11 CHAPTER 11 Areas Related to Circles

WHAT THIS CHAPTER BUILDS

You will learn to find the area and the arc length of a slice of a circle, and to subtract one shape from another to get the area of a shaded region.

MENSURATION UNIT

IN THIS CHAPTER

Bhavisha and Deo fold a paper circle along straight creases from its centre, and Deo holds scissors ready to cut one sector.

Getting started

Slices of a circle

A STORY

Folds from the centre

Bhavisha folds a large paper circle in half. Then Bhavisha folds the half into three equal parts and opens it out on the worktable. Straight creases now run from the centre to the rim.

"Every crease starts at the centre," Bhavisha says. "The paper between two creases is a sector."

Deo holds the scissors ready by one slice. "Three equal parts of a half," Deo says. "So this slice is one sixth of the whole circle."

"And the angle at its point?" Bhavisha asks.

"One sixth of 360°, so 60°," Deo says. "And one sixth of the paper sits inside it."

"Now suppose a slice had an angle of 50°," Bhavisha says. "No way of folding gives you that one."

Deo turns the scissors slowly. The creases do not help with this slice.

How would you find the area of a sector from its angle alone?

A circle can be cut into a smaller piece in two ways. This chapter uses both.

Let us picture a circular park. Draw two radii from its centre to two points on the boundary. The piece they cut off is a sector. Its area is the same fraction of the park's area as the angle between the radii is of a full turn. That fraction is $\theta/360°$. So the sector's area is $(\theta/360°) \cdot \pi r^2$. The arc is the curved edge along the park's own boundary. Its length is the same fraction, applied to the boundary instead of the area: $(\theta/360°) \cdot 2\pi r$.

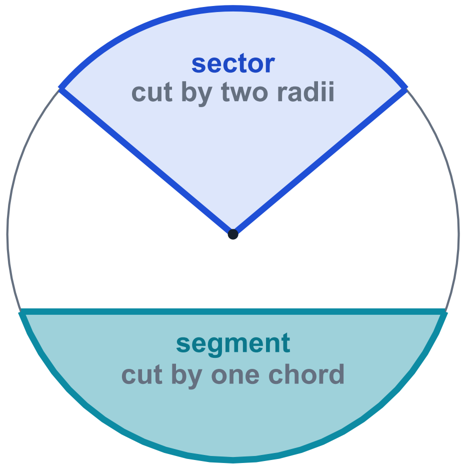
Now let us draw a single straight fence across the park, joining two points on the boundary directly. No radii are drawn at all. **That fence cuts off a segment, not a sector.** A segment is bounded by a straight chord and an arc, never by two radii. Its area is never read off a formula of its own. It is the sector built on the same arc, with the triangle formed by the two radii and the chord taken away.

One circle, two kinds of piece: a sector is bounded by two radii, a segment by one chord. Let's work out exactly how much of the circle each one covers.

RULE
Find the sector first, even when a segment is asked for. The segment is the sector minus its triangle.

KEY
A sector or a segment is not a new shape. Each is a fraction of the circle already known.

Two ways to cut one circle



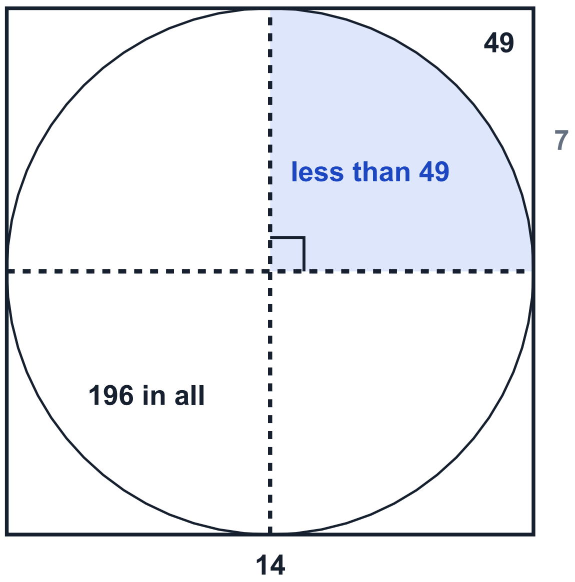
Above, a sector is cut by two radii; below, a segment is cut by one chord, both closed by an arc.

A SCHOLAR INDIA REMEMBERS

Before any formula, let us estimate. A circle of radius 7 *cm fits inside a square of side* 14 *cm. So the circle's area is less than* $14 \times 14 = 196$ *cm²*. *A quarter of the circle is a* $90°$ *sector. Its area must be less than* $196 \div 4 = 49$ *cm²*. *Keep that bound in mind as you work.*

Aryabhata · the hoopoe

A quarter of the circle is under forty-nine



The circle of radius 7 sits inside a square of side 14, so its area is under 196. A quarter of the circle, the 90° sector, sits inside a 7 by 7 quarter of that square, so its area is under 196 ÷ 4 = 49.

A circle of radius 7 sits inside a square of side 14, and its quarter sector sits inside a quarter of that square.

IN THE WILD

The minute hand of a clock sweeps from the 12 to the 4. That sweep cuts a sector, and its area is exactly the fraction of the whole face that 120 degrees is of 360: one third. A sector is always a fraction of the circle already known. A segment is that same sector, minus one triangle.

CHECK YOURSELF

1. A sector of a circle is the part cut off by

- **A.** a chord and the arc it cuts off
- **B.** two chords meeting at a point on the circle
- **C.** two tangents drawn from an external point
- **D.** two radii and the arc between them

2. A segment's area follows from its sector's area because a segment is

- **A.** that sector with its triangle removed
- **B.** that sector with its area doubled
- **C.** twice the sector's arc length
- **D.** the sector with its central angle halved

1 more in this set online.

IN EVERY CHAPTER

Before you start

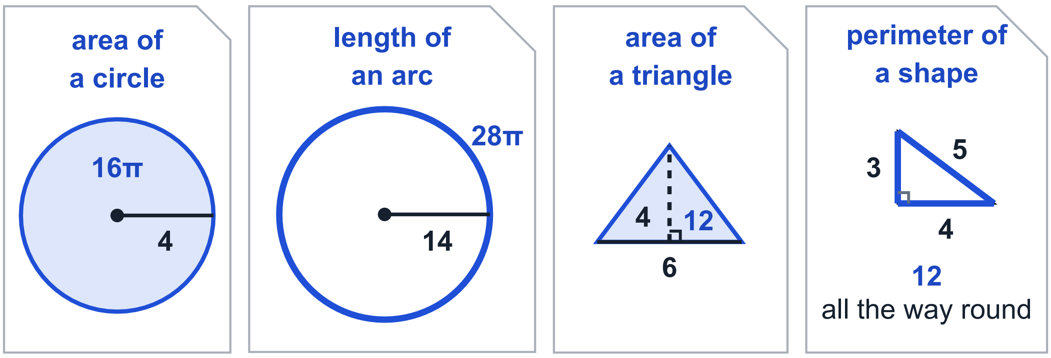
DO THIS FIRST

You already know how to measure a whole circle. Now measure a slice of one. Try each check below.

- **Area of a circle** (Class 9, Ganita Manjari, page 143).
  Here: a sector's area is a fraction of πr^2, the whole circle's area.
  Check: A circle of radius 4 has area $\pi \cdot 4^2 = 16\pi$.
- **Length of an arc** (Class 9, Ganita Manjari, page 125).
  Here: an arc's length is the same fraction, applied to $2\pi r$ instead.
  Check: A circle of radius 14 has circumference $2 \cdot \pi \cdot 14 = 28\pi$.
- **Area of a triangle** (Class 8, Ganita Prakash Part 2, page 154).
  Here: a segment's area subtracts this triangle from its sector.
  Check: A triangle with base 6 and height 4 has area $1/2 \cdot 6 \cdot 4 = 12$.
- **Perimeter of a shape** (Class 9, Ganita Manjari, page 119).
  Here: a sector's perimeter adds its two radii to the arc's length.
  Check: A shape with sides $3, 4, 5$ has perimeter $3 + 4 + 5 = 12$.
- **π is irrational** (Class 9, Ganita Manjari, page 123).
  Here: every area here uses $\pi \approx 3.14$, a rounded value.
  Check: $22/7 \approx 3.14$, both approximations to π.

If any of these felt new, read the page named before going on.

Four things you already know



Every formula in this chapter is built from these four: a circle's area, its rim, a triangle's area, and adding up the sides of a shape. A sector takes a fraction of the first two; a segment subtracts the third; a perimeter uses the fourth.

Four panels show a circle's area, an arc's length, a triangle's area and a shape's perimeter.

Sector of a circle

A sector of a circle is the region enclosed by two radii and the arc between them: two straight cuts from the centre, plus the curved edge trapped between them.

Let us draw two radii on a circle: they divide its interior into two regions, not one. **The smaller of the two is the minor sector, and the larger is the major sector.** The same two radii, read round the circle in two directions, give both.

The two sectors together make up the whole circle, so their angles always add to 360°. We can check this: a minor sector's angle of 90° leaves exactly 270° for the major sector cut by the same two radii.

Unless the two radii sit exactly opposite each other, giving two equal semicircular sectors, one of the two is always smaller. There is no tie to resolve.

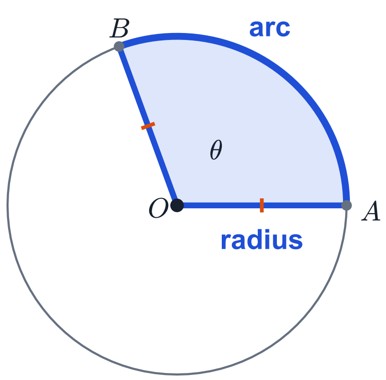
Your turn: a minor sector's angle is 130°. What is the major sector's angle? (Answer: 230°, since $360° - 130° = 230°$.)

BACK
Slices of a circle · p. 360

RULE
Minor and major sector angles always add to 360°.

KEY
Two radii plus the arc between them make a sector.

A sector: two radii and an arc



The shaded wedge is a sector, closed off by an arc between two tick-marked radii from the circle's centre.

CHECK YOURSELF

3. Which of these is a sector of a circle?

- **A.** two radii and the arc between them
- **B.** the region enclosed by a chord and an arc
- **C.** the straight line joining two points on the circle
- **D.** a line touching the circle at exactly one point

4. Of the two sectors the same two radii cut from a circle, the minor sector is told apart from the major sector by

- **A.** having a different radius from the major sector
- **B.** being the smaller of the two regions
- **C.** always containing the circle's centre
- **D.** having the shorter perimeter, whichever sector that is

1 more in this set online.

Segment of a circle

A segment of a circle is the region enclosed by a chord and the arc it cuts off. That is one straight cut, not two. No radii are drawn anywhere.

Naming the chord is the fastest way to tell a segment from a sector at a glance. A sector always has two straight edges meeting at the centre. A segment has exactly one straight edge. That edge touches the centre only when the chord happens to be a diameter.

Let us draw a chord on a circle. It divides the interior into two segments: the smaller is the minor segment, the larger the major segment. This is the same naming pattern a sector uses, carried over to a different pair of shapes.

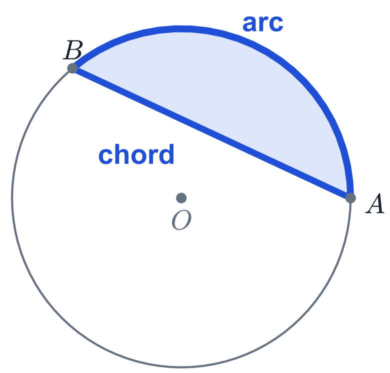
A sector question names two radii. A segment question names one chord. Read the word before you draw anything.

Your turn: a straight fence crosses a field, with no line to the centre. What shape does it cut off? (Answer: a segment, since only a chord and an arc bound it, with no radii at all.)

CAUTION
A sector is bounded by two radii; a segment is bounded by a chord. Do not mix up which figure a problem names.

KEY
A chord, not two radii, cuts off a segment.

A segment: one chord and an arc



The shaded region is a segment, closed off by a chord and the arc above it, with no line drawn to the circle's centre.

CHECK YOURSELF

5. A segment of a circle is the region enclosed by

- **A.** two radii and an arc
- **B.** two chords crossing inside the circle
- **C.** a tangent and a radius drawn to the point of contact
- **D.** a chord and the arc it cuts off

6. Of the two regions a single chord cuts from a circle, the smaller one is called the

- **A.** minor segment
- **B.** major segment
- **C.** minor sector
- **D.** triangle

1 more in this set online.

The ideas

Area of a sector

A sector's area is a fraction of the whole circle's area. The sector's own angle decides which fraction, and nothing else.

A full turn measures $360°$, and a circle of radius r has area πr^2. **A sector of angle θ takes the same fraction of that area as θ is of $360°$.** This gives sector area $(\theta/360°) \cdot \pi r^2$.

Let us try it on a quarter turn: $90°$ takes $90/360 = 1/4$ of the circle's area. A sector with $\theta = 180°$ is a semicircle. It needs no separate rule: the general formula already gives the expected $(1/2) \cdot \pi r^2$.

The angle always sits over $360°$ in this fraction, never over $180°$ or any other number. Check it on any angle you like: the same fraction rule holds every time.

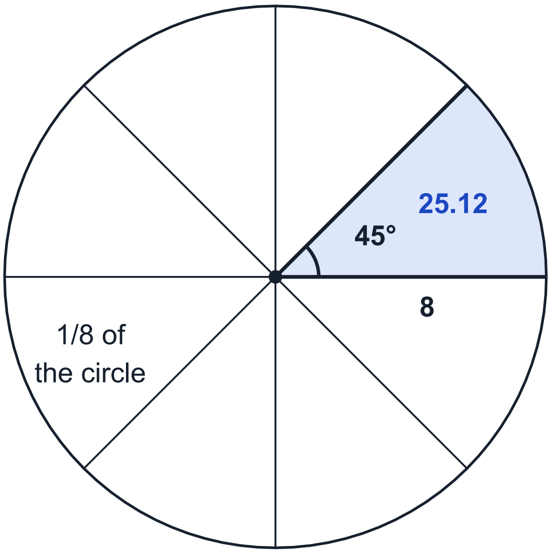
Your turn: a sector has radius 8 cm and angle $45°$. Using $\pi \approx 3.14$, what is its area? (Answer: 25.12 cm², since $(45/360) \cdot 3.14 \cdot 8^2 = (1/8) \cdot 3.14 \cdot 64 = 25.12$.)

TRY
What fraction of the circle is a sector of angle 90°?

Answer: $90/360 = 1/4$ of the circle.

KEY
Sector area is (angle over 360°) times the whole circle's area.

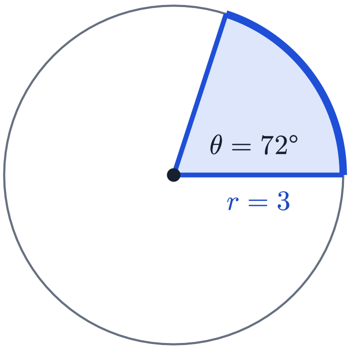
Forty-five degrees is an eighth of the circle



Eight sectors of 45° fill the circle, so one of them takes one eighth of the whole area: (1/8) · 3.14 · 8² = 25.12 cm².

A circle cut into eight equal 45 degree sectors has one shaded sector of area 25.12 square centimetres, one eighth of the whole.

A sector's fraction of the circle



Fraction of the circle: $\theta/360 = 0.2$.
Sector area $= (\theta/360) \times \pi r^2 \approx 5.65$.
Arc length $= (\theta/360) \times 2\pi r \approx 3.77$.

A 72 degree sector of radius 3 takes one fifth of the circle, with its area and arc length shown below.

A SCHOLAR INDIA REMEMBERS

A $60°$ *sector is what part of the whole circle? Count it.* $360 \div 60 = 6$*, so it is one sixth. With radius 7 cm, the circle's area is* $22/7 \times 7 \times 7 = 154$ *cm². One sixth of* 154 *is about* 25.7 *cm². Check your formula gives the same.*

Aryabhata · the hoopoe

CHECK YOURSELF

7. The area of a sector with angle θ in a circle of radius r is

- **A.** $(\theta/360) \cdot 2\pi r$
- **B.** $(\theta/180)\pi r^2$
- **C.** $\pi r^2 - \theta$
- **D.** $(\theta/360)\pi r^2$

8. A sector has radius 7 cm and angle 90°. Using $\pi = 22/7$, its area is

- **A.** 154 cm²
- **B.** 19.25 cm²
- **C.** 38.5 cm²
- **D.** 77 cm²

1 more in this set online.

Arc length of a sector

An arc's length follows the same fractional idea as a sector's area. It applies to the circle's boundary, not its interior.

The whole circle's circumference is $2\pi r$. **An arc of angle θ is the same fraction of that circumference as θ is of 360°.** So arc length works out to $(\theta/360°) \cdot 2\pi r$.

Sector area and arc length share the same fraction, $\theta/360°$. Only what the fraction multiplies changes: the circle's area for a sector, the circle's circumference for an arc. Let us compare the two: one number carries across both.

A semicircle's arc, at $\theta = 180°$, comes to exactly half the circumference, πr. This is the familiar fact that a semicircle's curved edge is half the boundary, reached here from the general formula, not memorised on its own.

Your turn: an arc has radius 7 cm and angle 45°. Using $\pi \approx 22/7$, what is its length? (Answer: 5.5 cm, since $(45/360) \cdot 2 \cdot (22/7) \cdot 7 = (1/8) \cdot 44 = 5.5$.)

RULE
The sector area formula and the arc length formula use the same fraction $\theta/360°$.

KEY
Arc length is (angle over 360°) times the whole circumference, $2\pi r$.

A SCHOLAR INDIA REMEMBERS

Is the arc longer or shorter than the straight chord below it? The arc bends, so it is longer. Take a 60° sector of radius 7 cm. Its chord is 7 cm, because the triangle is equilateral. Its arc is $1/6 \times 2 \times 22/7 \times 7 = 22/3$, about 7.33 cm. The arc is a little longer, as it should be.

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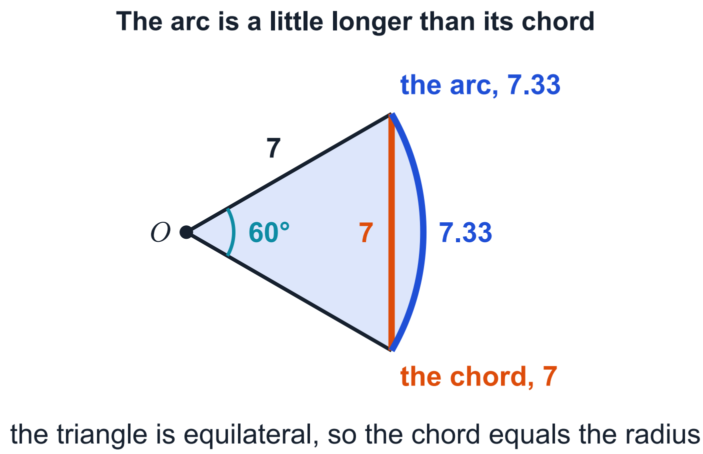


At 60° the two radii and the chord make an equilateral triangle, so the chord is 7, the same as the radius. The arc bends, and comes to 22/3, about 7.33: longer than the straight chord, as it should be.

A 60 degree sector of radius 7 has a chord of 7 and an arc of 7.33, the arc longer than the chord.

CHECK YOURSELF

9. The arc length of a sector with angle θ in a circle of radius r is

A. $(\theta/360)\pi r^2$

B. $(\theta/180) \cdot 2\pi r$

C. $(\theta/360) \cdot 2\pi r$

D. $2\pi r - \theta$

10. A sector has radius 21 cm and angle 60°. Using $\pi = 22/7$, its arc length is

A. 132 cm

B. 44 cm

C. 11 cm

D. 22 cm

1 more in this set online.

Minor and major sector together

A circle's two sectors, cut by the same two radii, are not two separate calculations. The second is always one subtraction away from the first.

The major sector's area is the whole circle's area, πr^2, minus the minor sector's area. Its angle is $360° - \theta$, where θ is the minor sector's own angle.

Let us take a minor sector of angle $72°$. It leaves a major sector of angle $360° - 72° = 288°$, cut by the identical two radii. Its area is πr^2 minus the minor sector's own area.

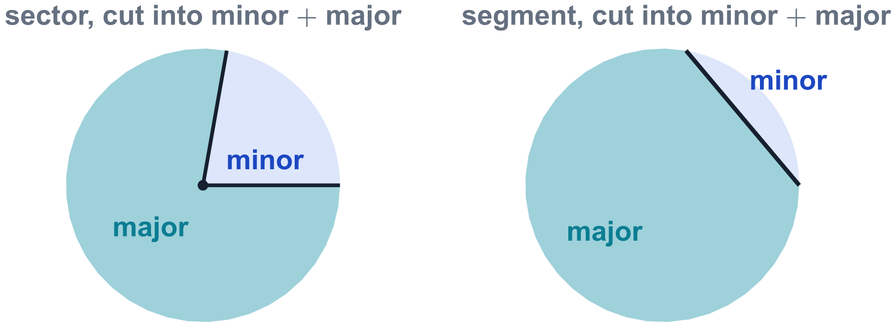
Find the minor sector first, then subtract. This is quicker and safer than working the major sector's own fraction, $(360° - \theta)/360°$, directly. We still get the same answer, with one fewer number to carry.

Your turn: a minor sector has area 45 cm² inside a circle of radius 18 cm. Using $\pi \approx 3.14$, what is the major sector's area? (Answer: 972.36 cm², since the circle's area is $3.14 \cdot 18^2 = 1017.36$ cm², and $1017.36 - 45 = 972.36$.)

BACK
Area of a sector · p. 365

RULE
Compute the minor sector first, then subtract it from πr^2 to get the major sector.

Minor + major = the whole circle



Two panels cut the same circle, one by two radii and one by a chord, each into a minor piece and a major piece.

CHECK YOURSELF

11. If a sector's angle is θ, the major sector cut by the same two radii has angle

- **A.** $180° - \theta$
- **B.** $2 \cdot \theta$
- **C.** $360° + \theta$
- **D.** $360° - \theta$

12. A circle of radius 7 cm (using $\pi = 22/7$) has a sector of angle 90°. The major sector's area is

- **A.** 38.5 cm²
- **B.** 154.0 cm²
- **C.** 115.5 cm²
- **D.** 77.0 cm²

1 more in this set online.

Perimeter of a sector or a segment

A sector and a segment are bounded by different edges. Let us look at each in turn. Their perimeters are built from different pieces entirely.

A sector's boundary is two straight radii plus one curved arc. **Its perimeter is $2r + (\theta/360°) \cdot 2\pi r$**: two radii, each of length r, plus the arc length already found.

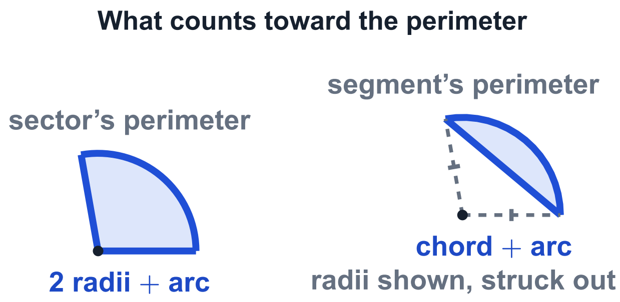
A segment's boundary looks similar at a glance: one straight edge, one curved edge. Let us look again: the straight edge is a chord, never a radius, and there is only one of it. A segment's perimeter is the chord's length plus the arc's length, with no radii counted at all.

Counting two radii into a segment's perimeter is a common slip. So is leaving the chord out of a sector's perimeter. Check which figure the question names before writing down any lengths: that one habit catches both mistakes.

Your turn: a sector has radius 5 cm and angle 144°. Using $\pi \approx 3.14$, what is its perimeter? (Answer: 22.56 cm, since the arc is $(144/360) \cdot 2 \cdot 3.14 \cdot 5 = 12.56$ cm, and $2 \cdot 5 + 12.56 = 22.56$.)

CAUTION
A sector's perimeter counts two radii; a segment's perimeter counts one chord and no radii.

BACK
Arc length of a sector · p. 367



A sector's perimeter is two radii plus an arc, while a segment's perimeter is just a chord plus an arc, with radii struck through.

CHECK YOURSELF

13. The perimeter of a sector consists of

- **A.** the chord plus the arc
- **B.** the two radii only
- **C.** the arc only
- **D.** two radii plus the arc

14. A student says a segment's perimeter equals its chord plus its two radii. What is wrong with this claim?

- **A.** nothing is wrong — that is the correct perimeter
- **B.** the perimeter should use one radius, not two
- **C.** a segment has no radii — just the chord and arc
- **D.** the chord should be counted twice

1 more in this set online.

Area of a segment

A segment sits inside its own sector, sharing the same arc. Its area is never found from a formula of its own. It is read off the sector's area instead.

Draw the chord bounding the segment, and the two radii bounding the matching sector. Together, the two radii and the chord enclose a triangle, sitting inside the sector but outside the segment. **A segment's area is its sector's area minus that triangle's area.** Picture the whole sector, with the triangular corner cut away.

For a radius r and central angle θ, this gives segment area equal to sector area minus triangle area. You work both out for the same r and the same θ. Skip the triangle, and the segment comes out too large, by exactly the triangle's own area.

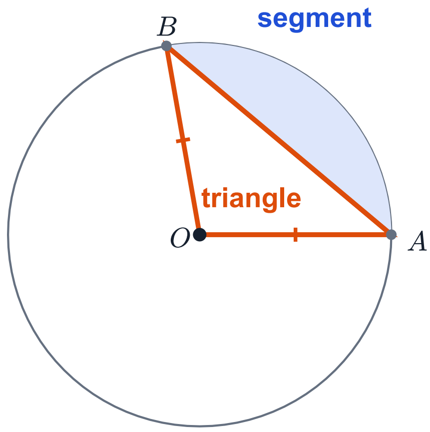
The triangle's area still needs a formula of its own. We work that out next.

Your turn: a sector has area 60 cm² and the triangle inside it has area 42 cm². What is the segment's area? (Answer: 18 cm², since $60 - 42 = 18$.)

BACK
Slices of a circle · p. 360

KEY
Segment area = sector area minus triangle area, always.

Segment = sector minus triangle



The wedge splits into an outlined triangle, made by the two radii and the chord, and the shaded segment that remains beside it.

CHECK YOURSELF

15. A segment's area, for the same radius and central angle as its sector, equals

- **A.** sector's area minus triangle's area
- **B.** the sector's area plus the triangle's area
- **C.** the sector's area alone
- **D.** the triangle's area alone

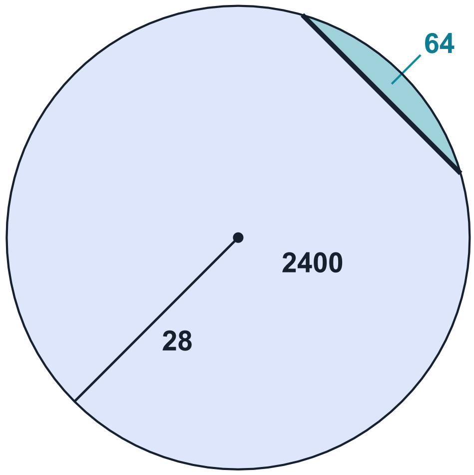
16. A sector has area 28.26 cm² and its triangle has area 18 cm². The segment's area is

- **A.** 10.26 cm²
- **B.** 46.26 cm²
- **C.** 28.26 cm²
- **D.** 18 cm²

2 more in this set online.

Minor and major segment together

Sixty-four off the rim leaves twenty-four hundred



the whole circle, 2464

One chord cuts a sliver of 64 off a circle of radius 28, whose whole area is (22/7) · 28² = 2464. The major segment is what is left: 2464 − 64 = 2400, one subtraction, no second formula.

A circle of radius 28 with area 2464 has a sliver of 64 cut off, leaving the major segment, 2400.

A circle's two sectors from the same two radii add to the whole circle. Its two segments from the same chord add to the whole circle too.

The major segment's area is the whole circle's area, πr^2, minus the minor segment's area. It is the same subtraction we already used for the major sector, run here on segments instead.

Let us run the numbers: a minor segment of area 20 cm² inside a circle of area 154 cm² leaves a major segment of area $154 - 20 = 134$ cm². No fresh calculation is needed for the larger piece.

Finding the minor segment first stays the practical order every time. Its own formula, sector minus triangle, is direct. The major segment's boundary, most of the circle plus the same chord, is far harder to measure directly.

Your turn: a minor segment has area 64 cm² inside a circle of radius 28 cm. Using $\pi \approx 22/7$, what is the major segment's area? (Answer: 2400 cm², since the circle's area is $(22/7) \cdot 28^2 = 2464$ cm², and $2464 - 64 = 2400$.)

RULE
Compute the minor segment first, then subtract it from πr^2 to get the major segment.

CHECK YOURSELF

17. The major segment cut by a chord equals

- **A.** circle's area minus the minor segment's area
- **B.** the minor segment's area plus the sector's area
- **C.** the minor segment's area doubled
- **D.** the full circle's area minus the sector's area

18. A circle of radius 10 cm (using $\pi = 3.14$) has a minor segment of area 28.5 cm². The major segment's area is

A. 78.5 cm²

B. 285.5 cm²

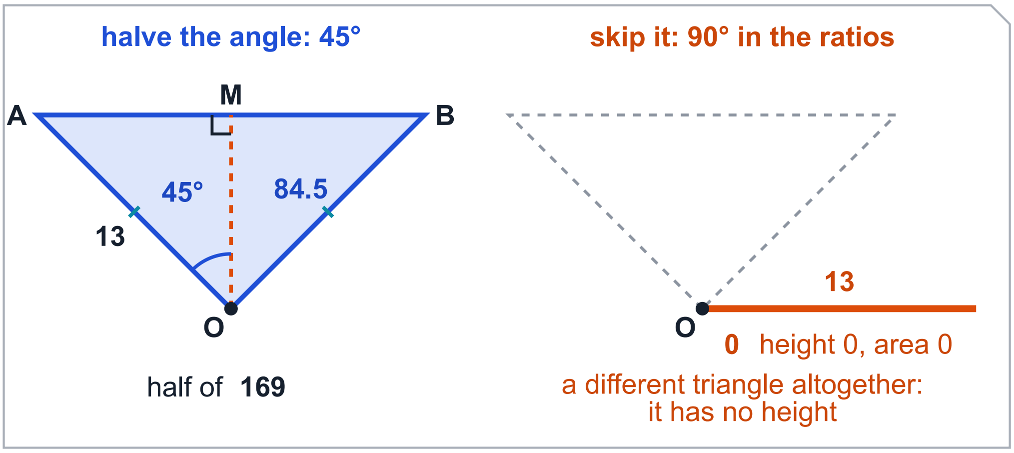
C. 342.5 cm²

D. 28.5 cm²

1 more in this set online.

The triangle inside a segment

Halve the angle, or the triangle collapses



The perpendicular from the centre bisects the angle, so 45°, not 90°, goes into the sine and cosine: 13² · sin 45° · cos 45° = 169 · 1/2 = 84.5. Put 90° in instead and cos 90° = 0 flattens the triangle to a line with no area.

Halving the angle to 45 degrees gives a triangle of area 84.5, but using 90 degrees instead flattens it to a line.

The triangle that a segment's area formula subtracts away has a fixed shape. It is worth working out once, not re-derived every time it appears.

Let us draw it: two of its sides are radii, each of length r, with the sector's own angle θ trapped between them. This is an isosceles triangle by construction, for any angle θ. **Drop a perpendicular from the centre straight down to the chord, and it bisects both the chord and the angle θ.** This splits the isosceles triangle into two identical right triangles, each carrying the half-angle $\theta/2$.

In either right triangle, the perpendicular is the side adjacent to $\theta/2$, and the half-chord is the side opposite it. The radius r is the hypotenuse. You now have perpendicular length $r \cdot \cos(\theta/2)$ and half-chord length $r \cdot \sin(\theta/2)$.

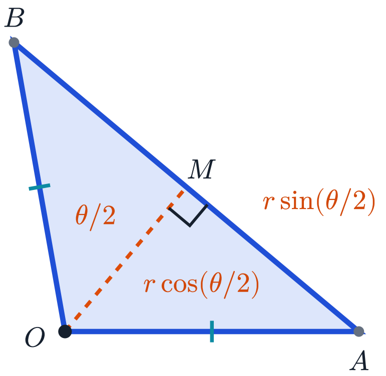
Doubling one right triangle's area gives the whole triangle's area: $r^2 \cdot \sin(\theta/2) \cdot \cos(\theta/2)$. Halve the angle first: every calculation with this triangle depends on that one step. Skip it, and θ itself ends up inside the ratios instead of $\theta/2$. That describes a different triangle altogether.

Your turn: a segment's central angle is $90°$ and its radius is 13 cm. What is the triangle's area? (Answer: 84.5 cm², since the half-angle is $45°$, and $13^2 \cdot \sin(45°) \cdot \cos(45°) = 169 \cdot 1/2 = 84.5$.)

BACK
Segment of a circle · p. 364

RULE
Halve the angle first: the perpendicular from the centre always bisects both the chord and the angle.

The perpendicular splits the triangle



A perpendicular from centre O meets the chord AB at a right angle, splitting the triangle by the half-angle theta over two.

CHECK YOURSELF

19. The triangle bounded by a segment's two radii and its chord has

- **A.** three unequal sides
- **B.** a right angle between the two radii, always
- **C.** two sides equal to the chord's length
- **D.** two radii as its equal sides

20. The perpendicular drawn from the centre to a segment's chord

- **A.** bisects the chord but not the angle θ
- **B.** bisects both the chord and the angle θ
- **C.** bisects the angle θ but not the chord
- **D.** extends the chord into a full diameter

1 more in this set online.

Three angles, three ratio pairs

Finding this triangle's area means finding a sine and a cosine at the segment's half-angle.

Every segment problem here uses a central angle of 60°, 90° or 120°. So the half-angle is always 30°, 45° or 60°: three angles worth knowing outright, rather than looked up each time.

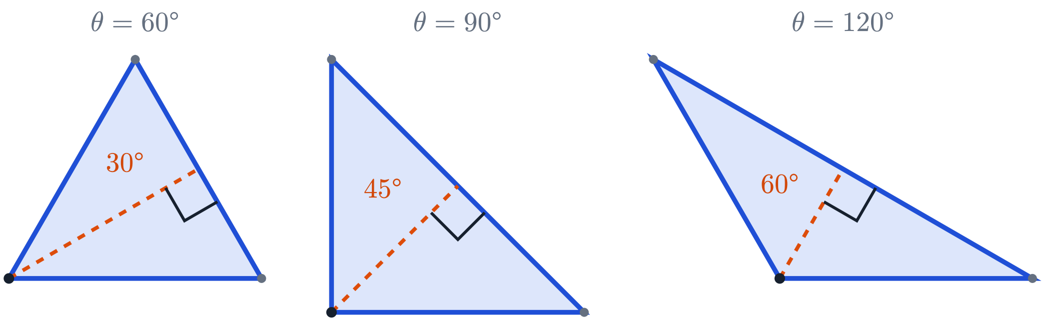
At 30°, $\sin(30°) = 1/2$ and $\cos(30°) = \sqrt{3}/2$. At 45°, $\sin(45°) = \cos(45°) = \sqrt{2}/2$. At 60°, $\sin(60°) = \sqrt{3}/2$ and $\cos(60°) = 1/2$. Let us look at the two outer pairs: sine and cosine swap between them, and that pattern makes the three pairs easier to hold as one rule.

A central angle outside 60°, 90° and 120° would need a calculator rather than memory, and none appears here. Once these three pairs are known, no calculator is ever needed for a segment like these.

Your turn: a segment's central angle is 120° and its radius is 9 cm. Using $\sqrt{3} \approx 1.73$, what is the triangle's area? (Answer: ≈ 35.03 cm², since the half-angle is 60°, and $9^2 \cdot \sin(60°) \cdot \cos(60°) \approx 81 \cdot 0.865 \cdot 0.5 \approx 35.03$.)

KEY
Half of 60°, 90° and 120° is 30°, 45° and 60°.

The same triangle at three capped angles



Three panels hold the two radii equal and open the angle between them to 60, 90 and 120 degrees, halving each for the triangle.

CHECK YOURSELF

21. For a segment problem with central angle θ = 90°, the triangle's half-angle used in the area formula is

- **A.** 90°
- **B.** 45°
- **C.** 180°
- **D.** 22.5°

22. At half-angle 60°, the standard values are

- **A.** $\sin(60°) = 1/2$ and $\cos(60°) = \sqrt{3}/2$
- **B.** $\sin(60°) = \sqrt{2}/2$ and $\cos(60°) = \sqrt{2}/2$
- **C.** $\sin(60°) = 1$ and $\cos(60°) = 0$
- **D.** $\sin(60°) = \sqrt{3}/2$ and $\cos(60°) = 1/2$

1 more in this set online.

Common mistakes

A segment is not its sector

It is tempting to read a sector's area and call that the segment's area too. The two shapes share the same arc. At a glance, they look like the same slice of the circle.

Let us look again: they are not the same slice. The triangle bounded by the two radii and the chord has to be subtracted first. Skip that subtraction, and the sector's own area gets reported as the segment's: an answer too large by exactly the triangle's area.

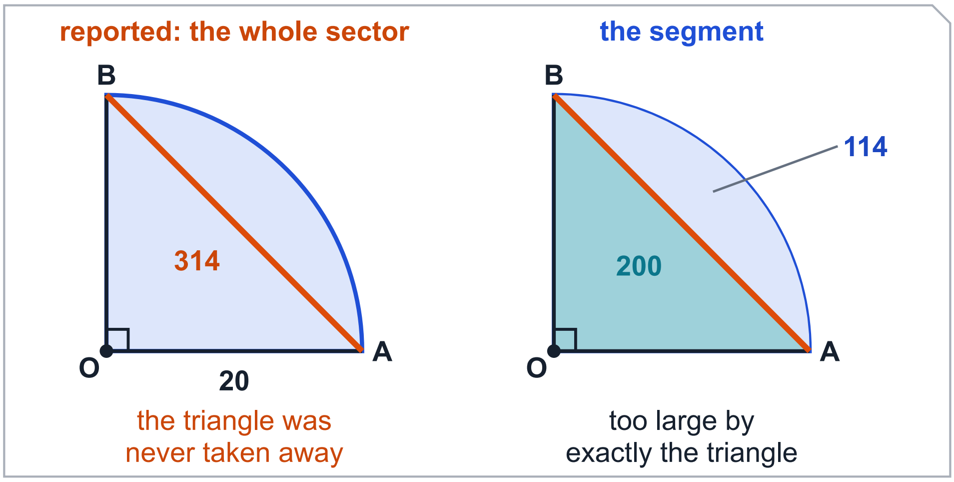
A segment lies inside its own sector, so its area must come out smaller than the sector's. An answer equal to the sector's area means the triangle was never taken away.

Your turn: a circle has radius 20 cm and a central angle of 90°. Using $\pi \approx 3.14$, what is the segment's area? (Answer: 114 cm², since the sector's area is 314 cm², the triangle's area is 200 cm², and $314 - 200 = 114$.)

CAUTION
Check the reverse trap too: a SECTOR needs no triangle subtracted at all.

BACK
Area of a segment · p. 370

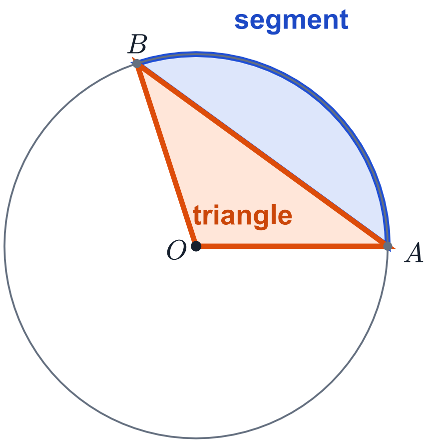
Too large by exactly the triangle



Reporting 314 for the segment reports the whole sector: the triangle between the two radii and the chord, 200, was never taken away. The segment is 314 − 200 = 114.

Reporting the whole sector, 314, skips subtracting the triangle, 200, leaving the true segment, 114.

A sector IS its triangle plus its segment



The wedge is filled in two colours, one for the triangle and one for the segment, showing both pieces that make up the whole shape.

FIRST TRY
~~For a chord at 90° in a circle of radius 10 cm, I found the sector area = (90/360) · 3.14 · 10² = 78.5 cm squared, and called that the segment's area.~~

SECOND LOOK
The segment is the sector minus the triangle the two radii make with the chord. The triangle's area is $10^2 \cdot \sin(45°) \cdot \cos(45°) = 50$ cm squared, so the segment's area is $78.5 - 50 = 28.5$ cm squared.

A segment is always the sector minus its triangle. Never stop at the sector.

IS A SEGMENT'S AREA EVER THE SAME AS ITS SECTOR'S AREA?

WEAK

A learner works out the sector's area: $(90/360) \cdot 3.14 \cdot 10^2 = 78.5$ cm². They report 78.5 cm² as the segment's area too.

Then they notice something wrong. A segment sits inside its sector, with the triangle sitting outside it. So the segment must come out smaller than 78.5 cm². An answer equal to the sector's area means the triangle was never taken away.

STRONG

The segment is the sector minus the triangle. The triangle's area is $10^2 \cdot \sin(45°) \cdot \cos(45°) = 50$ cm².

So the segment's area is $78.5 - 50 = 28.5$ cm², smaller than the sector, exactly as it must be.

A SCHOLAR INDIA REMEMBERS

Which is bigger, the sector or the segment? Picture them. The segment is the sector with the triangle cut away. So the segment must be smaller than the sector. If your segment comes out equal to the sector, the triangle has not been taken away.

Aryabhata · the hoopoe

CHECK YOURSELF

23. A student claims: "A segment's area equals its sector's area — subtracting the triangle is optional." At r = 10 cm, θ = 90° (sector = 78.5 cm², triangle = 50 cm²), what is wrong?

- **A.** the segment's area is 28.5 cm², not 78.5 cm² — the triangle must always be subtracted from a minor sector
- **B.** nothing is wrong — 78.5 cm² is correct
- **C.** the segment's area is 128.5 cm², since the triangle should be added instead
- **D.** the segment's area is 285.5 cm², found by subtracting the triangle from the full circle instead of the sector

24. A sector has angle 120° (minor) and its complementary major sector has angle 240°, both sharing the same chord and the same triangle of area T. To get the MINOR segment's area from the MINOR sector's area, a student should

- **A.** add T to the minor sector's area
- **B.** subtract T from the minor sector's area
- **C.** subtract T from the major sector's area
- **D.** add T to the major sector's area

3 more in this set online.

Worked examples

WORKED EXAMPLE · A sector and its major sector

1.	sector area $\approx (30/360) \cdot 3.14 \cdot 4^2$	*Substitute $\theta = 30°$, $\pi \approx 3.14$ and $r = 4$ cm into the sector-area formula.*
2.	$= (30/360) \cdot 3.14 \cdot 16 \approx 4.19$ cm²	*$4^2 = 16$, and $30/360$ simplifies to $1/12$.*
3.	whole circle's area $\approx 3.14 \cdot 16 = 50.24$ cm²	*This is the same πr^2, with no angle fraction, since the whole circle is the sector at $\theta = 360°$.*

WORKED EXAMPLE · **A sector and its major sector**

4. major sector's area $\approx 50.24 - 4.19 = 46.05 \approx 46.1$ cm²

The major sector is the whole circle's area minus the minor sector already found.

5. minor sector plus major sector $= 4.19 + 46.05 = 50.24$ cm²

CHECK ***Check by adding the two sectors back together.*** *The total should land on the whole circle's area,* 50.24 *cm², already found above.*

TRY
Find the area of a sector of radius 6 cm with angle 30°, and the area of the major sector. Using $\pi = 3.14$.

Answer: Sector $= 9.42$ cm²; full circle $= 113.04$ cm²; major sector $= 113.04 - 9.42 = 103.62$ cm².

BACK
Area of a sector · p. 365

RULE
Find the minor sector first, then subtract it from the whole circle for the major sector.

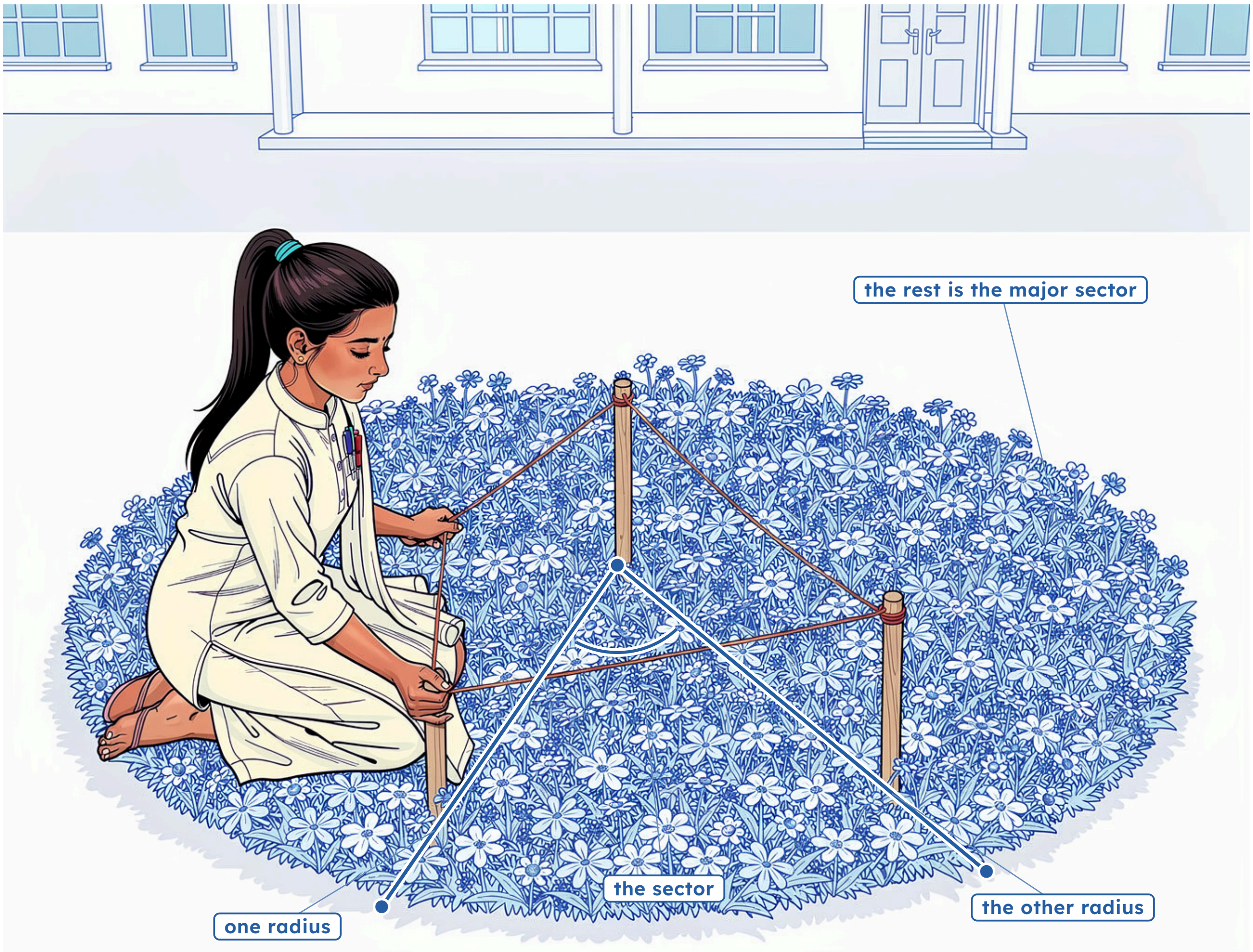
FIND A SECTOR'S AREA AND ITS MAJOR SECTOR, FROM THE RADIUS AND THE ANGLE.

1. Find the angle's fraction Divide the angle by 360°. For $\theta = 30°$, that fraction is $30/360 = 1/12$.

2. Scale the circle's area Multiply that fraction by πr^2. For $r = 4$ cm and $\pi \approx 3.14$, the sector's area is ≈ 4.19 cm².

3. Find the whole circle's area This is the same πr^2 with no fraction taken. For $r = 4$ cm, that is $3.14 \cdot 16 = 50.24$ cm².

4. Subtract for the major sector The major sector is the whole circle minus the minor sector, $50.24 - 4.19 = 46.05$ cm².



A girl kneels in a round flower bed where two radii from the centre peg mark off a sector and the major sector.

CHECK YOURSELF

25. For a sector of radius 4 cm and angle 30° (using $\pi = 3.14$), the sector's area is closest to

- **A.** 50.24 cm²
- **B.** 25.12 cm²
- **C.** 1.05 cm²
- **D.** 4.19 cm²

26. Continuing that sector (radius 4 cm, angle 30°, area 4.19 cm², $\pi = 3.14$), the major sector's area is closest to

- **A.** 4.19 cm²
- **B.** 46.05 cm²
- **C.** 54.43 cm²
- **D.** 25.12 cm²

1 more in this set online.

WORKED EXAMPLE · **Arc length and perimeter of a sector**

1. $\theta = (5/60) \cdot 360° = 30°$

A clock face is one full turn, 360°, and 5 minutes is 5/60 of the 60-minute face.

WORKED EXAMPLE · Arc length and perimeter of a sector

2. arc length $\approx (30/360) \cdot 2 \cdot (22/7) \cdot 14$

Substitute $\theta = 30°$, $\pi \approx 22/7$ and $r = 14$ cm into the arc-length formula.

3. $= 22/3 \approx 7.33$ cm

$30/360$ simplifies to $1/12$, and $2 \cdot (22/7) \cdot 14$ simplifies to 88, giving $88/12 = 22/3$.

4. perimeter $\approx 2 \cdot 14 + 22/3 = 106/3 \approx 35.33$ cm

A sector's perimeter is two radii plus the arc already found.

5. full circumference $\approx 2 \cdot (22/7) \cdot 14 = 88$ cm, one-twelfth of it ≈ 7.33 cm

CHECK ***Check the arc length against the full circumference.*** *One-twelfth of 88 cm matches the 7.33 cm already found, confirming the swept angle really is 30°.*

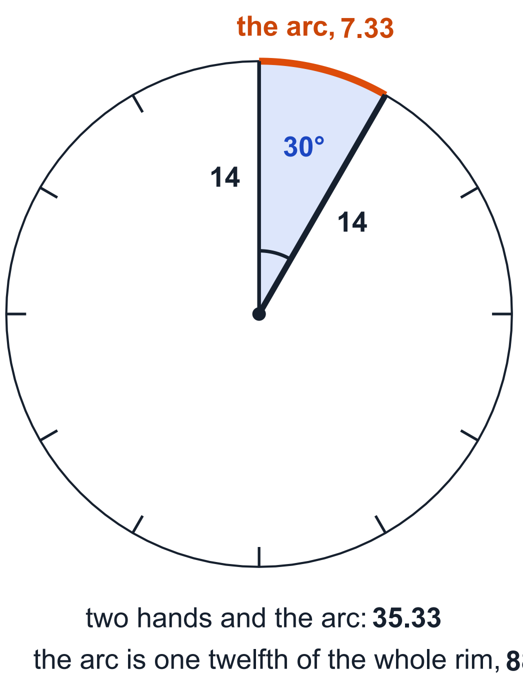
BACK
Arc length of a sector · p. 367

TRY
A clock's minute hand is 21 cm, sweeping $\theta = 60°$ in 10 minutes. Using $\pi \approx 22/7$, find the arc and perimeter.

Answer: Arc length $= 22$ cm; perimeter $= 2 \cdot 21 + 22 = 64$ cm.

KEY
5 minutes is one-twelfth of 60 minutes, so the swept angle is one-twelfth of 360°.

Five minutes sweeps one twelfth of the face



Five minutes is one twelfth of the hour, so the hand turns 30°, one twelfth of the face. Its tip traces one twelfth of the 88 cm rim, 7.33 cm; add the two 14 cm hands and the sector's perimeter is 35.33 cm.

A clock's hands from 12 to 1 mark a 30 degree sector of radius 14, with an arc of 7.33 and perimeter 35.33.

FIND AN ARC'S LENGTH AND A SECTOR'S PERIMETER, FROM THE RADIUS AND THE SWEPT ANGLE.

1. Find the angle from minutes A clock face is 360° for 60 minutes. For 5 minutes, the angle swept is $(5/60) \cdot 360° = 30°$.

2. Find the arc's length Multiply $\theta/360°$ by $2\pi r$. For $r = 14$ cm and $\pi \approx 22/7$, the arc is ≈ 7.33 cm.

3. Add two radii A sector's perimeter is two radii plus the arc, $2 \cdot 14 + 7.33 \approx 35.33$ cm.

4. Check against the circumference The full circumference is 88 cm. One-twelfth of it is ≈ 7.33 cm, matching the arc found above.

CHECK YOURSELF

27. A clock's minute hand, 14 cm long, sweeps through 30° in 5 minutes. Using $\pi = 22/7$, the arc length it traces is closest to

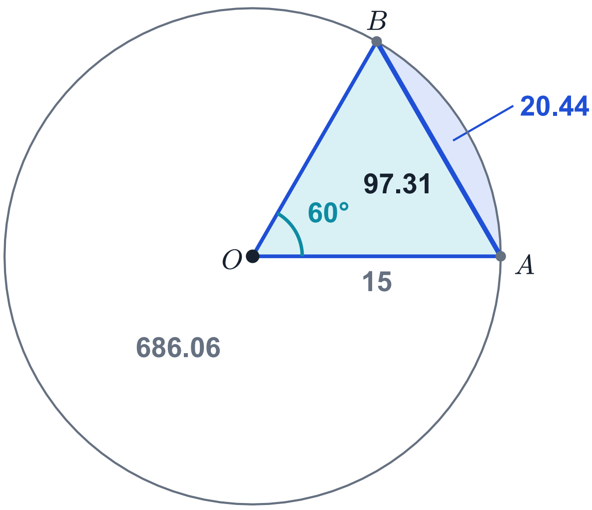
- **A.** 88 cm
- **B.** 44 cm
- **C.** 14.67 cm
- **D.** 7.33 cm

28. For that same sweep (radius 14 cm, arc length about 7.33 cm), the sector's perimeter is closest to

- **A.** 7.33 cm
- **B.** 21.33 cm
- **C.** 35.33 cm
- **D.** 28 cm

1 more in this set online.

A 60° segment is a thin sliver



Sector 117.75 minus triangle 97.31 leaves the minor segment, 20.44 cm².
The major segment, the rest of the circle, is 706.5 − 20.44 = 686.06 cm²;
together they are the whole circle again.

A small central angle leaves only a thin segment, because its triangle takes almost all of the sector.

WORKED EXAMPLE · **A segment at 60 degrees**

1.	sector area $\approx (60/360) \cdot 3.14 \cdot 15^2$	*Substitute $\theta = 60°$, $\pi \approx 3.14$ and $r = 15$ cm into the sector-area formula.*
2.	$= (1/6) \cdot 3.14 \cdot 225 = 117.75$ cm²	*$60/360$ simplifies to $1/6$, and $15^2 = 225$.*
3.	triangle area $= 15^2 \cdot \sin(30°) \cdot \cos(30°)$	*The half-angle of $60°$ is $30°$, so use $r^2 \cdot \sin(\theta/2) \cdot \cos(\theta/2)$.*
4.	$= 225 \cdot (1/2) \cdot (1.73/2) \approx 97.31$ cm²	*$\sin(30°) = 1/2$ and $\cos(30°) = \sqrt{3}/2 \approx 1.73/2$, from the standard pair.*
5.	minor segment area $\approx 117.75 - 97.31 = 20.44$ cm²	*Segment area is sector area minus triangle area.*
6.	major segment area $\approx 3.14 \cdot 225 - 20.44 = 706.5 - 20.44 = 686.06$ cm²	*The major segment is the whole circle's area minus the minor segment already found.*
7.	minor segment plus major segment $= 20.44 + 686.06 = 706.5$ cm²	CHECK ***Check by adding the two segments back together.*** *The total should land on the whole circle's area, $3.14 \cdot 225 = 706.5$ cm², matching the step above.*

RULE
At 60° the half-angle is 30°. Read its sine and cosine directly from the standard pair.

CHECK YOURSELF

29. A chord subtends 60° in a circle of radius 15 cm (using $\pi = 3.14$). The sector's area is closest to

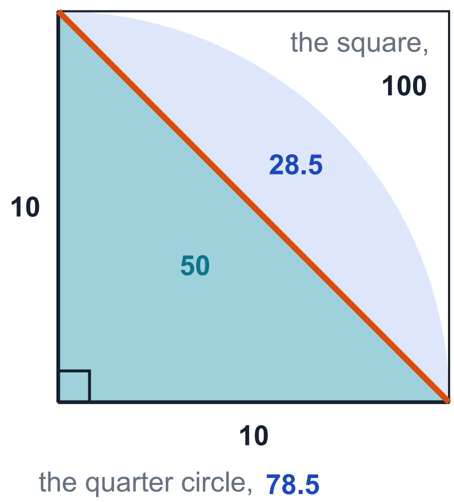
- **A.** 117.75 cm²
- **B.** 706.50 cm²
- **C.** 58.88 cm²
- **D.** 235.50 cm²

30. For that same chord (sector area 117.75 cm², triangle area about 97.31 cm²), the minor segment's area is closest to

- **A.** 215.06 cm²
- **B.** 117.75 cm²
- **C.** 20.44 cm²
- **D.** 97.31 cm²

1 more in this set online.

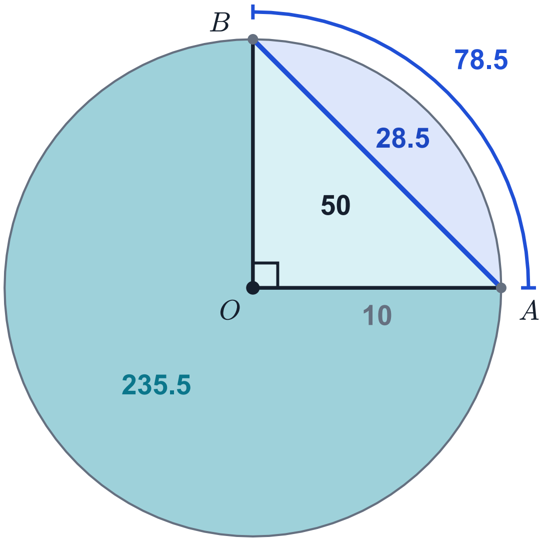
The triangle is half the square



At 90° the chord is the diagonal of a 10 by 10 square, so the triangle is half that square: 50, no sines needed. The quarter circle is 78.5, and what lies beyond the chord is 78.5 − 50 = 28.5.

A quarter circle of radius 10 sits on a square of side 10, with the diagonal as chord, triangle 50 and sliver 28.5.

A quarter and its three quarters make the circle



The quarter sector is 78.5 cm²: a 50 cm² triangle plus a 28.5 cm² segment. The other three quarters, the major sector, are 314 − 78.5 = 235.5 cm², and the two sectors add back to 314.

A sector and its complement always add back to the full circle, so find whichever is easier and subtract.

WORKED EXAMPLE · **A segment at 90 degrees**

1.	sector area $\approx (90/360) \cdot 3.14 \cdot 10^2$	*Substitute $\theta = 90°$, $\pi \approx 3.14$ and $r = 10$ cm into the sector-area formula.*
2.	$= (1/4) \cdot 3.14 \cdot 100 = 78.5\ \text{cm}^2$	*$90/360$ simplifies to $1/4$, and $10^2 = 100$.*
3.	triangle area $= 10^2 \cdot \sin(45°) \cdot \cos(45°)$	*The half-angle of $90°$ is $45°$.*

WORKED EXAMPLE · **A segment at 90 degrees**

4.	$= 100 \cdot \left(\sqrt{2}/2\right) \cdot \left(\sqrt{2}/2\right) = 100 \cdot (1/2) = 50$ cm²	$\sin(45°) = \cos(45°) = \sqrt{2}/2$, *and their product is exactly* $1/2$.
5.	minor segment area $\approx 78.5 - 50 = 28.5$ cm²	*Segment area is sector area minus triangle area.*
6.	major sector area $\approx 3.14 \cdot 100 - 78.5 = 314 - 78.5 = 235.5$ cm²	*This step pairs with the major-sector relation instead of the major-segment one: the whole circle's area minus the minor sector.*
7.	minor sector plus major sector $= 78.5 + 235.5 = 314$ cm²	**CHECK** ***Check by adding the two sectors back together.*** *The total should land on the whole circle's area,* $3.14 \cdot 100 = 314$ *cm², confirming both areas above.*

BACK
Area of a segment · p. 370

TRY
A chord subtends $\theta = 90°$ in a circle of radius 6 cm. Using $\pi = 3.14$, find the minor segment's area.

Answer: Sector $= 28.26$; triangle $= 18$; minor segment $= 28.26 - 18 = 10.26$ cm².

KEY
At 90° the two radii are perpendicular, so the triangle is right-angled and isosceles.

FIND A SEGMENT'S AREA, FROM THE SECTOR AND THE TRIANGLE INSIDE IT.

1. **Find the sector's area** Use $(\theta/360°) \cdot \pi r^2$. For $\theta = 90°$, $r = 10$ cm and $\pi \approx 3.14$, that is 78.5 cm².
2. **Halve the angle** The triangle's half-angle is $\theta/2$. For $\theta = 90°$, that is 45°.
3. **Find the triangle's area** Use $r^2 \cdot \sin(\theta/2) \cdot \cos(\theta/2)$. With $\sin(45°) = \cos(45°) = \sqrt{2}/2$, that is $100 \cdot (1/2) = 50$ cm².
4. **Subtract the triangle** Segment area is sector area minus triangle area, $78.5 - 50 = 28.5$ cm².
5. **Check the segment is smaller** A segment always sits inside its sector, so 28.5 cm² must be less than 78.5 cm², and it is.

CHECK YOURSELF

31. A chord subtends 90° in a circle of radius 10 cm (using $\pi = 3.14$). The sector's area is closest to

- **A.** 314 cm²
- **B.** 39.25 cm²
- **C.** 157 cm²
- **D.** 78.5 cm²

32. For that same chord (sector area 78.5 cm²), the triangle bounded by the two radii and the chord has area

- **A.** 50 cm²
- **B.** 100 cm²
- **C.** 70.7 cm²
- **D.** 78.5 cm²

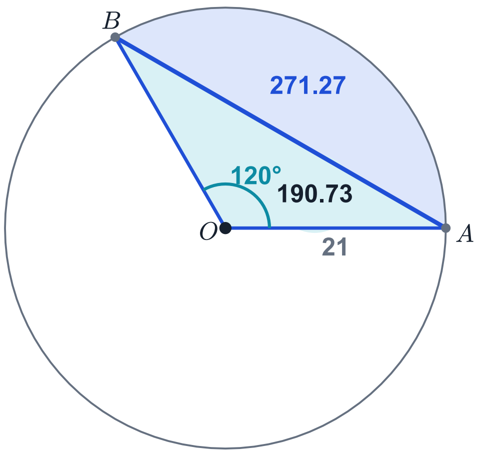
1 more in this set online.

WORKED EXAMPLE · A segment at 120 degrees

1.	sector area $\approx (120/360) \cdot (22/7) \cdot 21^2$	*Substitute $\theta = 120°$, $\pi \approx 22/7$ and $r = 21$ cm into the sector-area formula.*
2.	$= (1/3) \cdot (22/7) \cdot 441 = 462$ cm²	*$120/360$ simplifies to $1/3$, and $441/7 = 63$, so $(22/7) \cdot 441 = 22 \cdot 63 = 1386$, then $1386/3 = 462$.*
3.	triangle area $= 21^2 \cdot \sin(60°) \cdot \cos(60°)$	*The half-angle of $120°$ is $60°$.*
4.	$= 441 \cdot \left(\sqrt{3}/2\right) \cdot (1/2) \approx 441 \cdot 0.865 \cdot 0.5 \approx 190.73$ cm²	*$\sin(60°) = \sqrt{3}/2 \approx 0.865$ and $\cos(60°) = 1/2$, using $\sqrt{3} \approx 1.73$.*
5.	segment area $\approx 462 - 190.73 \approx 271.27$ cm²	*Segment area is sector area minus triangle area.*
6.	segment area 271.27 cm² is less than sector area 462 cm²	CHECK ***Check the size of the answer.*** *A segment sits inside its sector with a triangle removed, so it can never be larger than the sector, and 271.27 is less than 462, as it must be.*

KEY
No segment problem here needs a central angle wider than 120°.

The segment never outgrows its sector



Sector 462 minus triangle 190.73 leaves the segment, 271.27 cm². The segment is the sector with a triangle taken away, so 271.27 must be less than 462, and it is.

A 120 degree sector of a 21 cm circle has a triangle of 190.73 and a segment of 271.27 square centimetres.

CHECK YOURSELF

33. A chord subtends 120° in a circle of radius 21 cm (using $\pi = 22/7$). The sector's area is

- **A.** 1386 cm^2
- **B.** 231 cm^2
- **C.** 462 cm^2
- **D.** 924 cm^2

34. For that same chord (sector area 462 cm^2, triangle area about 190.73 cm^2), the minor segment's area is closest to

- **A.** 652.73 cm^2
- **B.** 271.27 cm^2
- **C.** 462 cm^2
- **D.** 190.73 cm^2

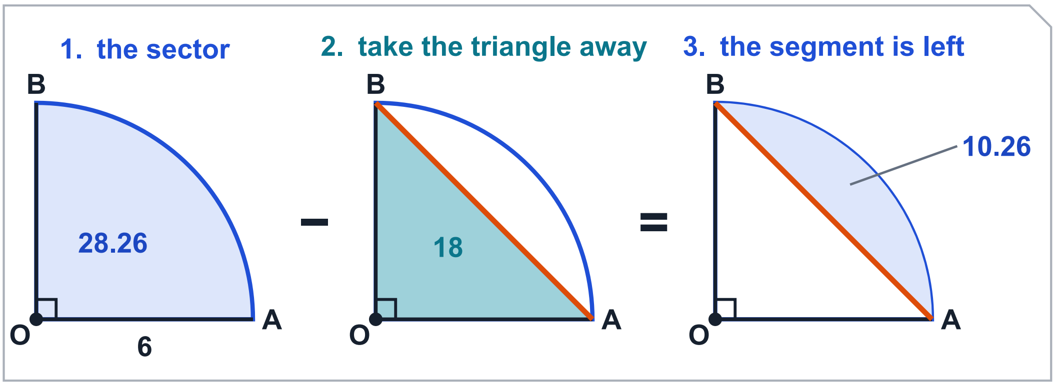
1 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap

Find the sector, then take the triangle away



The order never changes: the sector's formula is direct, 28.26 here; the triangle is one subtraction away, 18; what is left, 10.26, is the segment. A segment problem is a sector problem with one more step.

Read left to right, the strip is the whole method for any segment question: the sector first, because its formula is direct, then one subtraction.

- SECTOR AREA: $(\theta/360°) \cdot \pi r^2$. At $\theta = 90°$, $r = 10$ cm: 78.5 cm^2.
- ARC LENGTH: $(\theta/360°) \cdot 2\pi r$. At $\theta = 30°$, $r = 14$ cm: ≈ 7.33 cm.
- MAJOR FROM MINOR: whole circle minus minor. At $\theta = 30°$, $r = 4$ cm: $50.24 - 4.19 = 46.05$ cm^2.
- PERIMETER: sector is $2r$ plus arc, segment is chord plus arc. At $r = 14$ cm: sector perimeter $2 \cdot 14 + 7.33 \approx 35.33$ cm.
- SEGMENT AREA: sector minus triangle. At $\theta = 90°$, $r = 10$ cm: $78.5 - 50 = 28.5$ cm^2.
- TRIANGLE INSIDE: $r^2 \cdot \sin(\theta/2) \cdot \cos(\theta/2)$. At $\theta = 60°$, $r = 15$ cm: ≈ 97.31 cm^2.

Every sector and every segment is a slice cut from a circle, in one of two ways. A sector's area and arc length are both the same fraction of the circle, set by how its angle θ compares to 360°. A segment shares that same sector's arc, but trades the two radii for a single chord. We should reach for the sector first, even when the question asks for a segment. Its formula is direct, and the segment's formula is always just one subtraction away. Let us use this order on the next problem: find the sector, then take the triangle away.

BACK
Sector of a circle · p. 363

KEY
One circle, cut two ways: by two radii for a sector, by a chord for a segment.

CHECK YOURSELF

35. Every sector's area and arc length are both the same fraction of the circle's area and circumference as

- **A.** the radius r is of the diameter
- **B.** the chord is of the circumference
- **C.** the triangle is of the sector
- **D.** the angle θ is of 360°

36. A segment is best described, in one line, as

- **A.** a sector doubled in area
- **B.** an arc without a chord
- **C.** a triangle without a sector
- **D.** a sector minus its triangle

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

37. For a sector of angle θ in a circle of radius r, which pair of formulas is correct?

- **A.** area $= (\theta/360) \cdot 2\pi r$; arc length $= (\theta/360) \cdot \pi r^2$
- **B.** area $= (\theta/180) \cdot \pi r^2$; arc length $= (\theta/180) \cdot 2\pi r$
- **C.** area $= (\theta/360) \cdot \pi r^2$; arc length $= (\theta/360) \cdot \pi r^2$
- **D.** area $= (\theta/360) \cdot \pi r^2$; arc length $= (\theta/360) \cdot 2\pi r$

38. A sector has a central angle of $90°$ in a circle of radius r. What fraction of the whole circle's area does the sector cover?

- **A.** 1/2
- **B.** 1/4
- **C.** 1/3
- **D.** it depends on the value of r, not just on the angle

3 more in this set online.

IN EVERY CHAPTER

Where you will meet this

You divide round things by angle, or cut them with a straight line, more often than you think. Here are seven places a sector or a segment decides the numbers.

- **Drawing a pie chart.** A class survey asks 40 students their favourite sport. 10 choose cricket.
  The maths: cricket's slice is a sector, and every sector's angle is a share of the full 360°.
  10 out of 40 is $1/4$. So the slice's angle is $1/4$ of 360°, which is 90°.

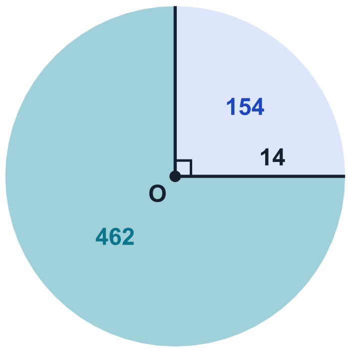
- **Designing a round park garden.** A round garden in a park, radius 7 m, is split into 6 equal sectors for 6 flowers.
  The maths: each sector's area is the same fraction of the bed's area as its angle is of 360°.
  Each sector's angle is 60°. Using $\pi \approx 22/7$, its area is $(60/360) \cdot (22/7) \cdot 7^2 \approx 25.67$ m².

- **Cutting a pizza into slices.** A round pizza, radius 14 cm, is cut into 8 equal slices for a party.
  The maths: each slice is a sector, and its angle is $1/8$ of the full 360°.
  Each slice's angle is $360/8 = 45°$. Using $\pi \approx 22/7$, one slice's area is $(45/360) \cdot (22/7) \cdot 14^2 = 77$ cm².

- **Pricing a wedge-shaped plot.** A wedge-shaped plot at a road junction is a sector of radius 21 m and angle 60°.
  The maths: the plot's area is the same fraction of a full circle's area as its angle is of 360°.
  Using $\pi \approx 22/7$, its area is $(60/360) \cdot (22/7) \cdot 21^2 = 231$ m². At ₹2000 per m², it costs ₹ 462000, or ₹4.62 lakh.

- **Cutting a cake for a party.** A round cake, radius 14 cm, is cut so one slice has a 90° angle.
  The maths: the rest of the cake is the major sector, the full circle's angle minus the slice's angle.
  Using $\pi \approx 22/7$, the slice's area is $(90/360) \cdot (22/7) \cdot 14^2 = 154$ cm². The rest of the cake is $616 - 154 = 462$ cm².

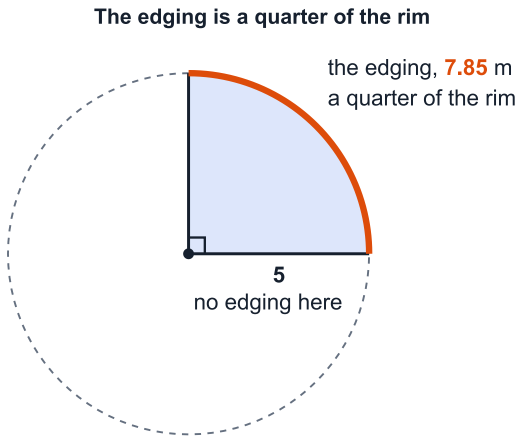
One slice and the rest of the cake



A 90° slice of a 14 cm cake is a quarter of it: 154 cm².
The rest is the major sector, 616 − 154 = 462 cm².

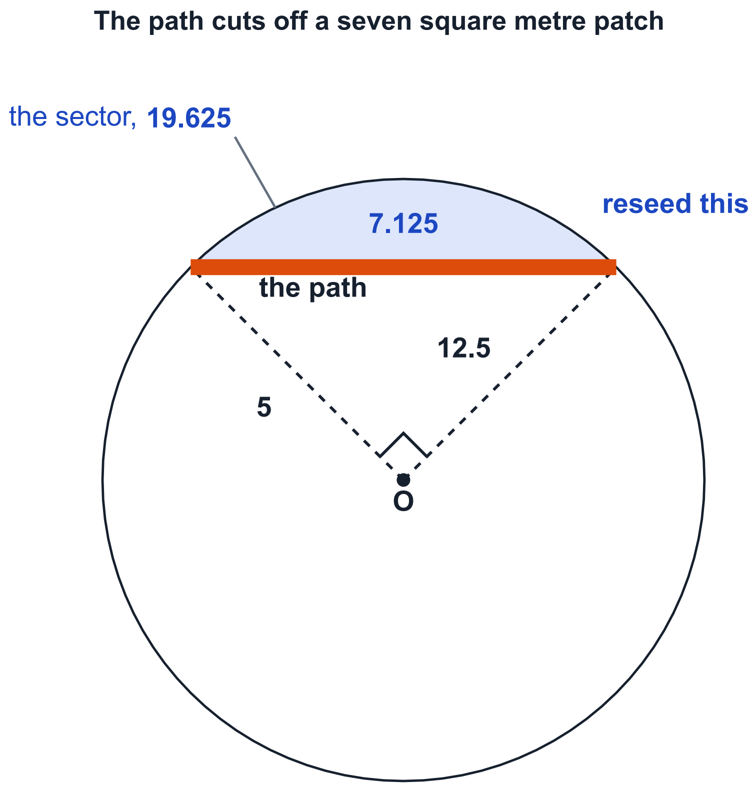
Find the smaller sector and subtract it from the whole circle: the major sector needs no formula of its own.

- **Edging a quarter-circle flower bed.** A flower bed in a corner is a quarter circle of radius 5 m. You buy plastic edging for its curved side.
  The maths: the curved side is an arc. It is the same fraction of the whole circle's edge as its angle is of 360°.
  A quarter circle has angle 90°. Using $\pi \approx 3.14$, the arc is $(90/360) \cdot 2 \cdot 3.14 \cdot 5 = 7.85$ m of edging.



Only the curved side needs edging. A quarter circle's arc is a quarter of the whole rim: (1/4) · 2 · 3.14 · 5 = 7.85 m, and the two straight sides of 5 m are the corner walls.

A garden corner's quarter circle of radius 5 needs edging only along its curved arc, 7.85 m, not the straight sides.



The path is a chord, so the strip of grass beyond it is a segment: the 90° sector, 19.625, minus the right-angled triangle between the path and the centre, 12.5, leaves 7.125 m² to reseed.

A path across a circle always makes a segment, and pricing the grass beyond it is the chapter's subtraction with real square metres attached.

- **Reseeding grass cut off by a path.** You run a landscaping business. A client's circular lawn, radius 5 m, is crossed by a straight path. Its two ends and the centre make a right angle.
  The maths: the small patch cut off is a segment. Its area is the sector's area minus the triangle's area.
  Using $\pi \approx 3.14$, the sector's area is $(90/360) \cdot 3.14 \cdot 5^2 = 19.625$ m². The triangle's area is 12.5 m². The patch is $19.625 - 12.5 = 7.125$ m².

Your turn. A round mural is painted with one coloured wedge, radius 6 m and angle 120°. Using $\pi \approx 3.14$, what is the wedge's area? *Answer:* Its area is $(120/360) \cdot 3.14 \cdot 6^2 = 37.68$ m².

Practice: Exercise 11.1

PRACTICE SET

1. PRACTICE A sector is cut from a circle of centre O and radius 7 cm, with an angle of 90° at the centre. Find the area of the sector. Use $\pi \approx 22/7$. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	the sector is $90/360 = 1/4$ of its circle	*a sector is a fraction of its own circle, and the fraction is the angle over 360° — write that fraction down before any π appears*
2.	the whole circle's area is $\pi r^2 = (22/7) \cdot 7^2 = (22/7) \cdot 49 = 154$ cm²	*with $\pi \approx 22/7$ a radius that is a multiple of 7 cancels the 7 underneath, so the whole circle comes out a whole number and nothing has been rounded yet*
3.	sector area $= (1/4) \cdot 154 = 38.5$ cm²	*take the fraction from the first line of the whole-circle area from the second*
4.	the check — four quarters would be $4 \cdot 38.5 = 154$ cm², the whole circle again	*a quarter has to fit into its own circle exactly four times; if it does not, either the fraction or the squaring went wrong*

2. PRACTICE The same circle, centre O and radius 7 cm, and the same 90° sector. Find the arc length of the sector. Use $\pi \approx 22/7$. (Same circle, same fraction — but an arc is a fraction of the circumference $2\pi r$, not of the area.)
3. PRACTICE The minute hand of a clock is 7 cm long. Find the area it sweeps in 15 minutes. Use $\pi \approx 22/7$. (The angle is not given. Ask what fraction of an hour 15 minutes is, then turn that fraction into degrees.)
4. PRACTICE Find the area of a sector of a circle with radius 6 cm if the angle of the sector is 60°. Use $\pi \approx 22/7$.
5. PRACTICE A sector has radius 21 cm and angle 60°. Find its arc length. Use $\pi \approx 22/7$.
6. PRACTICE The minute hand of a clock is 7 cm long. Find the area swept by it in 10 minutes. Use $\pi \approx 22/7$.
9. PRACTICE An umbrella has 8 ribs, equally spaced, each 45 cm long. Treating the umbrella as a flat circle, find the area between two consecutive ribs. Use $\pi \approx 22/7$.
10. PRACTICE A table cover is a circle of radius 35 cm. A chord cutting off a design segment subtends an angle of 90° at the centre. Find the area of the minor segment. Use $\pi \approx 22/7$.
11. PRACTICE A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding minor segment. Use $\pi = 3.14$ and $\sqrt{3} = 1.73$.
12. PRACTICE A car has two wiper blades, each 14 cm long, sweeping through an angle of 90° without overlapping. Find the total area cleaned by the two blades. Use $\pi \approx 22/7$.

7. PRACTICE A goat is tied to a peg at the centre of a circular field of radius 28 m by a rope, and can graze over a sector of angle $90°$. Find the area it can graze. Use $\pi \approx 22/7$.

8. PRACTICE A brooch is designed as a circle of radius 17.5 mm, divided into 9 equal sectors by wire radii from the centre. Find the angle of each sector, and the arc length of one sector. Use $\pi \approx 22/7$.

13. PRACTICE A lighthouse's rotating warning beam lights up a sector of angle $72°$ out to a radius of 17.5 m. Find the area lit up by the beam. Use $\pi \approx 22/7$.

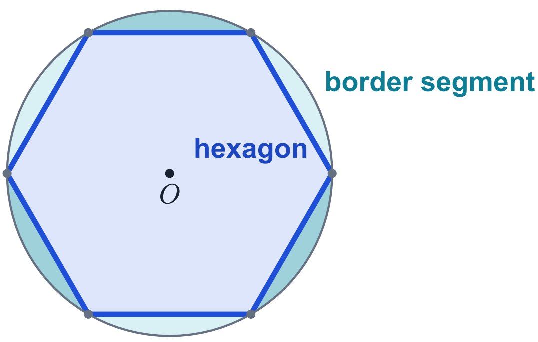
14. PRACTICE A circle has radius 10.5 cm. A minor sector cut from it has angle $60°$. Find the area of the major sector. Use $\pi \approx 22/7$.

ANSWERS **1.** $(1/4) \cdot (22/7) \cdot 7^2 = (1/4) \cdot 154 = 38.5$ cm².
2. $(1/4) \cdot 2 \cdot (22/7) \cdot 7 = (1/4) \cdot 44 = 11$ cm.
3. 15 minutes is a quarter of an hour, so $\theta = 90°$ and the area swept is 38.5 cm² — the same quarter circle as question 1.
4. $132/7 \approx 18.86$ cm². **5.** 22 cm. **6.** $77/3 \approx 25.67$ cm². **7.** 616 m².
8. Each sector's angle is $40°$; arc length $= 110/9 \approx 12.22$ mm.
9. $22275/28 \approx 795.54$ cm². **10.** 350 cm². **11.** 88.44 cm².
12. 308 cm² in total, from two sectors of 154 cm² each. **13.** 192.5 m².
14. Minor sector $= 57.75$ cm², so major sector $= 346.5 - 57.75 = 288.75$ cm².
Full solutions: **manthika.com/learn/math10-ch11**

BACK
Area of a sector · p. 365

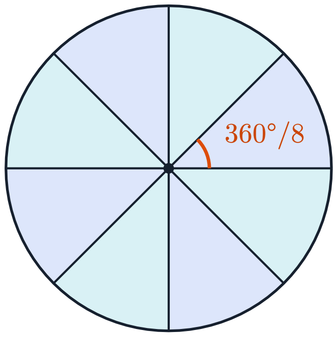
Further practice

A hexagon and its six border segments



A regular hexagon sits inside a circle, leaving six equal curved pieces between its sides and the circle, one of them labelled.

One circle, many equal sectors



A circle is divided into eight equal wedges in alternating colours, with one wedge's arc labelled 360 degrees divided by 8.

PRACTICE SET

15. PRACTICE Find the area of a sector of a circle with radius 42 cm if the angle of the sector is 60°. Use $\pi \approx 22/7$.

SOLUTION — Q15 · Worked in full

1. $A = (60/360) \cdot (22/7) \cdot 42^2$ — *substitute $\theta = 60°$, $\pi \approx 22/7$ and $r = 42$ cm into the sector-area formula*
2. $A = (1/6) \cdot (22/7) \cdot 1764 = 924$ — *60/360 simplifies to 1/6, and $42^2 = 1764$; multiplying these gives the sector's area*

16. PRACTICE A sector of a circle has radius 63 cm and angle 40°. Find the length of its arc. Use $\pi \approx 22/7$.

17. PRACTICE A circular lawn has radius 14 m. A gardener marks out a sector-shaped flower bed with a central angle of 108°. Find the area of the flower bed. Use $\pi \approx 22/7$.

18. PRACTICE A ceiling fan has a single blade of length 56 cm, fixed at the centre of rotation. As the blade turns through an angle of 45°, find the distance travelled by the tip of the blade. Use $\pi \approx 22/7$.

19. PRACTICE The minor sector of a circle has a central angle of 80°. What is the angle of the major sector cut off by the same two radii?

A. 80°
B. 180°
C. 280°
D. 300°

24. PRACTICE A chord of a circle of radius 30 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment. Use $\pi = 3.14$ and $\sqrt{3} = 1.73$.

25. PRACTICE A circle has radius 20 cm. A chord subtends a right angle at the centre. The sector so formed has area 314 cm², and the triangle formed by the two radii and the chord has area 200 cm². What is the area of the minor segment cut off by the chord?

A. 314 cm²
B. 200 cm²
C. 114 cm²
D. 514 cm²

26. PRACTICE A minor sector cut from a circle of radius 49 cm has an angle of 90° at the centre. Find the area of the major sector cut off by the same two radii. Use $\pi \approx 22/7$.

20. PRACTICE A sector is cut from a circle of radius 28 cm, with an angle of 45° at the centre. Find the perimeter of the sector. Use $\pi \approx 22/7$.

21. PRACTICE A chord of a circle of radius 20 cm subtends a right angle at the centre. Find the perimeter of the minor segment cut off by the chord. Use $\pi = 3.14$ and $\sqrt{2} = 1.41$.

22. PRACTICE Which two lengths make up the perimeter of a segment cut off by a chord?

A. The two radii and the arc
B. The chord and the two radii
C. The chord and the arc
D. The arc alone

23. PRACTICE A chord of a circle of radius 40 cm subtends a right angle at the centre. Find the area of the corresponding minor segment. Use $\pi = 3.14$.

27. PRACTICE A chord of a circle of radius 60 cm subtends an angle of 120° at the centre. Find the area of the corresponding minor segment. Use $\pi = 3.14$ and $\sqrt{3} = 1.73$.

28. PRACTICE A chord of a circle of radius 42 cm subtends an angle of 120° at the centre. Find the area of the corresponding major segment. Use $\pi \approx 22/7$ and $\sqrt{3} = 1.73$.

29. PRACTICE A circular tablecloth has radius 70 cm. A decorative sector-shaped patch stitched onto it covers an angle of 144° at the centre. Find the area of the tablecloth not covered by the patch. Use $\pi \approx 22/7$.

30. PRACTICE A sector of a circle has radius 21 cm. The length of its arc is 11 cm. Find the area of the sector. Use $\pi \approx 22/7$.

ANSWERS **15.** 924 cm². **16.** 44 cm. **17.** 184.8 m². **18.** 44 cm. **19.** C — 280°. **20.** 78 cm. **21.** 59.6 cm. **22.** C — The chord and the arc. **23.** 456 cm². **24.** 81.75 cm². **25.** C — 114 cm². **26.** 5659.5 cm². **27.** 2211 cm². **28.** 4458.93 cm². **29.** 9240 cm². **30.** 115.5 cm².
Full solutions: **manthika.com/learn/math10-ch11**