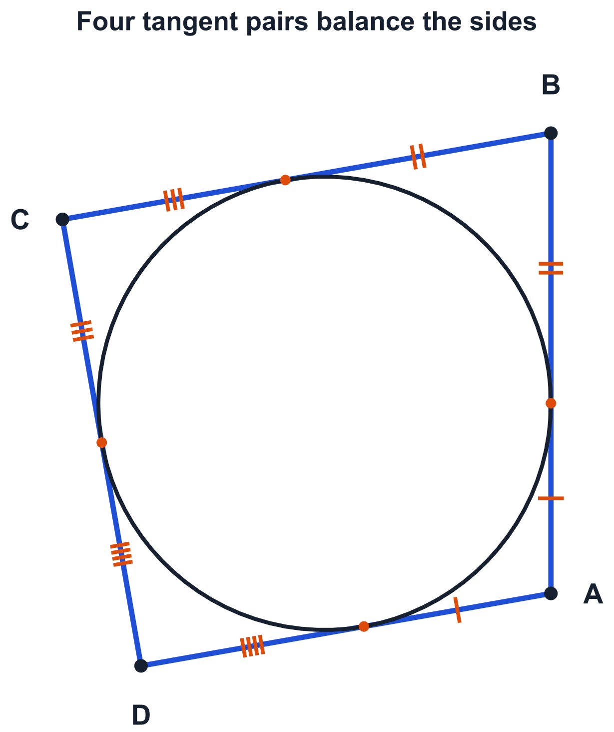
23. In proving that P bisects the chord AB, why is OP drawn?

- **A.** OP is the diameter of the smaller circle
- **B.** OP is equal in length to AB
- **C.** OP is needed only to measure the distance between the two circles
- **D.** OP is the radius here, and a centre-to-chord line there

24. Which two facts, one about tangents and one about chords, combine to prove $AP = BP$?

- **A.** the tangent-radius fact, and the Class 9 chord-bisection fact
- **B.** the equal-tangent-lengths theorem, and the fact that all chords of a circle are equal
- **C.** the angle-bisector fact, and the definition of a secant
- **D.** the RHS congruence criterion alone, with no chord fact needed

1 more in this set online.



A quadrilateral ABCD surrounds a circle touching all four sides, with matching tick counts of one, two, three and four at each corner.

WORKED EXAMPLE · **Opposite sides of a quadrilateral that circumscribes a circle**

1.	a quadrilateral $ABCD$ is drawn so all four sides touch a circle, at the points P, Q, R, S on sides AB, BC, CD, DA	*This is the given configuration.*
2.	$AP = AS,\ BP = BQ,\ CR = CQ,\ DR = DS$	*Tangents drawn from one external point to a circle are always equal in length.*
3.	$(AP + PB) + (CR + RD) = (AS + SD) + (BQ + QC)$	*Add the four equal pairs from step 2, grouped by vertex on each side.*
4.	$AB + CD = AD + BC$	$AP + PB = AB$, $CR + RD = CD$, $AS + SD = AD$ *and* $BQ + QC = BC$. *Each pair sits along one side.*
5.	for a square of side s, each tangent segment is $s/2$, so $AB + CD = 2s$ and $AD + BC = 2s$ too	CHECK ***Check the identity on a shape you know.*** *A square is a special circumscribing quadrilateral, and both sums come out equal there too.*

BACK
The two tangent lengths are equal · p. 333

RULE
Name the four points of contact first, then write every tangent-length equality before adding.

KEY
Four vertices, four pairs of equal tangents: add them the right way and the sides balance.

CHECK YOURSELF

25. In proving $AB + CD = AD + BC$ for a quadrilateral circumscribing a circle, what is the key step?

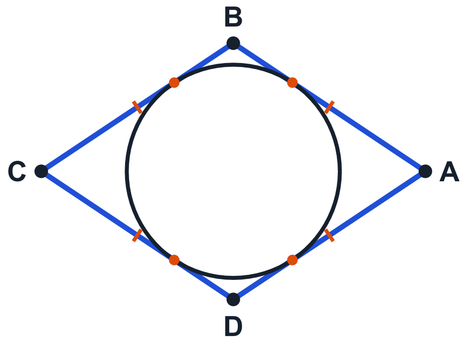
- **A.** naming the contact points and adding equal tangent pairs
- **B.** measuring all four sides directly with a ruler
- **C.** assuming the quadrilateral is a rectangle from the start
- **D.** applying the RHS congruence criterion directly to the whole quadrilateral

26. A quadrilateral $PQRS$ circumscribes a circle, with $PQ = 7$ cm, $QR = 5$ cm, $RS = 4$ cm. What is SP?

- **A.** 4 cm, the same as RS
- **B.** 8 cm, from a mismatched combination of the given sides
- **C.** 16 cm, from adding all three given sides
- **D.** 6 cm, from $PQ + RS = QR + SP$

2 more in this set online.

A circumscribing parallelogram is a rhombus



A parallelogram ABCD drawn as a diamond around a circle touching all four sides, with one matching tick mark on each side.

WORKED EXAMPLE · Why a parallelogram that circumscribes a circle must be a rhombus

1.	a parallelogram $ABCD$ circumscribes a circle	*This is the given configuration.*
2.	$AB = CD$ and $AD = BC$	*Opposite sides of a parallelogram are always equal.*
3.	$AB + CD = AD + BC$	*This is the previous result, since $ABCD$ circumscribes a circle.*
4.	$2 \cdot AB = 2 \cdot AD$	*Substitute $AB = CD$ and $AD = BC$ from step 2 into step 3.*
5.	$AB = AD$	*Divide both sides by 2.*
6.	$AB = BC = CD = AD$	*Combine $AB = AD$ with the parallelogram's own $AB = CD$ and $AD = BC$: all four sides come out equal.*

WORKED EXAMPLE · Why a parallelogram that circumscribes a circle must be a rhombus

7.	$ABCD$ is a rhombus	*A parallelogram with all four sides equal is a rhombus, by definition.*
8.	if $AB = 6$ cm in such a parallelogram, the proof forces $AD = 6$ cm too, so all four sides equal 6 cm	CHECK ***Check the conclusion with a number.*** *Once one side is fixed, the proof pins every other side to the same value, exactly what a rhombus requires.*

RULE
A parallelogram circumscribing a circle is always a rhombus. This is never true for a general parallelogram otherwise.

KEY
The side-sum identity plus "opposite sides equal" forces all four sides equal.

CHECK YOURSELF

27. Why does a parallelogram circumscribing a circle have to be a rhombus?

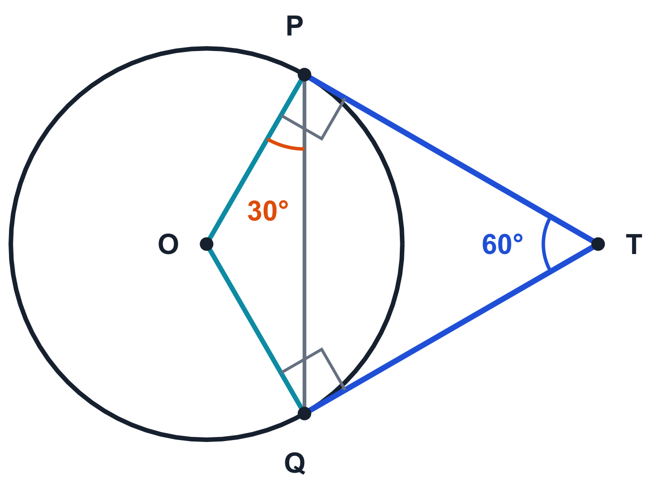
- **A.** every shape that circumscribes a circle is automatically a rhombus
- **B.** the identity plus equal opposite sides forces equal sides
- **C.** a circle can only be inscribed in a rhombus, never a general parallelogram
- **D.** the parallelogram's opposite angles must be right angles

28. A student argues a RECTANGLE that circumscribes a circle must also be a square, using the same argument as the rhombus proof. Is this valid?

- **A.** No — the argument only works for non-rectangular parallelograms
- **B.** Yes — a rectangle is a parallelogram too
- **C.** No — a circle can never be inscribed in a rectangle
- **D.** Yes, but only if the rectangle's diagonals are equal

1 more in this set online.

The angle at T doubles the angle at P



Two tangents from T touch a circle at P and Q, with angles of 30 degrees at P and 60 degrees at T.

WORKED EXAMPLE · **Relating the angle between two tangents to the angle at the point of contact**

1.	two tangents TP and TQ are drawn from an external point T to a circle with centre O, touching it at P and Q	*This is the given configuration.*
2.	$TP = TQ$	*The two tangent lengths from one external point are always equal.*
3.	$\triangle TPQ$ is isosceles	*Two of its sides, TP and TQ, are equal, from step 2.*
4.	$\angle TPQ = 90° - 1/2\angle PTQ$	*The two base angles of an isosceles triangle are equal, and all three angles of a triangle add to 180°.*
5.	$\angle OPT = 90°$	*The radius to the point of contact is always perpendicular to the tangent there.*
6.	$\angle OPQ = \angle OPT - \angle TPQ$	*Angle OPQ is what remains of the right angle at P, once angle TPQ is taken out of it.*
7.	$\angle OPQ = 90° - (90° - 1/2\angle PTQ) = 1/2\angle PTQ$	*Substitute step 5 and step 4 into step 6, then simplify.*
8.	$\angle PTQ = 2 \cdot \angle OPQ$	*Rearrange the previous step.*
9.	with $\angle OPQ = 30°$, the relation predicts $\angle PTQ = 2 \cdot 30° = 60°$	CHECK ***Check the relation with the numbers in the figure.*** *A 30° angle at P and a 60° angle at T fit exactly, since 60 is double 30.*

KEY
Equal tangents make an isosceles triangle. Combine that with the radius's right angle.

CHECK YOURSELF

29. In proving $\angle PTQ = 2 \cdot \angle OPQ$, why is $\triangle TPQ$ isosceles?

- **A.** $\angle PTQ$ is given as 90°
- **B.** O is equidistant from T, P, and Q
- **C.** P and Q both lie on the same circle
- **D.** $TP = TQ$, equal tangents from T

30. The proof combines the isosceles-triangle angle result with the tangent-perpendicular-to-radius fact. What does the second fact contribute?

- **A.** it proves $TP = TQ$ directly, without needing the equal-tangent-lengths theorem
- **B.** it fixes the value of $\angle PTQ$ directly
- **C.** it fixes $\angle OPT = 90°$, letting $\angle OPQ$ be found by subtraction
- **D.** it shows that O, P, and Q are collinear

2 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap

- TANGENT PERPENDICULAR TO RADIUS: the radius is perpendicular to the tangent at the point of contact. Radius 5 cm, tangent 12 cm, angle 90°.
- TANGENT LENGTH BY PYTHAGORAS: $OQ^2 = OP^2 + PQ^2$. For $OP = 5$ cm and $OQ = 13$ cm, $PQ = 12$ cm.
- EQUAL TANGENTS: the two tangents from one external point are always equal. Here $PQ = PR = 12$ cm.
- ANGLE BISECTOR: OP bisects the angle between the two tangents. A 60° angle at T pairs with 30° at P.
- CIRCUMSCRIBING SHAPES: opposite sides of a circumscribing quadrilateral add to the same total. $AB + CD = AD + BC$.

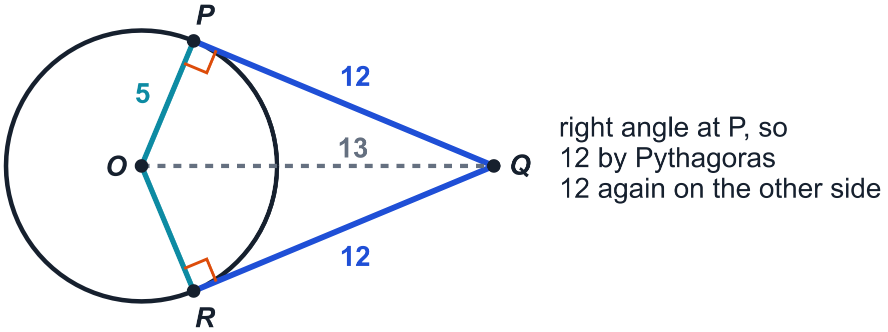
Let us look back over the whole chapter and see one fact doing almost all the work. The radius to a point of contact is always perpendicular to the tangent there. That single right angle forces OQ to be longer than OP in the first proof, and it supplies the right angle the RHS criterion needs in the second. Through that same congruence, we get both the equal tangent lengths and the angle bisector at the centre.

BACK
Tangent is perpendicular to the radius · p. 331

RULE
If a proof needs a right angle at the point of contact, it is using this same fact.

KEY
One right angle, radius perpendicular to tangent, drives both theorems here.

One right angle, and what follows from it



Find the right angle first: it gives Pythagoras his triangle, gives RHS its right angle, and so gives the two equal tangent lengths.

A right angle where the radius meets the tangent gives the 5, 12, 13 triangle, and RHS copies the 12 to the other tangent.

CHECK YOURSELF

31. This chapter's results — perpendicularity, equal tangent lengths, and the angle bisector — all trace back to which single fact?

- **A.** the radius is perpendicular to the tangent there
- **B.** every chord of a circle is bisected by a radius through its midpoint
- **C.** all tangents drawn from external points are equal in length
- **D.** the centre of a circle is equidistant from every point on it

32. If the radius were NOT perpendicular to the tangent at the point of contact, which of this chapter's results would still stand?

- **A.** none of them — both later results depend on this same right angle through RHS
- **B.** the equal-tangent-lengths result, since it only uses the definition of a tangent
- **C.** the angle-bisector result, since it only uses the equal tangent lengths
- **D.** all of them, since they are independent facts proved separately

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

33. A line touches a circle at exactly one point, P. Which of these statements about that line must be true?

- **A.** it must pass through the circle's centre
- **B.** it is perpendicular to the radius drawn to P
- **C.** it must be exactly twice as long as the radius
- **D.** it cannot be extended beyond the point P in either direction

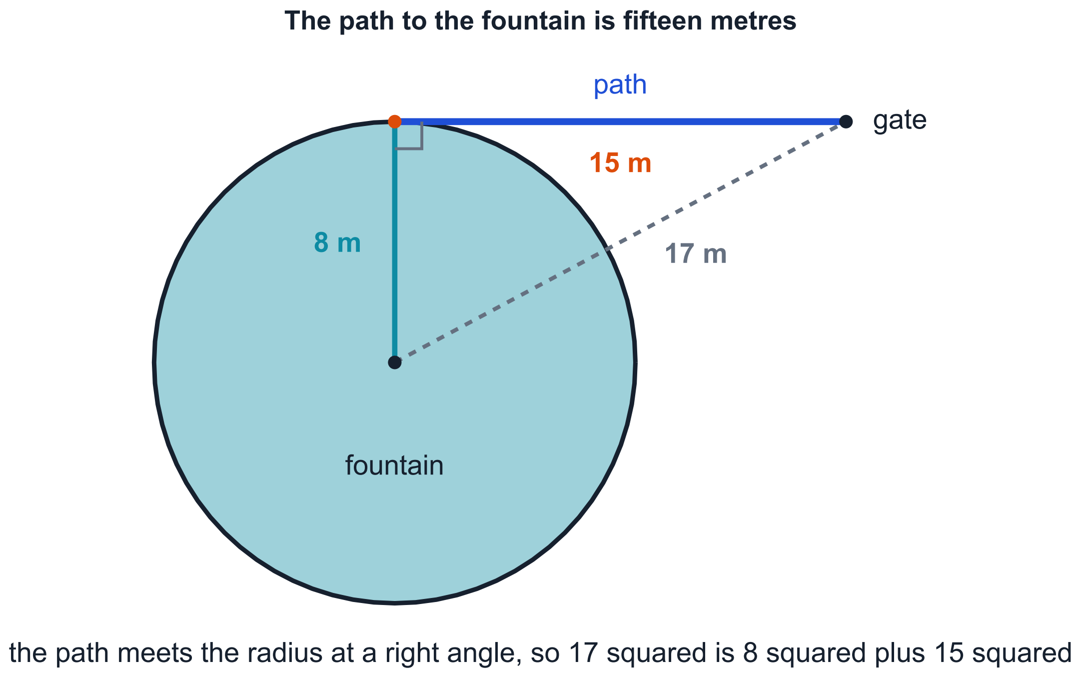
34. In the standard tangent-perpendicular-to-radius proof, O is the centre, P the point of contact, and Q any other point on the tangent. Why is OQ longer than OP?

- **A.** because Q lies inside the circle, and every point inside is closer to O than the circle's boundary is
- **B.** because the tangent line is drawn longer than the radius, by convention
- **C.** because Q, being a point on the tangent other than P, lies outside the circle, while P lies on it
- **D.** because OQ and OP are corresponding sides of congruent triangles, which forces them to differ by a fixed ratio

3 more in this set online.

IN EVERY CHAPTER

Where you will meet this



The path just touches the fountain, so it is a tangent, and the radius to that touching point meets it at a right angle. That right angle turns the two given distances into the third by Pythagoras.

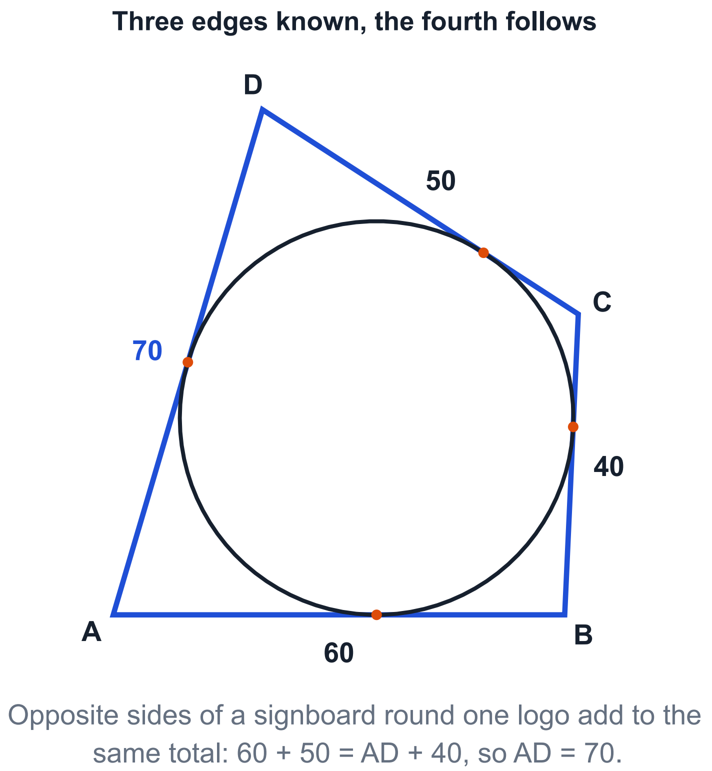
Spot the tangent and the right angle comes free, turning a word problem into a Pythagoras sum.

You see round shapes with straight lines near them all the time. Here are seven places a tangent, or its length, decides how something is built or measured.

- **A bypass road and a roundabout.** A bypass road runs past a circular roundabout, touching its edge at exactly one point.
  The maths: the road that touches the circle once is a tangent. A road that crosses it twice would be a secant.

- **Laying a path to a fountain.** A circular fountain has radius 8 m. A gate stands 17 m from the fountain's centre. A path from the gate touches the fountain's edge without crossing it.
  The maths: the path's length is the tangent length from the gate. It follows from the right angle between the radius and the tangent.
  $17^2 = 8^2 + \text{path}^2$, so $\text{path}^2 = 289 - 64 = 225$. The path is 15 m.

BACK
Tangent, secant and point of contact · p. 326



Four tangent lengths pair up around the circle, so three edges of the board fix the fourth.

- **Roping off a circular display.** A shop ropes off a round stand, radius 0.9 m. Two ropes run from a post 1.5 m from the centre, just touching the edge.
  The maths: the two tangents drawn from one outside point are always equal in length.
  Each rope is $\sqrt{1.5^2 - 0.9^2} = \sqrt{1.44} = 1.2$ m, so both ropes measure 1.2 m.
- **Drawing a rangoli around a circle.** You draw a circle for a rangoli. Then you draw a parallelogram around it, each side just touching the circle.
  The maths: a parallelogram whose four sides all touch one circle is a rhombus. So all four sides come out equal.
  If one side is 30 cm, all four are 30 cm, and the border is $4 \cdot 30 = 120$ cm long.
- **Where a running track meets its bend.** A running track has straight sections and curved bends. Each straight meets a bend without a kink, touching it at exactly one point.
  The maths: the straight is a tangent to the bend, and a tangent touches a circle at exactly one point.
- **Roping off a pond from the bank.** You want one straight rope from where you stand to just touch a circular pond, without crossing into the water.
  The maths: from outside the pond you can draw two such ropes. From a point on the bank, only one. From inside the water, none at all.
- **Cutting a signboard around a round logo.** A four-sided signboard $ABCD$ is cut so each edge just touches a round logo. You know three edges.
  The maths: when all four sides touch one circle, opposite sides add to the same total. So $AB + CD = AD + BC$.
  With $AB = 60$, $BC = 40$ and $CD = 50$ cm: $60 + 50 = AD + 40$, so $AD = 70$ cm.

Your turn. You stand 25 m from the centre of a circular skating rink, radius 7 m. A straight rope from where you stand just touches the rink's edge without crossing it. How long is the rope? *Answer:* $25^2 - 7^2 = 625 - 49 = 576 = 24^2$. The rope is 24 m.

Practice: Exercise 10.1

PRACTICE SET

1. PRACTICE A tangent touches a circle of centre O and radius 3 cm at the point P. An external point Q is 5 cm from the centre. Find the tangent length PQ. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	mark the right angle at P, where the radius OP meets the tangent, so triangle OPQ is right-angled at P and OQ is the side facing that right angle	*draw and mark before calculating — the side facing the right angle is the hypotenuse, and knowing which side that is decides whether the next line adds or subtracts*
2.	$OQ^2 = OP^2 + PQ^2$, so $5^2 = 3^2 + PQ^2$	*write Pythagoras with the hypotenuse by itself on one side before any number goes in*
3.	$PQ^2 = 25 - 9 = 16$, so $PQ = 4$ cm	*subtract, then take the square root*
4.	the check — 4 cm is shorter than the 5 cm from Q to the centre	*the tangent length is a leg of the triangle and OQ is the hypotenuse, so the answer has to come out smaller; if it came out bigger, the two squares were added instead of subtracted*

2. PRACTICE A tangent touches a circle of centre O and radius 10 cm at the point P. The tangent length from an external point Q is $PQ = 24$ cm. Find the distance OQ. (Same triangle, the other way round — this time the unknown is the hypotenuse, so the two squares are added.)

3. PRACTICE From an external point Q, the tangent to a circle with centre O touches it at P and has length $PQ = 20$ cm. The distance OQ is 25 cm. Find the radius of the circle. (Mark the right angle at P first, then ask which of the three lengths is facing it.)

4. PRACTICE How many tangents can a circle have at one point on it?

5. PRACTICE A circle can have at most how many parallel tangents?

6. PRACTICE A tangent touches a circle of radius 6 cm at the point P. The distance from the centre O to an external point Q is $OQ = 10$ cm. Find the tangent length PQ.

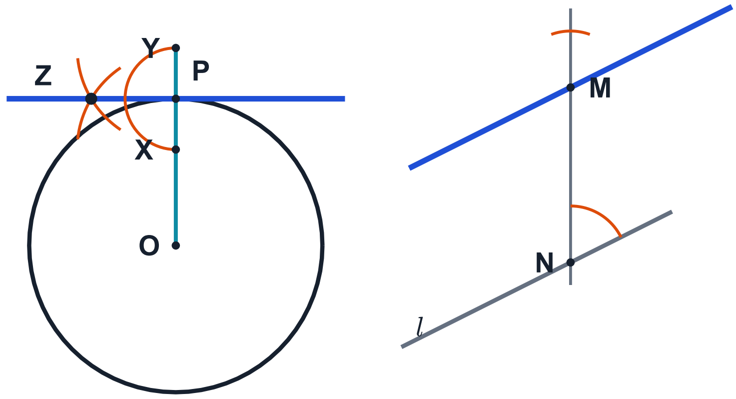
7. PRACTICE Draw a circle, then a line lying outside it. Construct two lines parallel to that line — one a tangent to the circle, and the other a secant.

ANSWERS 1. $PQ^2 = 5^2 - 3^2 = 16$, so $PQ = 4$ cm.
2. $OQ^2 = 10^2 + 24^2 = 100 + 576 = 676$, so $OQ = 26$ cm.
3. OQ faces the right angle, so $OP^2 = 25^2 - 20^2 = 625 - 400 = 225$, and the radius is 15 cm.
4. Exactly one. 5. Two — one at each end of a diameter, both perpendicular to it.

6. $PQ^2 = OQ^2 - OP^2 = 10^2 - 6^2 = 64$, so $PQ = 8$ cm.
7. Draw the diameter perpendicular to the given line, and place the tangent at the end of that diameter. Then draw any second line parallel to the given line that still cuts the circle in two points — that second line is the secant.
Full solutions: **manthika.com/learn/math10-ch10**

Further practice

Constructing a tangent, and the parallel a secant starts from



Two panels show compass constructions: one builds a tangent at P using equal arcs, the other draws a line parallel to a given one.

PRACTICE SET

8. PRACTICE A line lies in the same plane as a circle and meets it at exactly two points. Name this kind of line, and explain in one sentence how a tangent is different from it.

9. PRACTICE A line and a circle lie in the same plane and do not meet at all. Such a line is:
- **A.** a tangent to the circle
- **B.** a secant of the circle
- **C.** neither a tangent nor a secant
- **D.** a chord of the circle

10. PRACTICE True or false, with a reason: a line meeting a circle at two points that are extremely close together is a tangent to the circle.

11. PRACTICE A tangent touches a circle of radius 9 cm at the point P. The length of the tangent from an external point Q is 12 cm. Find the distance OQ from the centre to Q.

12. PRACTICE From a point Q outside a circle with centre O, the length of the tangent to the circle is 15 cm, and the distance OQ is 17 cm. Find the radius of the circle.

13. PRACTICE A tangent touches a circle at the point P. A second line is drawn through P, perpendicular to this tangent. Prove that this second line passes through the centre of the circle.

14. PRACTICE A tangent to a circle of radius 20 cm touches it at the point P. An external point Q is at a distance $OQ = 29$ cm from the centre. Find the tangent length PQ.

15. PRACTICE A circle has radius 7 cm and centre O. The tangent from an external point Q has length 24 cm. Which of these is the distance OQ?

A. 25 cm
B. 26 cm
C. 30 cm
D. 31 cm

ANSWERS

8. This line is a secant. A tangent is different because it meets the circle at exactly one point, not two.
9. C — neither a tangent nor a secant.
10. False. However close the two points are, the line stays a secant as long as they remain two distinct points. It becomes a tangent only when the two points coincide exactly into one point.
11. $OQ = 15$ cm **12.** $OP = 8$ cm
13. By the point-of-contact theorem, the radius OP is perpendicular to the tangent at P. At a given point on a line, only one perpendicular to that line can be drawn, so the second line described is exactly the line OP — which passes through the centre O.
14. $PQ = 21$ cm **15.** A — 25 cm.
Full solutions: **manthika.com/learn/math10-ch10**

Practice: Exercise 10.2

PRACTICE SET

1. PRACTICE Two tangents TP and TQ are drawn to a circle with centre O from an external point T, touching the circle at P and at Q. If $\angle POQ = 120°$, find $\angle PTQ$ and $\angle OTP$. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	draw the two radii OP and OQ and the line OT, and mark $\angle OPT = \angle OQT = 90°$	*each radius meets its own tangent at a right angle, and marking BOTH right angles first is what turns the picture into a quadrilateral whose angles can be added*
2.	$\angle POQ + \angle OPT + \angle OQT + \angle PTQ = 360°$	*$OPTQ$ is a quadrilateral, and the four angles of any quadrilateral add to 360°*
3.	$120° + 90° + 90° + \angle PTQ = 360°$, so $\angle PTQ = 60°$	*put in the given angle and the two right angles, then subtract*
4.	$\angle OTP = 1/2 \cdot 60° = 30°$	*OT bisects the angle between the two tangents, so half of $\angle PTQ$ sits on each side of it*

SOLUTION — Q1 · Worked in full

5. the check — $\angle POQ + \angle PTQ = 120° + 60° = 180°$

the two right angles always use up 180° of the quadrilateral, so the angle at the centre and the angle between the tangents must always make up the other 180°

2. PRACTICE A quadrilateral $ABCD$ is drawn so that all four of its sides touch a circle. If $AB = 10$ cm, $BC = 8$ cm and $CD = 7$ cm, find AD. (Name the four points of contact first. One pair of opposite sides added together equals the other pair added together.)

3. PRACTICE Two circles share the same centre O. The larger has radius 17 cm and the smaller has radius 8 cm. A chord of the larger circle touches the smaller circle. Find the length of that chord. (The chord is a tangent to the smaller circle, so the radius to its point of contact is perpendicular to it and cuts it in half — find half the chord first, then double it.)

4. PRACTICE Two tangents TP and TQ are drawn to a circle with centre O from an external point T. If $\angle POQ = 110°$, find $\angle PTQ$.

5. PRACTICE Prove that the tangents drawn at the two ends of a diameter of a circle are parallel to each other.

6. PRACTICE Two concentric circles have radii 5 cm and r. A chord of the larger circle, of length 8 cm, touches the smaller circle. Find r.

7. PRACTICE A quadrilateral $ABCD$ circumscribes a circle, touching side AB at P, with $AP = 4$ cm and $PB = 3$ cm. If $BC = 9$ cm and $CD = 6$ cm, find AD.

8. PRACTICE A triangle ABC circumscribes a circle, touching side BC at P, side CA at Q, and side AB at R. Show that $AR + BP + CQ$ equals half the triangle's perimeter.

9. PRACTICE Prove that a parallelogram circumscribing a circle is a rhombus.

10. PRACTICE Two tangents PA and PB are drawn from an external point P to a circle with centre O. If $\angle APB = 80°$, find $\angle POA$.

ANSWERS

1. In quadrilateral $OPTQ$ the two radii give right angles at P and Q, so $\angle PTQ = 360° - 120° - 90° - 90° = 60°$, and since OT bisects it, $\angle OTP = 30°$.

2. $AB + CD = AD + BC$, so $10 + 7 = AD + 8$ and $AD = 9$ cm.

3. Half the chord is $\sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15$ cm, so the whole chord is 30 cm.

4. In quadrilateral $OPTQ$, $\angle OPT = \angle OQT = 90°$ (radius perpendicular to tangent). The four angles sum to 360°, so $\angle POQ + \angle PTQ = 180°$, giving $\angle PTQ = 180° - 110° = 70°$.

5. Let AB be a diameter, with tangents drawn at A and at B. Each tangent is perpendicular to the radius at its own point of contact, and both radii lie along the same diameter AB. Two lines perpendicular to the same line are parallel, so the two tangents are parallel to each other.

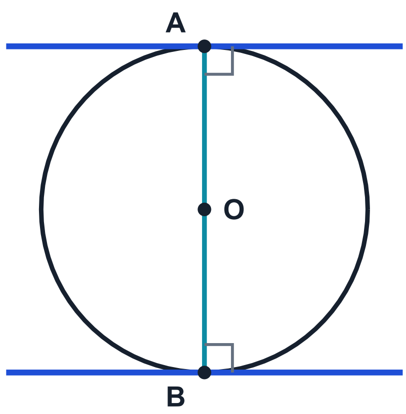
6. The perpendicular from the common centre to the chord bisects it, giving a half-chord of 4 cm. In the right triangle this forms, $r^2 + 4^2 = 5^2$, so $r^2 = 9$ and $r = 3$ cm.

7. $AB = AP + PB = 4 + 3 = 7$ cm. Since $AB + CD = AD + BC$ for any quadrilateral circumscribing a circle, $7 + 6 = AD + 9$, giving $AD = 4$ cm.
8. Let the tangent lengths from A, B, C be x, y, z. Then $AR = AQ = x$, $BR = BP = y$, and $CP = CQ = z$, since tangents from one point are always equal. The perimeter is $AB + BC + CA = (x + y) + (y + z) + (z + x) = 2(x + y + z)$. So $AR + BP + CQ = x + y + z$ is exactly half the perimeter.
9. Let the parallelogram be $ABCD$. Tangents drawn from one point are equal, so adding the two tangent lengths at each vertex around the figure gives $AB + CD = BC + AD$. In a parallelogram $AB = CD$ and $BC = AD$, so $2AB = 2BC$, and therefore $AB = BC$. Adjacent sides are equal, so all four sides are equal and the parallelogram is a rhombus.
10. OP bisects $\angle APB$, so $\angle APO = 40°$. Triangle OAP is right-angled at A (radius perpendicular to tangent), so $\angle POA = 180° - 90° - 40° = 50°$.
Full solutions: **manthika.com/learn/math10-ch10**

BACK
The two tangent lengths are equal · p. 333

Further practice

Parallel tangents at the ends of a diameter



A circle with a diameter from A to B, and a tangent line resting on the circle at each of the two ends.

PRACTICE SET

11. PRACTICE Two tangents PA and PB are drawn from an external point P to a circle with centre O. If $\angle APB = 64°$, find $\angle APO$.

SOLUTION — Q11 · **Worked in full**

1.	$\angle APO = 1/2\angle APB$	*OP bisects the angle between the two tangents from P*
2.	$\angle APO = 1/2 \cdot 64° = 32°$	*substitute the given angle and simplify*

12. PRACTICE Two tangents TP and TQ are drawn to a circle with centre O from an external point T. If $\angle OPQ = 40°$, find $\angle PTQ$.

13. PRACTICE A parallelogram $PQRS$ circumscribes a circle. If $PQ = 8$ cm, find QR.

14. PRACTICE Two tangents TP and TQ are drawn to a circle with centre O from an external point T. If $\angle PTQ = 54°$, find $\angle OPQ$.

15. PRACTICE A quadrilateral $PQRS$ circumscribes a circle, touching all four sides. If $PQ = 12$ cm, $QR = 15$ cm and $RS = 14$ cm, find SP.

16. PRACTICE A rectangle $ABCD$ circumscribes a circle, so all four of its sides touch the circle. Prove that $ABCD$ must be a square.

17. PRACTICE A circle is inscribed in a triangle PQR, touching all three sides. The line from the vertex P to the centre O always:

A. bisects the opposite side QR
B. bisects the angle at P
C. is perpendicular to QR
D. is equal in length to the inradius

18. PRACTICE Two concentric circles have centre O. The larger circle has radius 13 cm and the smaller has radius 5 cm. Find the length of the chord of the larger circle that touches the smaller circle.

19. PRACTICE A triangle PQR circumscribes a circle, touching side QR at S, side RP at T, and side PQ at U. If $QS = 5$ cm, $RS = 4$ cm and $PT = 6$ cm, find the perimeter of triangle PQR.

20. PRACTICE Two tangents TP and TQ are drawn from an external point T to a circle with centre O and radius 5 cm. If $TO = 13$ cm, find the perimeter of the quadrilateral $OPTQ$.

21. PRACTICE Two parallel tangents to a circle with centre O touch it at the points M and N. A third tangent to the same circle, touching it at a point C that lies between the two parallel tangents, meets the tangent at M at a point A and meets the tangent at N at a point B, so A, C and B are three points on this third tangent line. Prove that $\angle AOB = 90°$.

22. PRACTICE Two tangents TP and TQ are drawn from an external point T to a circle with centre O and radius 9 cm. The perimeter of quadrilateral $OPTQ$ is 42 cm. Find TO.

ANSWERS **11.** $\angle APO = 32°$ **12.** $\angle PTQ = 80°$ **13.** $QR = 8$ cm **14.** $\angle OPQ = 27°$
15. $SP = 11$ cm
16. In a rectangle, opposite sides are equal: $AB = CD$ and $AD = BC$. Since $ABCD$ circumscribes the circle, $AB + CD = AD + BC$. Substituting the equal pairs gives $2 \cdot AB = 2 \cdot AD$, so $AB = AD$. Combined with $AB = CD$ and $AD = BC$, all four sides are equal — and a rectangle with all four sides equal is a square.
17. B — bisects the angle at P. **18.** $AB = 24$ cm **19.** 30 cm **20.** 34 cm
21. OA bisects the angle between the two tangents from A, so $\angle OAB = 1/2\angle MAB$. Likewise OB bisects the angle at B, so $\angle OBA = 1/2\angle NBA$. The tangents at M and N are parallel and AB is a transversal, so $\angle MAB + \angle NBA = 180°$ (co-interior angles). Adding the two bisected halves, $\angle OAB + \angle OBA = 90°$. In triangle OAB the three angles add to $180°$, so $\angle AOB = 90°$.
22. $TO = 15$ cm
Full solutions: **manthika.com/learn/math10-ch10**