12 CHAPTER 12 Surface Areas and Volumes

WHAT THIS CHAPTER BUILDS
You will learn to find the surface area and the volume of a solid built from two simpler solids, counting only the faces that stay exposed.

MENSURATION UNIT, BOARD CORE

IN THIS CHAPTER

Nila and Aarav compare their ice cream cones, each a cone topped with one round scoop.

Getting started

Solids built from solids

A STORY

One scoop on a cone

Nila and Aarav stand by the canteen window, each holding an ice cream cone with one round scoop on top.

"Suppose the ice cream fills the cone right to the rim," Aarav says, "and the part above the rim is half a ball."

"Then the ice cream is two solids," Nila says. "A cone and a hemisphere. Add their volumes and you have all of it."

"So adding works," Aarav says. "What if I wanted to wrap the whole thing in foil?"

"Then you cannot just add," Nila says. "The flat circle where the scoop sits on the cone is hidden inside. No foil goes there."

"So the foil covers the side of the cone and the round top," Aarav says. "Nothing else."

"Volumes add outright," Nila says. "Surfaces count only what stays outside."

Aarav looks at his scoop. "Then I need the volume and the surface of each basic solid first."

You will learn them for the cube, cuboid, cylinder, cone, sphere and hemisphere, and then put the solids together.

Many everyday objects are made of two basic solids joined at one face. A test tube is a cylinder closed by a hemisphere. A tent is a cone sitting on a cylinder. A pen stand is a cylinder with a conical hole bored into it. Look around your own desk, and you can probably spot a fourth.

Six basic solids build every combination here: cube and cuboid, sphere and hemisphere, cylinder and cone. Each one's own surface area and volume comes first, then two of them join together.

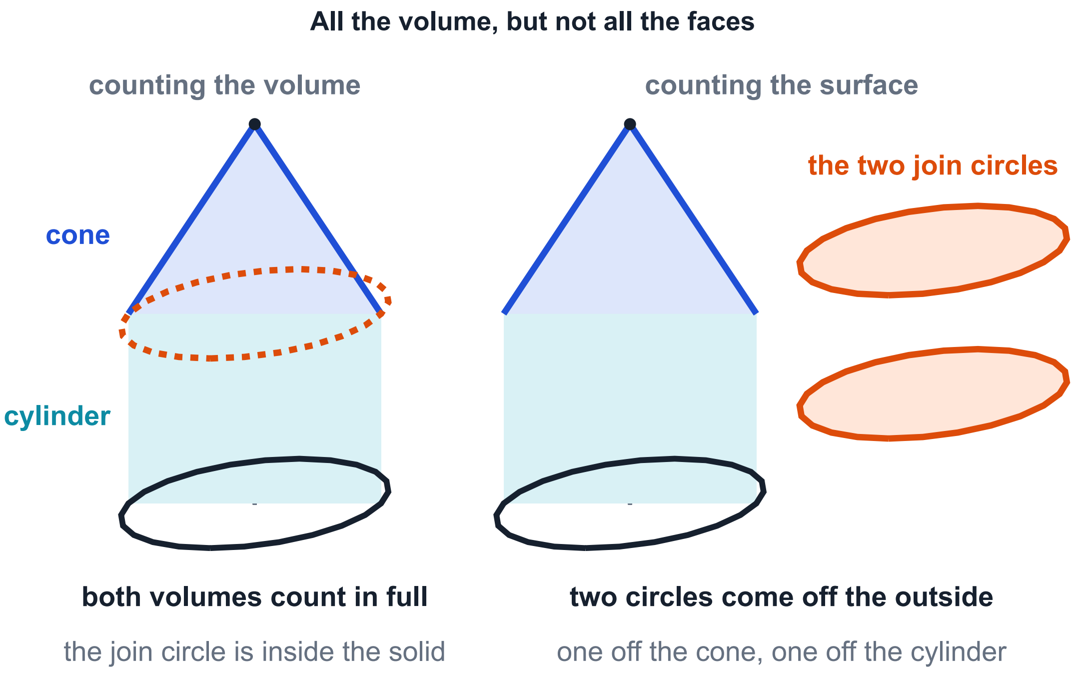
A combination's volume always adds or subtracts outright, but its surface area counts only the faces still exposed after the join. We never count a hidden face again once a join covers it, on either side.

RULE
Before adding surface areas, name the faces the join hid, then count only what still shows.

KEY
Joining two solids creates no new faces: it only hides ones that were already there.

A join hides a face on both sides

Two solids drawn apart still show both circles where they will meet, but drawn joined as one solid, neither circle is drawn or counted.



One join counted twice: both solids keep their whole volume, but the outside gives up two circles, one from each solid.

A SCHOLAR INDIA REMEMBERS

Kanada is named as the founder of the Vaisheshika school. The name means the school of distinction. It sorted all the things that exist into six kinds. We need that care with solids. A toy top is a cone joined to a hemisphere. Some of its surface shows, and some is hidden at the join. Those are two different kinds of surface.

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IN THE WILD

An ice-cream cone with a scoop on top is a cone joined to a hemisphere. Its volume is the cone's plus the hemisphere's, added outright. Its surface is not the cone's plus the hemisphere's, since the flat circle where they meet is hidden, not exposed. Knowing this, we can find the paint or wrapping any joined shape needs: add the separate surfaces, then subtract what the join hid.

CHECK YOURSELF

1. A toy is made by fixing a cone on top of a hemisphere of the same radius. To find its volume and total surface area, this chapter's rule is to

- **A.** add the two volumes outright, but count only the faces still exposed for the surface area
- **B.** add both the volumes and the total surface areas of the two solids outright
- **C.** average the two volumes and the two surface areas
- **D.** measure the whole combination as one new solid from scratch, ignoring the two original solids

2. When two solids are joined, why does the combination's volume simply add the two volumes, while its surface area is not simply the sum of the two surface areas?

- **A.** volume is always a larger number than surface area, so only volume survives a join
- **B.** volume counts space enclosed; surface area counts faces, and the join hides one
- **C.** surface area only applies to solids with a curved face, and volume applies to every solid
- **D.** surface area depends on which solid is named first in the problem, so order matters

1 more in this set online.

IN EVERY CHAPTER
Before you start

DO THIS FIRST

You already know how to measure flat shapes. Now measure solids built from them. Try each check below.

- **A solid's faces** (Class 8, Ganita Prakash Part 2, page 78).
  Here: a combination's hidden face is still one of these, just not exposed.
  Check: A cube has 6 faces. Hiding 1 leaves 5 exposed.
- **The net of a cylinder** (Class 8, Ganita Prakash Part 2, page 82).
  Here: unfolding shows the flat circles and the curved part separately.
  Check: A cylinder of radius 7 unfolds to a curved width of $2 \cdot 22/7 \cdot 7 = 44$.
- **Pythagoras' theorem** (Class 8, Ganita Prakash Part 2, page 44).
  Here: the slant height $l = \sqrt{r^2 + h^2}$ comes from this same relation.
  Check: $7^2 + 24^2 = 25^2$, since $49 + 576 = 625$.
- **Area of a rectangle** (Class 9, Ganita Manjari, page 130).
  Here: each of a cuboid's six faces is a rectangle, measured this same way.
  Check: A rectangle 5 cm by 3 cm has area $5 \cdot 3 = 15$.
- **Area of a circle** (Class 9, Ganita Manjari, page 143).
  Here: a cylinder's flat ends are circles, measured with this same formula.
  Check: A circle of radius 7 has area $22/7 \cdot 7^2 = 154$.

If any of these felt new, read the page named before going on.

Two ways to measure a surface

A solid's curved surface area counts only its curved part. It leaves out every flat face, if the solid has one.

The total surface area counts every face instead, flat and curved together. A solid with at least one flat face always has a total surface area bigger than its curved surface area.

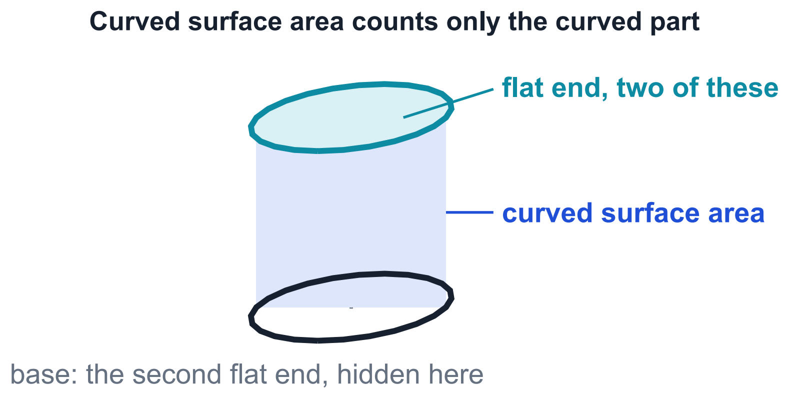
A sphere has no flat face, so its curved and total surface areas are the same number. We check whether a solid has a flat face before deciding which surface area a question wants.

Your turn: which is bigger for a cylinder, its curved surface area or its total surface area? (Answer: the total surface area, since it also counts the two flat circular ends the curved surface area leaves out.)

BACK
Solids built from solids · p. 395

RULE
A sphere has no flat face at all, so its curved and total surface areas are the same number.

KEY
Curved surface area skips the flat ends, but total surface area counts them too.



A cylinder's flat ends are its top and bottom circles, and the curved band between them is its curved surface.

CHECK YOURSELF

3. For a solid cylinder, what is the difference between its curved surface area and its total surface area?

- **A.** the curved surface area adds the two flat circular ends to the total surface area
- **B.** the total surface area adds the two flat circular ends to the curved surface area
- **C.** they are two different names for exactly the same measurement
- **D.** the curved surface area includes the volume enclosed, and the total does not

4. A sphere has no flat faces anywhere on it. What does this mean for its curved surface area and its total surface area?

- **A.** the total surface area is still larger, because every solid's total exceeds its curved measure
- **B.** a sphere has no total surface area, only a curved one
- **C.** they are the same number, since every part of a sphere's surface is curved
- **D.** the curved surface area is larger, since it is measured along the curve

1 more in this set online.

A combination of two solids

A combination of solids is a single object made from two basic solids joined at one matching face. The join can work two ways.

One solid can sit on top of another, the way a hemisphere sits on a cone. Or one solid's shape can be hollowed out from inside another, the way a conical hole is bored into a cylinder.

A test tube is a cylinder closed by a hemisphere at one end. We name the two solids and their one shared face first for every new combination, and the working gets easier after that.

Your turn: is a cube alone a combination of solids? (Answer: no, since a combination needs two basic solids joined at one shared face, and a cube is only one solid.)

TRY
A test tube is a cylinder closed at one end by what shape?

Answer: A hemisphere.

KEY
Two solids share one face: added on, or hollowed out.

A SCHOLAR INDIA REMEMBERS

Name the parts before you measure anything. A capsule is one cylinder and two hemispheres. Write that as a list: cylinder, hemisphere, hemisphere. Then every length you need belongs to one part on the list.

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CHECK YOURSELF

5. What makes an object a 'combination of solids' in this chapter's sense?

A. it is any object built from more than one material

B. it is any solid with both a curved and a flat face

C. joining two basic solids at one matching face

D. it is any solid with more than one axis of symmetry

6. A conical hole is bored into a solid cylinder. Is the result a 'combination of solids' in the chapter's sense?

A. no, because nothing was joined onto the cylinder from outside

B. no, because a hole is empty space, not a solid

C. yes — a solid's shape is removed from inside another

D. yes, but only if the hole passes all the way through the cylinder

1 more in this set online.

The slant height of a cone

A cone has two different lengths that are easy to confuse. Its height runs straight up the axis, from the base to the apex.

Its slant height runs along the curved surface instead, from a point on the rim of the base up to the apex. For a cone of base radius r and height h, Pythagoras' theorem gives the slant height as $l = \sqrt{r^2 + h^2}$.

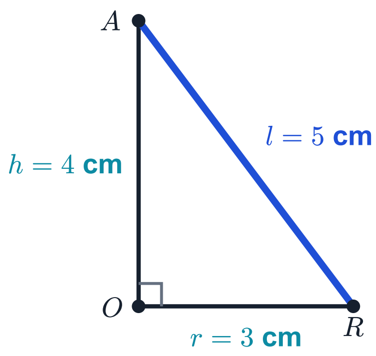
Because $l = \sqrt{r^2 + h^2}$, the slant height l is always longer than both r and h. We check which length a question wants before picking a value off the diagram. Reaching for the height when a question needs the slant height is a common slip.

Your turn: a cone has base radius 6 cm and height 8 cm. What is its slant height? (Answer: 10 cm, since $l = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$.)

RULE
$l = \sqrt{r^2 + h^2}$ always makes l longer than both r and h.

KEY
Slant height runs along the curved surface, not straight up the axis.

Slant height l is not the vertical height h



A right triangle shows a cone's height, radius and slant height as its three sides, and the slant height is the longest of the three.

CHECK YOURSELF

7. For a cone, what is the slant height?

- **A.** the straight-line distance from the apex straight down to the centre of the base
- **B.** the distance around the circular rim of the base
- **C.** the straight-line distance from the apex to a point on the rim of the base
- **D.** the radius of the circular base

8. Why is a cone's slant height l never equal to its height h, for any cone with a real base radius?

- **A.** l and h measure the same line, just named differently by different textbooks
- **B.** l equals h whenever the cone's radius is a whole number
- **C.** l is smaller than h whenever the base radius is small
- **D.** $l = \sqrt{r^2 + h^2}$ always exceeds h, since $r^2 > 0$

2 more in this set online.

The ideas

Surface area and volume of boxes

A cuboid is a box-shaped solid with three pairs of matching rectangular faces. For a cuboid of length l, breadth b and height h, each pair of opposite faces has area lb, bh and hl.

Adding all three pairs gives the total surface area of a cuboid: $2lb + 2bh + 2hl$. Its volume is the product of the three edges, lbh.

A cube is a cuboid whose three edges are all equal, $l = b = h = a$. We measure length, breadth and height for any box, and find both totals the same way.

Your turn: a cube has edge 5 cm. What are its total surface area and volume? (Answer: 150 cm² and 125 cm³, since $6 \cdot 5^2 = 150$ and $5^3 = 125$.)

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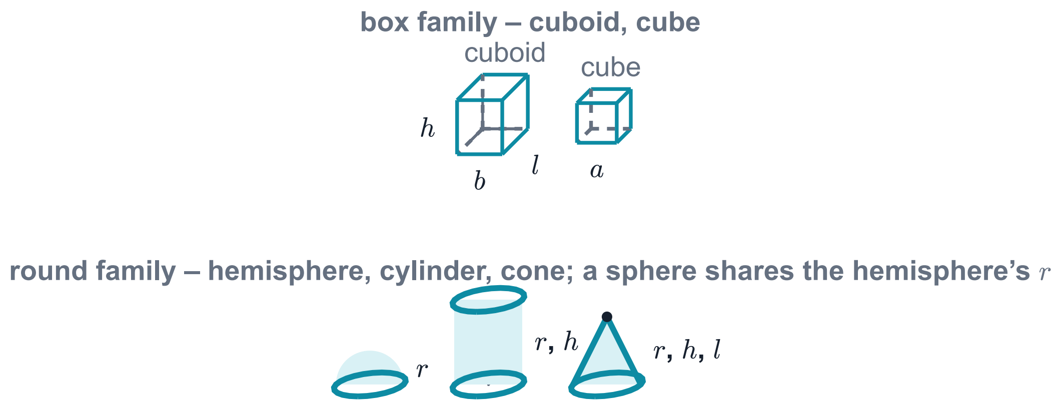
RULE
Total surface area of a cuboid is $2lb + 2bh + 2hl$, from three pairs of matching rectangular faces.

KEY
A cube is just a cuboid with all three edges equal.

Six solids, one labelled dimension each



Five basic solids are drawn, each labelled with only the letters its own formulas need.

CHECK YOURSELF

9. For a cuboid of length l, breadth b and height h, which expression gives its total surface area?

- **A.** lbh — the cuboid's volume, not its surface area
- **B.** $2(l+b+h)$ — twice the sum of the three edge lengths
- **C.** $2lb+2bh+2hl$
- **D.** $6lbh$ — six times the cuboid's own volume

10. A cube is 'a cuboid with $l=b=h=a$.' Using this, why does $2lb+2bh+2hl$ simplify to $6a^2$ for a cube?

- **A.** the formula changes completely for a cube, since a cube is a different solid
- **B.** each term becomes $2a^2$, and three copies sum to $6a^2$
- **C.** 6 comes from multiplying the three edges together
- **D.** the formula only works for a cube if a is measured in centimetres

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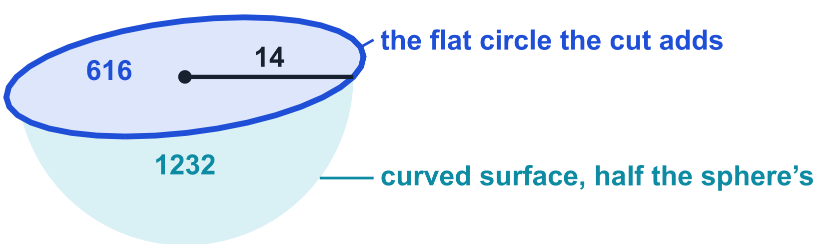
Surface area and volume, sphere family

A hemisphere keeps its flat circle



Halving a sphere halves the curved surface: 2 · 22/7 · 14² = 1232 cm². The cut adds a flat circle of 616 cm² that the sphere never had, so the total is not half the sphere's.

Cutting a solid creates a new face, so the surface area of a half is more than half of the whole.

A sphere is a perfectly round solid with every point on its surface the same distance from its centre. For a sphere of radius r, the surface area is $4 \cdot \pi r^2$ and the volume is $4/3 \cdot \pi r^3$.

A hemisphere is exactly half a sphere, cut through its centre. Its curved surface area and its volume are both half of the whole sphere's: curved surface area $2 \cdot \pi r^2$, volume $2/3 \cdot \pi r^3$.

A hemisphere's total surface area is not half the sphere's: the flat circular base has to be added on, giving $3 \cdot \pi r^2$ in total. We can halve the curved surface and the volume with confidence, but never the total surface area the same way.

Your turn: a hemisphere has radius 14 cm. What is its curved surface area? (Answer: 1232 cm², since $2 \cdot 22/7 \cdot 14^2 = 2 \cdot 616 = 1232$.)

CAUTION
A hemisphere's TOTAL surface area is not half the sphere's: it needs the flat circle added on.

KEY
Sphere formulas halve neatly for a hemisphere's curved surface and its volume.

CHECK YOURSELF

11. For a sphere of radius r, which pair correctly gives its surface area and volume?

- **A.** surface area $2\pi r^2$, volume $2/3\pi r^3$
- **B.** surface area πr^2, volume πr^3
- **C.** surface area $4\pi r^3$, volume $4/3\pi r^2$
- **D.** surface area $4\pi r^2$, volume $4/3\pi r^3$

12. A hemisphere's volume is exactly half a sphere's, so why is a hemisphere's TOTAL surface area more than half a sphere's surface area?

- **A.** the hemisphere adds a flat circular base that the sphere never had
- **B.** it is not more than half — $3\pi r^2$ is exactly half of $4\pi r^2$
- **C.** curved surfaces always measure larger once a solid is cut in half
- **D.** the hemisphere's radius is larger than the sphere's, so its surface area is larger

2 more in this set online.

Surface area and volume: cylinder

A right circular cylinder has two flat circular ends and one curved surface joining them. For radius r and height h, the curved surface area is $2 \cdot \pi rh$.

Adding the two flat circular ends gives the total surface area, $2 \cdot \pi rh + 2 \cdot \pi r^2$. The volume is $\pi r^2 h$, the area of one circular end times the height.

Curved surface area, by contrast, needs the height and the radius together, not the base area. If a question only asks for volume, we can skip the curved surface area completely.

Your turn: a cylinder has radius 7 cm and height 15 cm. What is its curved surface area? (Answer: 660 cm², since $2 \cdot 22/7 \cdot 7 \cdot 15 = 2 \cdot 22 \cdot 15 = 660$.)

RULE
Volume $\pi r^2 h$ never needs the curved surface area: only the base area and the height matter.

KEY
Curved surface area plus two circles gives the total surface area.

CHECK YOURSELF

13. For a cylinder of radius r and height h, which expression gives its total surface area?

- **A.** $\pi r^2 h$ — the cylinder's volume, not its surface area
- **B.** $2\pi rh + 2\pi r^2$
- **C.** $2\pi rh$ — the curved part alone, missing the two flat ends
- **D.** $4\pi rh$ — doubling the curve instead of adding the ends

14. A cylinder's total surface area formula adds $2\pi rh$ to two copies of πr^2. What does each of the two πr^2 pieces represent?

- **A.** the two flat circular ends, top and bottom
- **B.** the curved surface, counted in two different ways
- **C.** one end of the cylinder, counted twice by mistake
- **D.** the inside and outside of the curved surface

1 more in this set online.

Surface area and volume: cone

A right circular cone has one curved surface that slants from a circular base up to a single apex. Its slant height is $l = \sqrt{r^2 + h^2}$, for base radius r and height h.

The curved surface area of a cone is πrl, built from the slant height, never the vertical height. Adding the flat circular base gives the total surface area, $\pi rl + \pi r^2$.

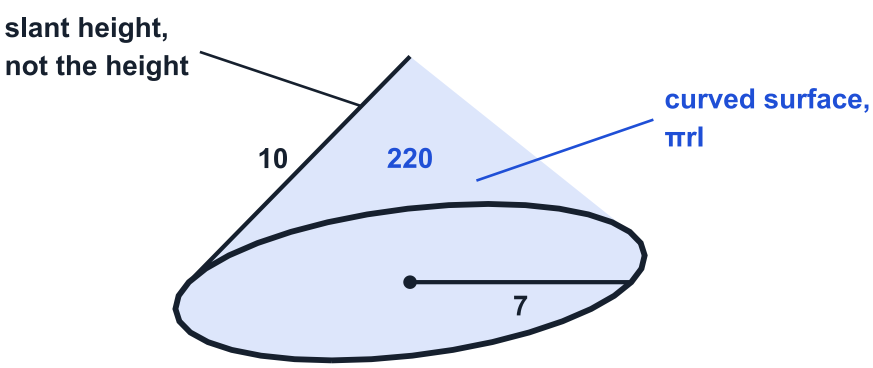
The volume of a cone, $1/3 \cdot \pi r^2 h$, uses the vertical height h: the slant height plays no part in it at all. We reach for h when we need the volume, and for l only when we need a curved surface.

Your turn: a cone has radius 7 cm and slant height 10 cm. What is its curved surface area? (Answer: 220 cm², since $\pi rl \approx 22/7 \cdot 7 \cdot 10 = 220$.)

CAUTION
Volume uses h, not l: the two formulas each use a different length.

KEY
Curved surface area of a cone always uses slant height l, never the vertical height h.

The curved surface uses the slant height



Curved surface area = πrl ≈ 22/7 × 7 × 10 = 220 cm².
The volume would use h, not l.

A cone with radius 7 and slant height 10 has curved surface area pi r l, about 220 square centimetres.

CHECK YOURSELF

15. For a cone of base radius r and slant height l, which expression gives its curved surface area?

- **A.** $\pi r h$
- **B.** $1/3\pi r^2 h$
- **C.** $\pi r l$
- **D.** $2\pi r l$

16. A cone's total surface area formula is $\pi r l + \pi r^2$, using the slant height l for one term but the radius r for the other. Why does the first term need l while the second needs only r?

- **A.** both terms could use either l or r; the choice makes no difference to the answer
- **B.** l belongs to the base and r belongs to the curved surface, the reverse of what the formula shows
- **C.** the curved part slants and needs l; the flat base needs only r
- **D.** the flat base also needs l, but the formula simplifies it away

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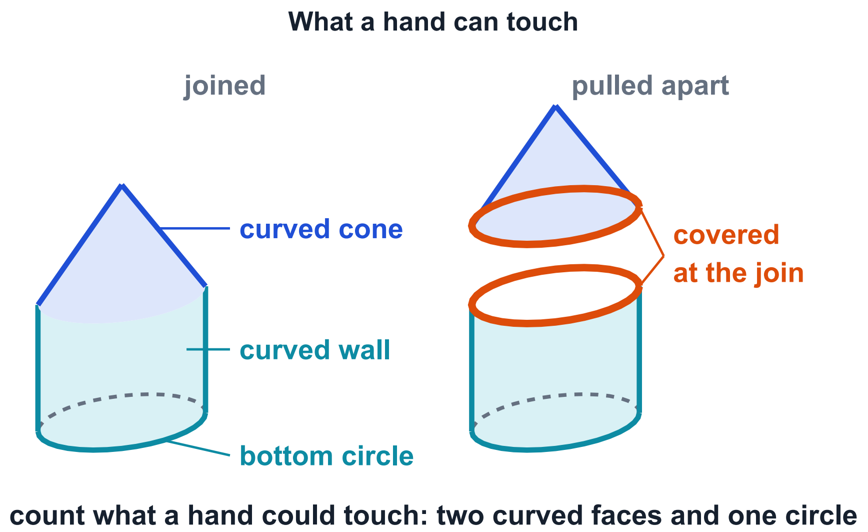
Only exposed faces count



Picture the finished object. The cone's curved surface, the cylinder's curved wall and its bottom circle can be touched; the two circles pressed together at the join cannot, so they are left out.

A cone joined to a cylinder hides the two circles at their join, leaving only the curved and flat faces exposed.

A combination of two solids has more faces than either solid alone, but not every one of them counts. The surface area of a combination adds up only the faces still visible from outside the finished object.

A face where the two solids meet is covered over. It drops out of the total, no matter which of the two solids it originally belonged to.

Checking whether a face is covered at the join settles which faces belong in a combination's surface area and which do not.

Your turn: does the face where two solids touch belong in the combination's surface area? (Answer: no, since that face is covered and no longer visible from outside.)

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A combination of two solids · p. 398

RULE
Picture the finished, joined object and count only what a hand could touch from outside.

KEY
Covered-up faces do not count, no matter which solid they came from.

A SCHOLAR INDIA REMEMBERS

Look at each face and ask what kind it is. Can you touch it from outside, or is it hidden inside the join? Take a cone standing on a hemisphere. The two flat circles are pressed together and hidden. We count the curved surface of the cone and the curved surface of the hemisphere. The hidden circles do not count.

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CHECK YOURSELF

17. When two solids are joined to form a combination, which faces count toward the combination's total surface area?

- **A.** every face of both original solids, whether hidden or visible
- **B.** only the faces that belonged to the larger of the two solids
- **C.** only the faces still visible from outside the combined solid
- **D.** only the curved faces, never the flat ones

18. A hemisphere is fixed onto one flat end of a cylinder. Why does the cylinder's OTHER flat end still count fully toward the total surface area?

- **A.** it was never touched by the join, so it remains just as exposed as before
- **B.** it does not count fully — every face of a joined solid loses some area
- **C.** it counts fully only if the hemisphere is smaller than the cylinder's radius
- **D.** it stops counting once any face of the solid is joined to another solid

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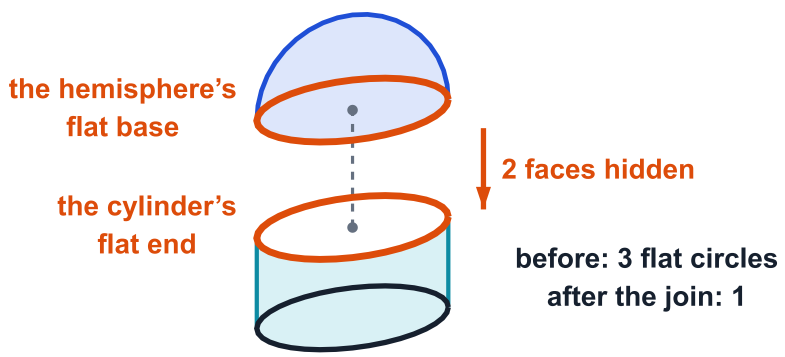
A join hides two faces

Two circles vanish at one join



one join, two faces gone, one from each solid

The join sits exactly where the cylinder's end and the hemisphere's base both were, so both are covered. Neither counts toward the surface area of the finished solid.

Count the flat circles before and after: three become one, because a single join swallows a face from each of the two solids.

A join between two solids hides a face on each side of it, not just one. Fixing a hemisphere onto the flat end of a cylinder removes two faces at once.

The cylinder's own flat end disappears, because the hemisphere now sits exactly where it used to be. The hemisphere's flat base disappears too, because it sits flush against the cylinder's end.

Adding the two full total surface areas counts the joined faces twice, before anyone has subtracted anything. One fix adds only the curved surfaces that remain. A second fix subtracts the hidden circle from a total that still includes it. We check our own working the same way before trusting any sum of two totals.

Your turn: when a hemisphere is glued onto a cylinder, how many faces does the join hide? (Answer: two, since both the cylinder's flat end and the hemisphere's flat base disappear together.)

CAUTION
Adding the two full totals counts the joined faces twice before anyone subtracts anything.

KEY
One join can remove two faces, one from each solid, never from just one.

CHECK YOURSELF

19. When a hemisphere is fixed onto a flat end of a cylinder of the same radius, how many faces does the join hide?

- **A.** two — the cylinder's end and the hemisphere's base, both at once
- **B.** one — only the cylinder's flat end disappears
- **C.** one — only the hemisphere's flat base disappears
- **D.** none — both flat faces are still there, simply touching each other

20. Why does a join between two solids always hide a face on BOTH sides of it, rather than on just one side?

- **A.** because the smaller solid's face always covers the larger solid's face
- **B.** it does not always hide both sides — only curved solids hide both faces
- **C.** one side is hidden by the join, and the other side is hidden by the paint or material added later
- **D.** the two faces used to sit in the very same place once joined, so both are covered at once

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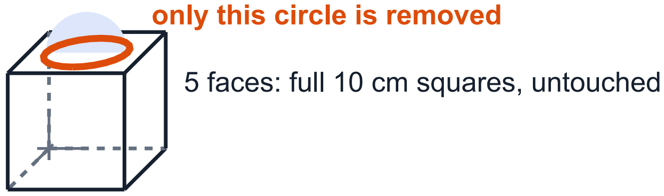
Only the shared face disappears

Five faces stay whole – one loses a circle



cube's total surface: $6 \times 10^2 = 600$ cm²
circle removed: $22/7 \times 3.5^2 = 38.5$ cm²
hemisphere's curved surface: $2 \times 22/7 \times 3.5^2 = 77$ cm²

A hemisphere on a cube's face removes only the circle beneath it, and the cube's other five faces stay complete.

A join changes exactly one face of each solid, never the whole solid. Every other face stays fully exposed and fully counted, exactly as it was before the join.

Fix a hemisphere onto one face of a cube. The cube's other five faces are untouched, and all five still count in full.

Only the small circle directly under the hemisphere is removed from the cube's total surface area. We name which single face the join touches before doing any arithmetic. Every other face then stays safe from being lost or doubled by mistake.

Your turn: a hemisphere is fixed to one face of a solid. Do the solid's other faces change? (Answer: no, since a join changes only the one face it touches, and every other face stays untouched.)

TRY
A hemisphere sits on one face of a cube. How many of the cube's faces still count in full?

Answer: Five: only the face under the hemisphere loses its circle.

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Only exposed faces count · p. 404

KEY
A join changes one face, not the whole solid.

CHECK YOURSELF

21. A hemisphere sits on one face of a cube. What happens to the cube's other five faces?

- **A.** they stay fully exposed and fully counted, untouched by the join
- **B.** each loses a small circle of area, matching the hemisphere's base
- **C.** they are halved in area, since the cube now shares its shape with the hemisphere
- **D.** they disappear along with the joined face, since the cube is no longer a separate solid

22. Why does only the small circle under the hemisphere disappear from the cube's face, rather than the cube's whole face disappearing?

- **A.** the cube's whole face does disappear, but the hemisphere's curved surface exactly replaces its full area
- **B.** the circle disappears because it is inside the cube, hidden from the hemisphere's own view
- **C.** the hemisphere's base only covers a circle matching its own radius
- **D.** the circle disappears only if the hemisphere's radius equals the cube's edge

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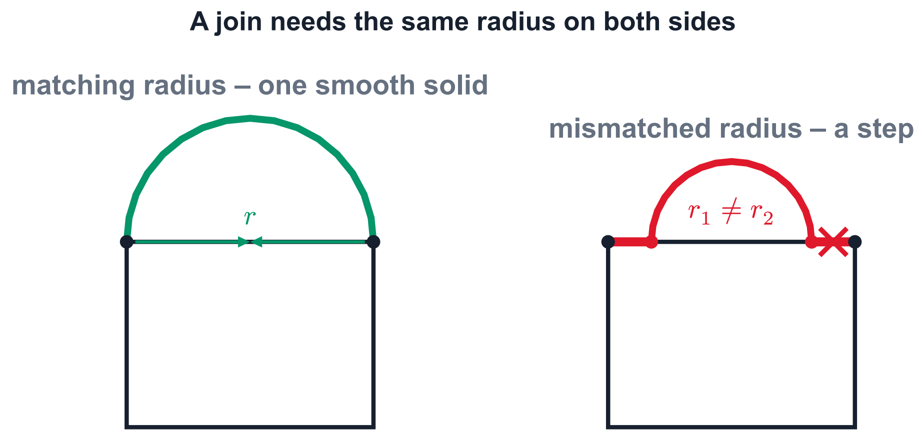
A join needs matching radii



A dome whose base matches the block's width joins smoothly, but a narrower dome leaves a flat ring exposed at the join.

Two solids join smoothly only when they share the same radius at the face where they meet. A cone mounted on a hemisphere needs the cone's base radius to match the hemisphere's radius exactly.

The same rule holds for a hemisphere fixed on a cylinder. It also holds for a hemisphere fixed on a flat face of matching size. A mismatched radius leaves a step or a gap at the join, instead of one smooth surface.

We check that the two radii match before writing down any formula.

Your turn: can a cone of radius 5 cm sit smoothly on a hemisphere of radius 6 cm? (Answer: no, since a smooth join needs the two radii to match exactly.)

CAUTION
A problem giving two different radii at the join describes two solids side by side, not one combined solid.

KEY
A matching radius at the join is what makes the combination one smooth solid.

CHECK YOURSELF

23. For a cone mounted on a hemisphere to join smoothly, what must be true of the two solids?

- **A.** they must share the same height
- **B.** they must share the same total surface area
- **C.** the same radius at the touching face
- **D.** they must both be made of the same material

24. Why does a mismatched radius at a join leave a step or a gap between the two solids?

- **A.** because the two solids would then have different volumes
- **B.** because mismatched radii make the combination's total surface area negative
- **C.** a larger circle cannot meet a smaller one edge-to-edge
- **D.** because the taller solid always pushes the shorter one out of alignment

1 more in this set online.

Spotting a combination in real life

Many everyday objects are combinations of exactly two basic solids, joined at one matching face. Naming the two solids first turns an unfamiliar object into two familiar formulas.

A test tube is a cylinder with a hemisphere at one end. A capsule-shaped tablet is a cylinder with a hemisphere at each end. An ice-cream cone with a scoop on top is a cone with a hemisphere.

Get the naming of the two solids and their shared face wrong, and every next formula is built on the wrong shape.

Your turn: what two solids make up a pencil sharpened to a point at one end? (Answer: a cylinder and a cone, joined at the pencil's flat circular end.)

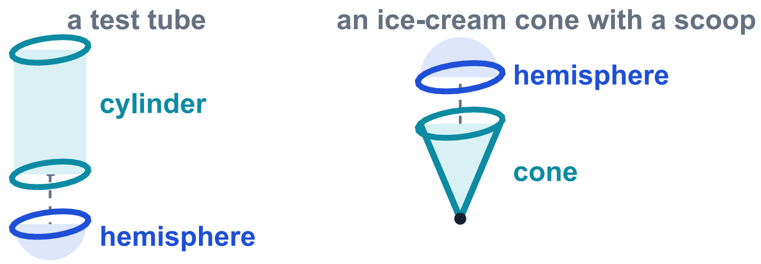
BACK
A combination of two solids · p. 398

TRY
What two solids make up a capsule-shaped tablet?

Answer: A cylinder with a hemisphere at each end.

KEY
Naming the two solids first turns an unfamiliar object into two familiar formulas.

Two real objects, each two named solids



A test tube and an ice-cream cone are each pulled apart into the two solids that join to make them.

CHECK YOURSELF

25. A capsule of medicine is described in this chapter as a combination of which basic solids?

- **A.** a cylinder with a cone fixed at each end
- **B.** a cylinder with a hemisphere fixed at each end
- **C.** two hemispheres joined directly to each other, with no cylinder
- **D.** a sphere with a cylinder passing through its centre

26. An ice-cream cone with one scoop of ice cream on top is treated in this chapter as a combination of which two solids?

- **A.** a cone and a sphere
- **B.** a cone and a hemisphere
- **C.** a cylinder and a cone
- **D.** a cuboid and a hemisphere

1 more in this set online.

Volume simply adds or subtracts

The volume of a combination is the plain sum of its two parts' volumes. Nothing is lost to a hidden face, unlike surface area.

When one solid is hollowed out of another instead of joined onto it, the volume works the other way. It becomes the difference of the two volumes, the larger solid's volume minus the smaller one's.

We decide whether a solid is joined on or hollowed out. That single choice tells us at once whether its volume adds or subtracts.

Your turn: does the volume of a scooped-out ice-cream tub add or subtract? (Answer: it subtracts, since the scooped part is hollowed out, not joined on.)

RULE
Hollowing out a solid subtracts a volume, and joining two solids together adds one.

KEY
Volume never subtracts for a join: only surface area does that.

A SCHOLAR INDIA REMEMBERS

Surface and volume are different kinds of thing. Keep them apart. When two solids join, some surface is hidden. No volume is hidden. So for a cone on a hemisphere, we simply add the two volumes.

Kanada · the sparrow

CHECK YOURSELF

27. A toy is a cone fixed on top of a hemisphere. How is the toy's total volume found?

- **A.** add the volume of the cone to the volume of the hemisphere
- **B.** add the two volumes, then subtract the volume of the hidden circle at the join
- **C.** average the two volumes
- **D.** multiply the two volumes together

28. A conical cavity is hollowed out of a solid cylinder. How is the volume of the remaining solid found?

- **A.** add the cone's volume to the cylinder's volume
- **B.** subtract the cylinder's volume from the cone's volume
- **C.** the remaining solid's volume cannot be found without knowing the surface area first
- **D.** subtract the cone's volume from the cylinder's volume

2 more in this set online.

Common mistakes

Adding both totals overcounts

A tempting shortcut: add the total surface areas of the two separate solids to get the combination's total surface area.

That shortcut is wrong. The face where the two solids meet is hidden inside the join. Adding the two totals counts that face twice, once from each solid, when the combination's real surface does not carry it at all.

We subtract the hidden faces before trusting any sum of two totals, so this mistake never reaches the final answer.

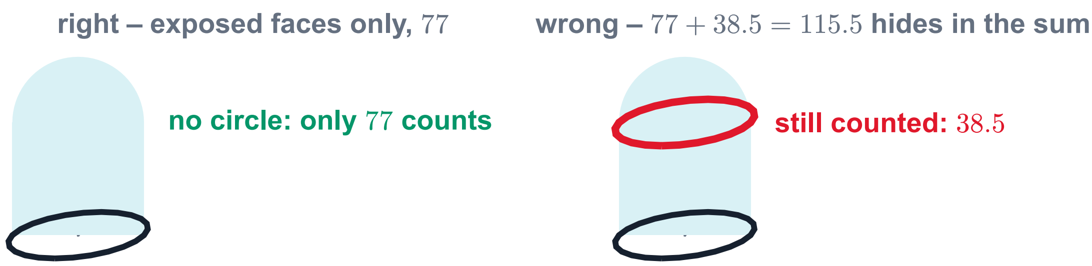
Your turn: a hemisphere of radius 7 cm sits on a cylinder of radius 7 cm, taking $\pi \approx 22/7$. By how much does adding the two totals overcount? (Answer: 308 cm², since the join circle is $\pi r^2 \approx 22/7 \cdot 7 \cdot 7 = 154$ cm², and adding both totals double-counts it: $2 \cdot 154 = 308$.)

BACK
A join hides two faces · p. 405

CAUTION
Adding both totals always overcounts, by exactly the faces at the join.

KEY
Subtract the hidden faces before trusting a sum of two totals.

The wrong sum keeps a circle that is not there



The correct panel counts 77 once, and the wrong panel hides an extra circle of 38.5 inside its 115.5, red-ringed at the join.

FIRST TRY
~~Hemisphere of radius 3.5 cm fixed onto one flat end of a cylinder of the same radius. I added the hemisphere's whole surface, $3 \cdot \pi r^2 = 115.5$ cm², straight onto the cylinder's own total.~~

SECOND LOOK
The join hides one circle of area $\pi r^2 = 38.5$ cm² inside the cylinder's flat end. That circle drops out of the cylinder's total, and only the hemisphere's curved part, $2 \cdot \pi r^2 = 77$ cm², is added back.

At a join, take away the hidden circle once. Never add a whole surface twice.

WHAT IS THE TOTAL SURFACE AREA OF A CYLINDER WITH A HEMISPHERE FIXED ON ONE END?

WEAK
A hemisphere of radius 3.5 cm sits on a cylinder of the same radius and height 6 cm, taking $\pi \approx 22/7$. The cylinder's total surface area is $2 \cdot 22/7 \cdot 3.5 \cdot 6 + 2 \cdot 38.5 = 132 + 77 = 209$ cm². The hemisphere's total surface area is $3 \cdot 38.5 = 115.5$ cm². Adding the two totals gives $209 + 115.5 = 324.5$ cm².

But the join hides a circle on the cylinder and a circle on the hemisphere, so the true surface must be smaller than 324.5 cm² by two circles, $2 \cdot 38.5 = 77$ cm². An answer equal to the sum of the two totals means the join was never taken away.

STRONG
Count only what is outside the joined solid: the cylinder's curved surface, its one remaining flat end, and the hemisphere's curved surface. That gives $132 + 38.5 + 77 = 247.5$ cm².

And $324.5 - 247.5 = 77$, exactly the two hidden circles.

CHECK YOURSELF

29. A hemisphere of radius 3.5 cm is fixed onto one flat end of a cylinder of the same radius. A student finds the combination's total surface area by adding the cylinder's total surface area to the hemisphere's total surface area. What is wrong with this method?

- **A.** the joined face is hidden on both sides and must be dropped from both totals
- **B.** nothing is wrong — adding the two total surface areas is exactly the right method
- **C.** the method is wrong because the hemisphere's total surface area formula is incorrect
- **D.** the method is wrong because the cylinder's radius does not match the hemisphere's

30. For the hemisphere and cylinder of radius 3.5 cm above, adding the two total surface areas overcounts the true answer. By how much does it overcount, and why?

- **A.** by 38.5 cm² — only the cylinder's hidden face is double-counted
- **B.** by 115.5 cm² — the hemisphere's whole total surface area is the overcount
- **C.** by 77 cm² — two hidden flat circles, one on each side
- **D.** by 0 cm² — adding the two total surface areas gives the correct answer

3 more in this set online.

Worked examples

WORKED EXAMPLE · **A cylindrical pipe's surface and volume**

1.	radius $r = 7$ cm, height $h = 20$ cm, taking $\pi \approx 22/7$	*These are the pipe's own numbers, and $\pi \approx 22/7$ holds for the whole problem.*
2.	curved surface area $= 2 \cdot \pi r h \approx 2 \cdot 22/7 \cdot 7 \cdot 20 = 880$ cm²	*Put r and h into the cylinder's curved surface area formula.*
3.	total surface area $= 880 + 2 \cdot \pi r^2 \approx 880 + 2 \cdot 22/7 \cdot 49 = 880 + 308 = 1188$ cm²	*Add the two flat circular ends, $2 \cdot \pi r^2$, to the curved surface area from the step before.*
4.	volume $= \pi r^2 h \approx 22/7 \cdot 49 \cdot 20 = 3080$ cm³	*The volume formula uses only the base area and the height. It never needs the curved surface area.*
5.	$2 \cdot 22/7 \cdot 49 = 308$, and $880 + 308 = 1188$	CHECK ***Check the total by re-adding the curved surface and the two flat ends on their own.*** *The pieces must add back to the 1188 cm² found above.*

BACK
Surface area and volume: cylinder · p. 402

TRY
Radius 7 cm, height 10 cm, $\pi \approx 22/7$: find the curved surface area, total surface area and volume.

Answer: Curved surface area 440 cm², total surface area 748 cm², volume 1540 cm³.

RULE
Find curved surface area first: total surface area and volume both reuse the same πr^2.

A CYLINDER'S SURFACE AND VOLUME

1. **Find the volume first** Multiply the base area by the height, $\pi r^2 h$, using the pipe's radius 7 cm and height 20 cm. That gives 3080 cm³.
2. **Find the curved surface** Multiply $2 \cdot \pi r h$ using the same radius and height. That gives 880 cm².
3. **Add the two flat ends** Add $2 \cdot \pi r^2$, here 308 cm², to the curved surface for the total surface area, 1188 cm².
4. **Check the curved surface against the volume** The curved surface is always $2/r$ times the volume, since $2\pi r h = (2/r) \cdot \pi r^2 h$. Here $2 \cdot 3080/7 = 880$, which matches.

CHECK YOURSELF

31. A cylindrical pipe has radius 7 cm and height 20 cm. Taking $\pi = 22/7$, what is its curved surface area?

- **A.** 3080 cm³, using the volume formula instead
- **B.** 880 cm²
- **C.** 1188 cm², including the two flat ends
- **D.** 440 cm², forgetting to double the curved surface term

32. For the same pipe, why is the total surface area, 1188 cm², greater than the curved surface area, 880 cm², by exactly $2 \cdot 22/7 \cdot 49$?

- **A.** because the total surface area formula includes an extra safety margin for real pipes
- **B.** the total adds two flat ends, each $22/7 \cdot 49$, left out of the curved measure
- **C.** because the pipe's volume is added into the total surface area
- **D.** because the curved surface area formula was applied incorrectly the first time

1 more in this set online.

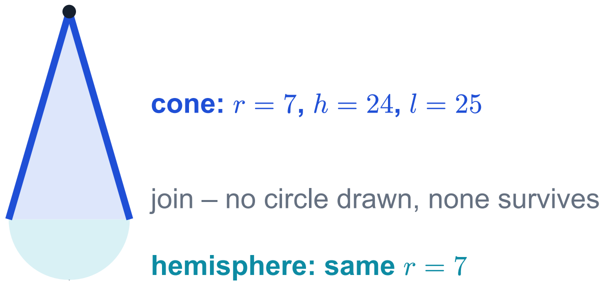
WORKED EXAMPLE · **A toy's total surface, cone on hemisphere**

1.	cone radius $r = 7$ cm, cone height $h = 24$ cm, hemisphere of the same radius, taking $\pi \approx 22/7$	*These are the toy's own numbers, and the hemisphere's radius matches the cone's so the two join smoothly.*
2.	slant height $l = \sqrt{r^2 + h^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ cm	*Pythagoras' theorem gives the slant height from the cone's radius and height.*
3.	curved surface area of the cone $= \pi r l \approx 22/7 \cdot 7 \cdot 25 = 550$ cm²	*Put r and the slant height l into the cone's curved surface area formula.*
4.	curved surface area of the hemisphere $= 2 \cdot \pi r^2 \approx 2 \cdot 22/7 \cdot 49 = 308$ cm²	*Put the same radius into the hemisphere's curved surface area formula.*
5.	total surface area of the toy $= 550 + 308 = 858$ cm²	*Both flat circular faces vanish at the join, so the total is just the two curved surfaces added.*
6.	$7^2 + 24^2 = 49 + 576 = 625 = 25^2$	CHECK ***Check the slant height by confirming that 7, 24 and 25 form a Pythagorean triple.*** *Squaring 25 must return the same 625 we started with.*

BACK
Surface area and volume: cone · p. 403

KEY
Same radius, curved-only join: no flat circle survives anywhere on this toy.

A cone on a hemisphere: no flat circle survives



A cone of radius 7 and height 24 sits on a hemisphere of the same radius, with no circle drawn at their join.

CHECK YOURSELF

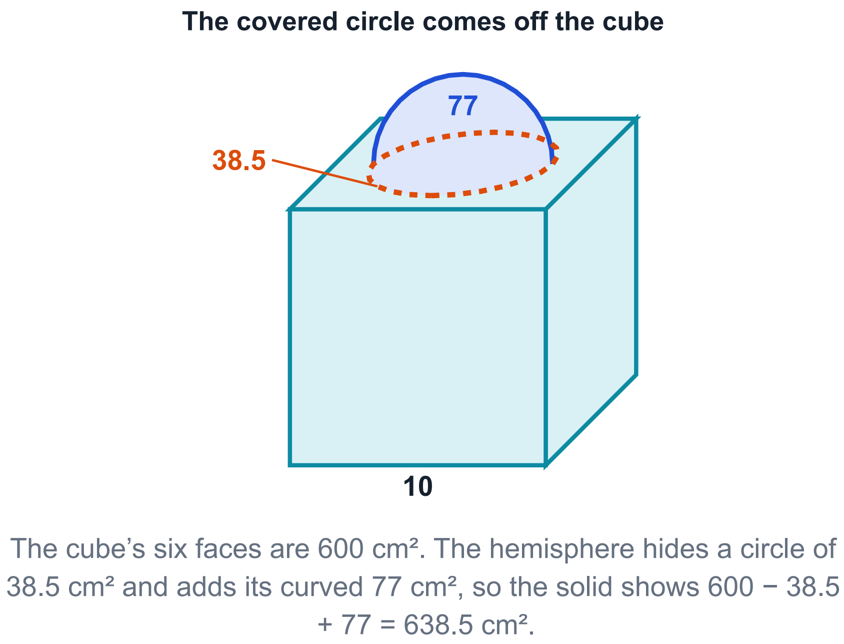
33. A toy is a cone of radius 7 cm and height 24 cm mounted on a hemisphere of the same radius. What is the cone's slant height?

- **A.** 25 cm
- **B.** 24 cm, the same as the cone's height
- **C.** 31 cm, from adding the radius and height directly
- **D.** 625 cm, forgetting to take the square root

34. For this toy, why does the total surface area add the cone's CURVED surface area to the hemisphere's CURVED surface area, rather than either solid's total surface area?

- **A.** because a toy's surface area is always measured using curved surfaces only
- **B.** because the cone and hemisphere have no flat faces to begin with
- **C.** both flat bases are pressed together at the join and vanish
- **D.** because using total surface area would give a smaller, and therefore wrong, answer

1 more in this set online.



The hemisphere covers 38.5 cm² of the cube's top face and adds a curved 77 cm², so the surface is 600 − 38.5 + 77 = 638.5 cm².

WORKED EXAMPLE · A hemisphere fixed on a cube

1. cube of edge $a = 10$ cm, hemisphere of radius $r = 3.5$ cm fixed on one face, taking $\pi \approx 22/7$	*These are the block's own numbers, with the hemisphere sitting entirely on one of the cube's six faces.*
2. total surface area of the cube $= 6a^2 = 6 \cdot 100 = 600$ cm²	*Find the cube's total surface area before the hemisphere is added at all.*
3. area covered by the hemisphere's base $= \pi r^2 \approx 22/7 \cdot 3.5 \cdot 3.5 = 38.5$ cm²	*The circle where the hemisphere meets the cube is hidden, so its area comes off the cube's total.*
4. curved surface area of the hemisphere $= 2 \cdot \pi r^2 \approx 77$ cm²	*The hemisphere gives back its own curved surface in exchange for the circle it covers.*
5. total surface area of the combination $= 600 - 38.5 + 77 = 638.5$ cm²	*Remove the covered circle, then add the hemisphere's curved surface, matching only the one face the join touches.*
6. $2 \cdot 38.5 = 77$	CHECK ***Check that the hemisphere's curved surface is exactly double the flat circle it replaces.*** *$2 \cdot \pi r^2$ is always twice πr^2, for any radius.*

BACK
Surface area and volume of boxes · p. 400

TRY
A cube of edge 12 cm has a hemisphere of radius 3.5 cm on one face. Find the total surface area, taking $\pi \approx 22/7$.

Answer: $6 \cdot 12^2 = 6 \cdot 144 = 864$ cm² for the cube, then $864 - 38.5 + 77 = 902.5$ cm².

KEY
The cube's other five faces stay whole: only the one under the hemisphere loses its circle.

A HEMISPHERE FIXED ON A CUBE'S FACE

1. **Find the cube's total surface** Use $6a^2$ with edge $a = 10$ cm. That gives 600 cm².
2. **Find the circle the hemisphere covers** Use πr^2 with the hemisphere's radius $r = 3.5$ cm. That gives 38.5 cm².
3. **Find the hemisphere's curved surface** Use $2 \cdot \pi r^2$ at the same radius. That gives 77 cm².
4. **Take the circle away, add the curved surface** Subtract 38.5 cm² from the cube's total, then add 77 cm². The combination's total is 638.5 cm².
5. **Check the curved surface against the circle** The hemisphere's curved surface is always twice the circle it covers. Here $2 \cdot 38.5 = 77$, which matches.

CHECK YOURSELF

35. A cube of edge 10 cm has a hemisphere of radius 3.5 cm fixed on one face. What is the cube's own total surface area, before the hemisphere is added?

- **A.** 600 cm²
- **B.** 1000 cm³, from 10^3, its volume
- **C.** 100 cm², the area of one face only
- **D.** 60 cm², from $6 \cdot 10$

36. Fixing the hemisphere removes a circle of area 38.5 cm² from the cube's top face and adds the hemisphere's curved surface area of 77 cm². Why does the combination's total surface area, 638.5 cm², come from $600 - 38.5 + 77$ rather than just $600 + 77$?

- **A.** the hemisphere's base covers, and removes, that circle of the face
- **B.** because $600 + 77$ would give too large a number for a toy this size
- **C.** because the cube's formula $6a^2$ already overcounts by 38.5 cm² on its own
- **D.** because the hemisphere's curved surface area formula already includes a subtraction

1 more in this set online.

WORKED EXAMPLE · A capsule's total surface area

1.	cylinder of radius $r = 3.5$ cm and height $h = 8$ cm, with a hemisphere of the same radius fixed on each flat end, taking $\pi \approx 22/7$	*These are the capsule's own numbers, with two joins instead of one.*
2.	curved surface area of the cylinder $= 2 \cdot \pi r h \approx 2 \cdot 22/7 \cdot 3.5 \cdot 8 = 176$ cm²	*Put r and h into the cylinder's curved surface area formula.*
3.	curved surface area of one hemisphere $= 2 \cdot \pi r^2 \approx 77$ cm², so both hemispheres give 154 cm²	*Put the same radius into the hemisphere's curved surface area formula, then double it for the two hemispheres.*
4.	total surface area of the capsule $= 176 + 154 = 330$ cm²	*Both flat ends of the cylinder and both flat faces of the hemispheres vanish at the two joins, so only curved surfaces remain.*

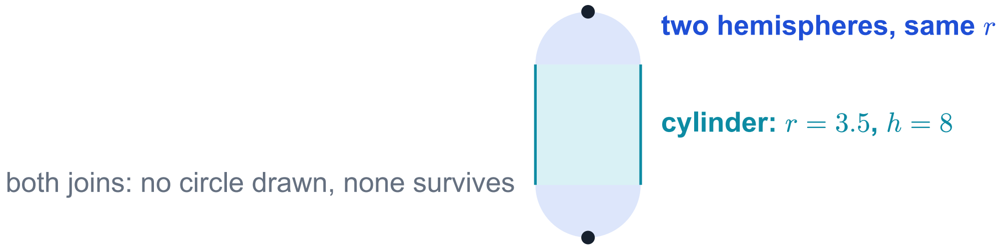
WORKED EXAMPLE · **A capsule's total surface area**

5. $4 \cdot 22/7 \cdot 3.5 \cdot 3.5 = 154$

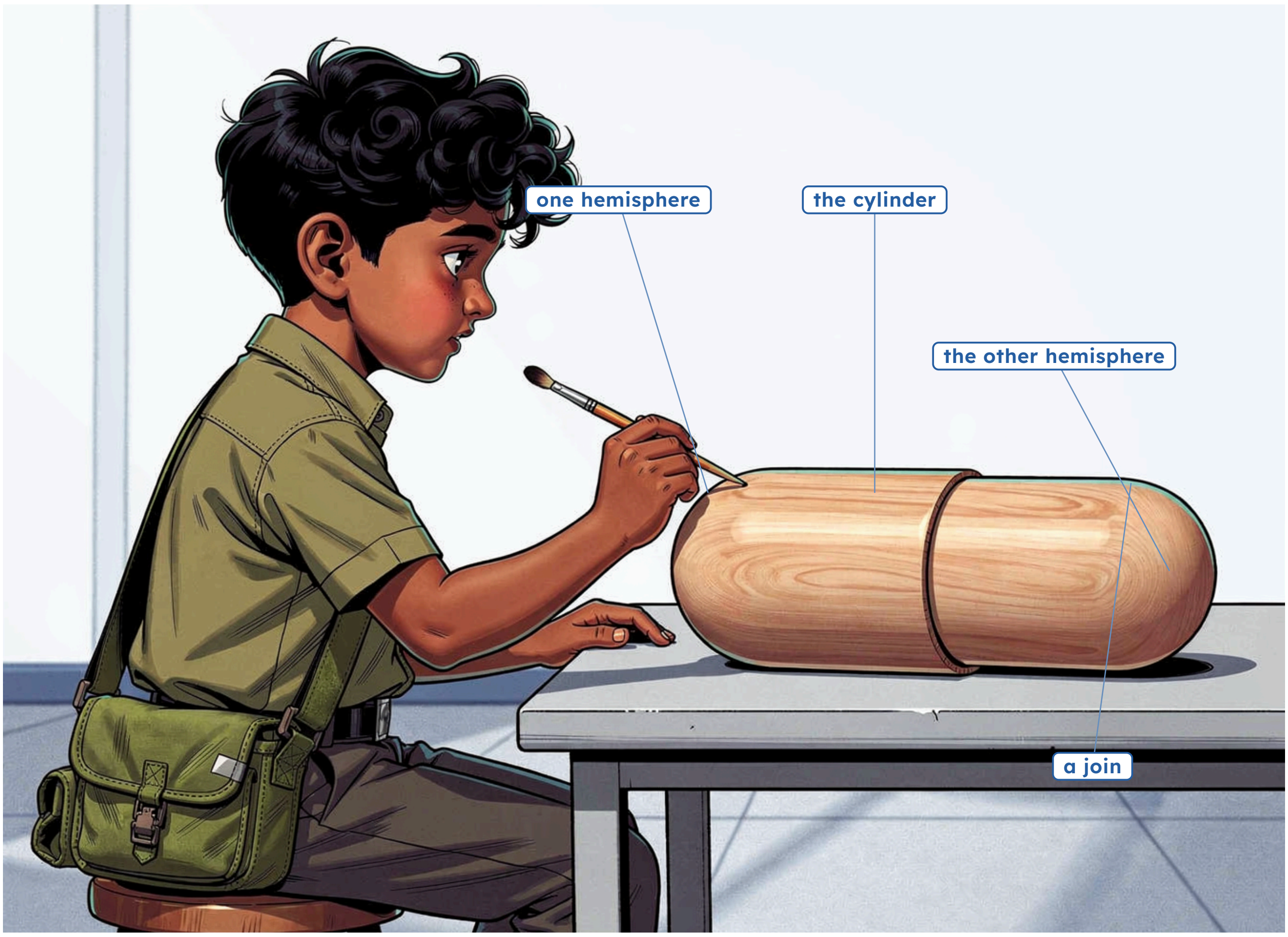
CHECK ***Check the two curved surfaces against a full sphere of the same radius.***
Two hemispheres together always carry a sphere's own surface area, $4 \cdot \pi r^2$.

KEY
Two joins, four vanished faces: nothing flat is left on a capsule.

A capsule: two joins, nothing flat anywhere



A capsule's cylinder and its two rounded ends meet at joins where no circle is drawn, since none survives.



Kabir paints a wooden capsule: a cylinder with a hemisphere of the same width on each end.

CHECK YOURSELF

37. A capsule is a cylinder of radius 3.5 cm and height 8 cm with a hemisphere of the same radius fixed on each flat end. How many flat faces does this capsule have left, once both hemispheres are fixed on?

- **A.** none — all four flat faces are hidden at the two joins
- **B.** two — the cylinder's two flat ends are still exposed underneath the hemispheres
- **C.** two — each hemisphere's own flat base is still exposed, facing outward
- **D.** four — both cylinder ends and both hemisphere bases are all still exposed

38. Why does the capsule's total surface area use ONLY curved surfaces — the cylinder's curved surface plus both hemispheres' curved surfaces — and no flat-face term at all?

- **A.** because a capsule, by definition, has no flat faces anywhere in its formula
- **B.** every flat face is hidden at one of the two joins
- **C.** because flat faces do not count toward surface area unless a solid stands on a table
- **D.** because curved surfaces always outnumber flat surfaces in any combination

1 more in this set online.

WORKED EXAMPLE · **A toy's volume, hemisphere and cone**

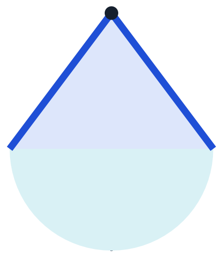
1.	hemisphere of radius $r = 3$ cm, topped by a cone of the same radius and height $h = 4$ cm, taking $\pi \approx 3.14$	*These are the toy's own numbers, and $\pi \approx 3.14$ holds for this problem instead of 22/7.*
2.	volume of the cone $= 1/3 \cdot \pi r^2 h \approx 1/3 \cdot 3.14 \cdot 9 \cdot 4 = 37.68$ cm³	*Put the cone's own radius and height into the cone's volume formula.*
3.	volume of the hemisphere $= 2/3 \cdot \pi r^3 \approx 2/3 \cdot 3.14 \cdot 27 = 56.52$ cm³	*Put the same radius into the hemisphere's volume formula.*
4.	volume of the toy $= 37.68 + 56.52 = 94.2$ cm³	*The two parts are simply joined, so their volumes add with nothing subtracted.*
5.	$3.14 \cdot 9 \cdot 4 = 113.04$, and $113.04/3 = 37.68$	CHECK ***Check the cone against the cylinder sharing its base and height.*** *A cone always holds exactly a third of that cylinder, and 113.04 divided by 3 returns the 37.68 found above.*

TRY
A hemisphere of radius 3 cm is topped by a cone of the same radius, height 6 cm. Find its volume, $\pi = 3.14$.

Answer: Cone $= 1/3 \cdot 3.14 \cdot 9 \cdot 6 = 56.52$ cm³, hemisphere is also 56.52 cm³, total 113.04 cm³.

KEY
Volume never subtracts for a plain join: both parts' volumes just add.

Two volumes that simply add



A cone of radius 3 and height 4 sits on a hemisphere of radius 3, joined with no face subtracted from either volume.

A HEMISPHERE-AND-CONE TOY'S VOLUME

1. **Find the cone's volume** Use $1/3 \cdot \pi r^2 h$ with the cone's own radius $r = 3$ cm and height $h = 4$ cm, taking $\pi = 3.14$. That gives 37.68 cm³.
2. **Find the hemisphere's volume** Use $2/3 \cdot \pi r^3$ at the same radius, $r = 3$ cm. That gives 56.52 cm³.
3. **Add them, nothing taken away** The two parts are joined, not hollowed out, so simply add the volumes. $37.68 + 56.52 = 94.2$ cm³.
4. **Check the cone against a cylinder** A cone always holds a third of the cylinder sharing its base and height. $3.14 \cdot 9 \cdot 4 = 113.04$, and a third of that is 37.68.

CHECK YOURSELF

39. A toy is a hemisphere of radius 3 cm topped by a cone of the same radius and height 4 cm. Taking $\pi = 3.14$, what is the cone's volume?

- **A.** 113.04 cm³, from $3.14 \cdot 9 \cdot 4$, forgetting the $1/3$
- **B.** 37.68 cm³
- **C.** 56.52 cm³, using the hemisphere's volume formula instead
- **D.** 47.1 cm², treating the answer as a surface area

40. For this toy, the total volume is found as $37.68 + 56.52 = 94.2$ cm³, simply adding the cone's and hemisphere's volumes. Why is no term ever subtracted here, unlike some surface area calculations for the very same toy?

- **A.** joining never removes any enclosed space
- **B.** because volume calculations never involve subtraction, even for a hollowed-out solid
- **C.** because this particular cone and hemisphere happen to have the same radius
- **D.** because a subtraction term is needed, but it happens to equal zero for this toy

1 more in this set online.

WORKED EXAMPLE · **Volume after hollowing out a cavity**

1. solid cylinder of radius $r = 3.5$ cm and height $h = 10$ cm, with a conical cavity of the same radius and depth 6 cm hollowed from one end, taking $\pi \approx 22/7$

These are the block's own numbers, with the cavity's radius matching the cylinder's own radius.

WORKED EXAMPLE · Volume after hollowing out a cavity

2.	volume of the cylinder $= \pi r^2 h \approx 22/7 \cdot 3.5 \cdot 3.5 \cdot 10 = 385$ cm³	*Find the full cylinder's volume before any cavity is taken out.*
3.	volume removed by the cavity $= 1/3 \cdot \pi r^2 h \approx 1/3 \cdot 38.5 \cdot 6 = 77$ cm³	*The cavity is a cone, so use the cone's volume formula with its own depth as the height.*
4.	volume of the remaining solid $= 385 - 77 = 308$ cm³	*A hollowed-out cavity takes its own volume away from the whole. It is never added.*
5.	$22/7 \cdot 3.5 \cdot 3.5 \cdot 6 = 231$, and $231/3 = 77$	CHECK ***Check the cavity against the cylinder it was cut from.*** *A cone is always a third of a cylinder with the same radius and height, and 231 divided by 3 returns the 77 cm³ removed.*

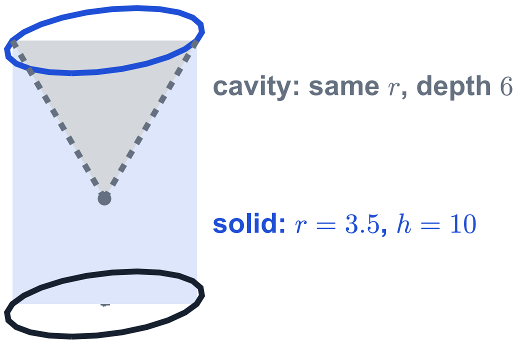
BACK
Surface area and volume: cylinder · p. 402

TRY
Radius 3.5 cm, height 12 cm, cavity of the same radius, depth 3 cm: find the remaining volume, $\pi \approx 22/7$.

Answer: Cylinder $= 38.5 \cdot 12 = 462$ cm³, cavity $= 1/3 \cdot 38.5 \cdot 3 = 38.5$ cm³, remaining 423.5 cm³.

KEY
Hollowing out is a join in reverse: subtract the cavity's volume instead of adding it.

Hollowing out is a join in reverse



A cone-shaped cavity of depth 6 is hollowed from the top of a cylinder of radius 3.5 and height 10.

THE VOLUME LEFT AFTER HOLLOWING A CYLINDER

1. **Find the whole cylinder's volume** Use $\pi r^2 h$ with radius $r = 3.5$ cm and height $h = 10$ cm. That gives 385 cm³.
2. **Find the cavity's volume** The cavity is a cone, so use $1/3 \cdot \pi r^2 h$ with the same radius and its own depth, 6 cm. That gives 77 cm³.
3. **Subtract the cavity** Take the cavity's volume away from the whole cylinder. $385 - 77 = 308$ cm³.
4. **Check the cavity against the cylinder** A cone is always a third of a cylinder sharing its radius and height. $22/7 \cdot 3.5 \cdot 3.5 \cdot 6 = 231$, and a third of that is 77.

CHECK YOURSELF

41. A solid cylinder of radius 3.5 cm and height 10 cm has a conical cavity of the same radius and depth 6 cm hollowed from one end. What is the volume of material removed by the cavity?

- **A.** 231 cm³, from $22/7 \cdot 3.5 \cdot 3.5 \cdot 6$, forgetting the $1/3$
- **B.** 385 cm³, the same as the cylinder's own full volume
- **C.** 0 cm³, since a cavity is empty space and has no volume of its own
- **D.** 77 cm³

42. Why is the remaining solid's volume found as $385 - 77 = 308$ cm³, rather than $385 + 77$?

- **A.** because $385 + 77$ would exceed the volume of the original block, which is impossible
- **B.** because subtraction always applies whenever two different solids appear in one problem
- **C.** because the cavity's depth, 6 cm, is less than the cylinder's height, 10 cm
- **D.** the cavity removes material, so its volume is subtracted

1 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap

- BOXES: total surface area is $2lb + 2bh + 2hl$, and volume is lbh. A cube of edge 10 cm gives 600 cm² and 1000 cm³.
- ROUND SOLIDS: cylinder volume is $\pi r^2 h$, and cone volume is $1/3 \cdot \pi r^2 h$. A cone always holds exactly a third of the cylinder sharing its base and height.
- SLANT HEIGHT: $l = \sqrt{r^2 + h^2}$. For $r = 7$ cm and $h = 24$ cm, $l = 25$ cm.
- EXPOSED FACES: a join hides one face on each side. A hemisphere on a cylinder of radius 3.5 cm hides two circles of 38.5 cm² each.
- VOLUME: it always adds or subtracts outright. A hemisphere-and-cone toy gives $56.52 + 37.68 = 94.2$ cm³, with nothing removed for the join.

We can see every combination as one idea, split into two halves.

Surface area counts only the faces still exposed once two solids are joined. The shared face disappears from both sides of the join, not just one.

Volume never loses anything to a hidden face, whether it adds two parts together or subtracts one from the other after hollowing out.

Name the two basic solids and their one shared face first, and the formulas you need are already familiar.

BACK
Only exposed faces count · p. 404

KEY
Surface area drops the shared face, but volume never does.

CHECK YOURSELF

43. This chapter's whole idea splits into two halves, one for surface area and one for volume. What is the volume half of that idea?

- **A.** volume counts only the faces still exposed once two solids are joined
- **B.** volume is always found by multiplying the two solids' surface areas together
- **C.** volume adds the two parts, or subtracts one when hollowed out
- **D.** volume drops the shared face at the join, exactly like surface area does

44. Why does this chapter organise every combination problem around the SAME one idea — exposed faces for area, plain addition or subtraction for volume — instead of a separate rule for each pair of solids?

- **A.** because there are too many possible pairs of solids to give each one its own rule
- **B.** because the six basic solids all have the same formulas for surface area and volume
- **C.** because CBSE examinations only ever test one specific pair of solids
- **D.** the same reasoning about hidden faces and enclosed space applies to any pair

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

45. A toy is made by fixing a hemisphere on top of a cylinder of the same radius. Which statement about finding its volume and total surface area is correct?

- **A.** both the volume and total surface area are found by adding the two solids' own totals. No adjustment for the hidden join is needed
- **B.** the volume is found by adding the two solids' volumes. The surface area must leave out the faces hidden where the two solids meet
- **C.** the surface area is found by adding the two solids' total surface areas. The volume must leave out the space hidden where the two solids meet
- **D.** neither the volume nor the surface area can be found without measuring the finished solid directly. Joining changes both formulas entirely

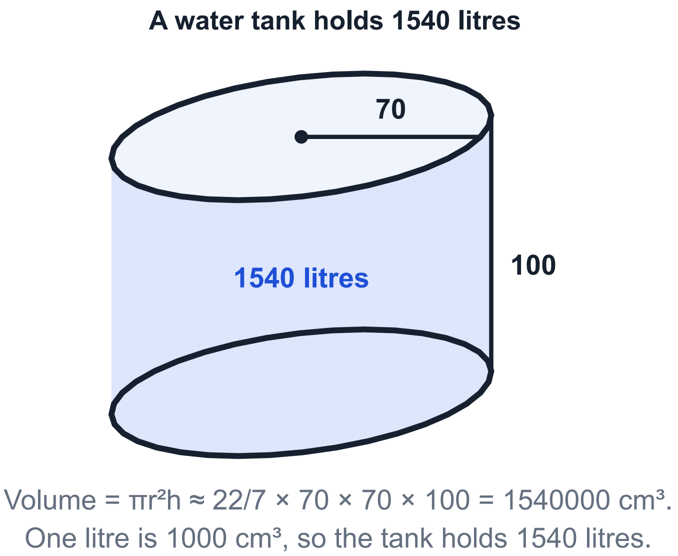
46. A cone is mounted on top of a cylinder of the same radius. When finding the total surface area of this combined solid, the circular face where the cone's base meets the cylinder's top should be

- **A.** counted twice, once for the cylinder and once for the cone, since both solids have that face
- **B.** counted only once, since it appears in both solids' own surface area totals, treated as if it still belonged to one of the two shapes
- **C.** left out of the total entirely, since it is hidden inside the join and no longer visible from outside
- **D.** replaced by the average of the cylinder's and cone's own surface areas at that face

3 more in this set online.

IN EVERY CHAPTER

Where you will meet this

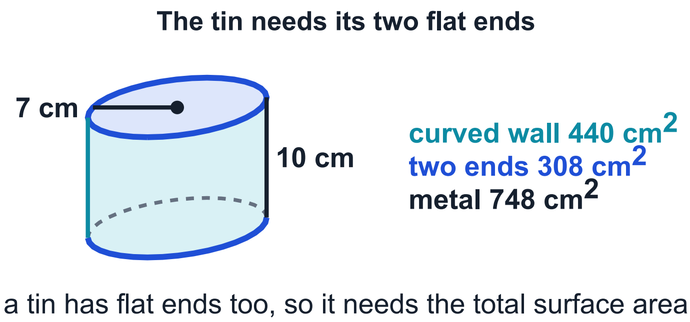


A tank of radius 70 cm and height 100 cm holds 22/7 × 70² × 100 = 1540000 cm³, which is 1540 litres.

You meet cylinders, cones and their combinations in ordinary objects all the time. Here are eight places their surface area or volume decides a real number.

- **How much a water tank holds.** A cylindrical water tank has radius 70 cm and height 100 cm.
  The maths: a cylinder's volume tells you how much water it holds.
  Using $\pi \approx 22/7$, its volume is $(22/7) \cdot 70^2 \cdot 100 = 1540000$ cm³, which is 1540 litres.
- **Quoting a room-painting job.** You run a painting business. A client's room is 4 m by 3 m by 3 m high. One door takes up 2 m². Your paint covers 10 m² per litre.
  The maths: the walls' area to paint comes from the room's length, breadth and height.
  The walls measure $2 \cdot 3 \cdot (4 + 3) = 42$ m². Minus the door, that is 40 m² to paint. At 10 m² per litre, you need 4 litres.
- **Making a sweets tin.** A cylindrical sweets tin has radius 7 cm and height 10 cm.
  The maths: a tin needs its total surface area, not just its curved surface area, since it has flat ends too.
  Using $\pi \approx 22/7$, the curved wall is $2 \cdot (22/7) \cdot 7 \cdot 10 = 440$ cm². The two flat ends add $2 \cdot (22/7) \cdot 7^2 = 308$ cm². The tin needs $440 + 308 = 748$ cm² of metal.

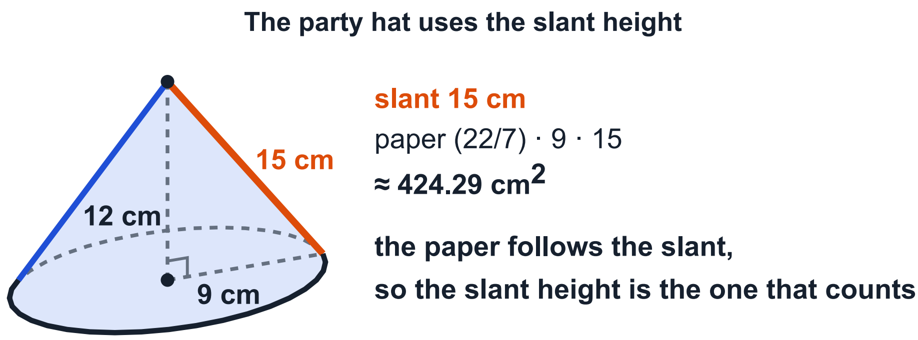
BACK
Two ways to measure a surface · p. 397



Curved wall plus both ends: 440 and 308 make 748 square centimetres of metal. Leave out the ends and the tin has no lid and no bottom.

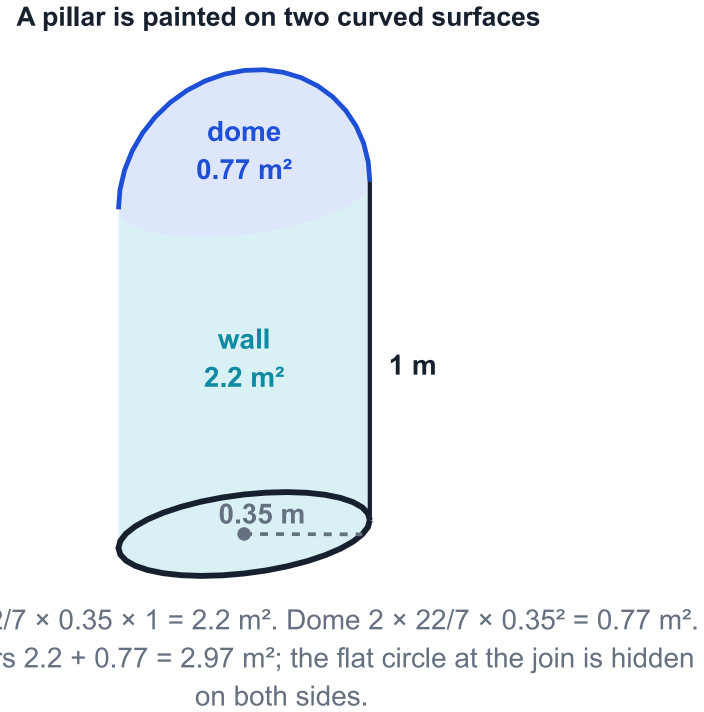
A cylinder with radius 7 cm and height 10 cm has a curved wall of 440, ends of 308, total metal 748 square centimetres.

- **Making a party hat.** A cone-shaped party hat has base radius 9 cm and height 12 cm.
  The maths: the curved paper needed comes from the cone's slant height, not its straight height.
  The slant height is $\sqrt{9^2 + 12^2} = \sqrt{225} = 15$ cm. Using $\pi \approx 22/7$, the paper needed is $(22/7) \cdot 9 \cdot 15 \approx 424.29$ cm².
- **Sharpening a pencil.** A pencil's body is a cylinder. Its sharpened tip is a cone cut from the same wood.
  The maths: a cone tip and a cylinder body join smoothly only when they share the same radius.



The straight height, 12, is not on the paper at all. The paper runs along the slant, 15, and 22/7 times 9 times 15 is the 424.29 square centimetres needed.

Pythagoras comes first: turn the radius and the height into the slant height, and only then does the curved-surface formula apply.



Only what paint can reach counts, so the join between two solids hides a face on each side of it.

- **Painting a garden pillar.** A garden pillar is a cylinder of radius 0.35 m and height 1 m, topped by a hemisphere of the same radius.
  The maths: the paint needed uses each solid's curved surface area. The flat circle where they join is hidden on both sides.
  Using $\pi \approx 22/7$, the cylinder's curved area is $2 \cdot (22/7) \cdot 0.35 \cdot 1 = 2.2$ m². The hemisphere's curved area is $2 \cdot (22/7) \cdot 0.35^2 = 0.77$ m². The paint covers $2.2 + 0.77 = 2.97$ m².
- **Making a candle.** A candle has a cylindrical base, radius 3 cm and height 8 cm. Its cone-shaped tip has the same radius and height 4 cm.
  The maths: the wax needed is simply the cylinder's volume plus the cone's volume.
  Using $\pi = 3.14$, the cylinder's volume is $3.14 \cdot 3^2 \cdot 8 = 226.08$ cm³. The cone's volume is $(1/3) \cdot 3.14 \cdot 3^2 \cdot 4 = 37.68$ cm³. The wax needed is $226.08 + 37.68 = 263.76$ cm³.
- **Cutting a length of steel pipe.** A hollow steel pipe has outer radius 5 cm, inner radius 3 cm and length 20 cm.
  The maths: the material left is the outer cylinder's volume minus the inner cylinder's volume.
  Using $\pi = 3.14$, the material is $3.14 \cdot (5^2 - 3^2) \cdot 20 = 1004.8$ cm³.

Your turn. A wooden dowel is a cylinder of radius 2 cm and length 25 cm. It has a cone-shaped tip of the same radius and height 3 cm glued on one end. Using $\pi = 3.14$, what is its total volume? *Answer:* The cylinder's volume is $3.14 \cdot 2^2 \cdot 25 = 314$ cm³. The cone's volume is $(1/3) \cdot 3.14 \cdot 2^2 \cdot 3 = 12.56$ cm³. The total is $314 + 12.56 = 326.56$ cm³.

Practice: Exercise 12.1

PRACTICE SET

1. PRACTICE A cylinder of radius 3.5 cm and height 10 cm stands on its flat base, with a hemisphere of the same radius fixed on its top end. Find the total surface area, taking $\pi \approx 22/7$. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · Worked in full

1.	list the surfaces that are still exposed — the curved side of the cylinder, the curved surface of the hemisphere, and the flat base the solid stands on	*the chapter's one rule is that only exposed faces count, so the list comes before any formula; the cylinder's top circle is under the hemisphere and is not on the list*
2.	curved surface of the cylinder $= 2\pi rh = 2 \cdot (22/7) \cdot 3.5 \cdot 10 = 220$ cm²	*22/7 times 3.5 is exactly 11, so this line has no rounding in it*
3.	curved surface of the hemisphere $= 2\pi r^2 = 2 \cdot 38.5 = 77$ cm², and the flat base $= \pi r^2 = 38.5$ cm²	*both use the same $\pi r^2 = 38.5$ cm², so work that number out once and reuse it*
4.	total surface area $= 220 + 77 + 38.5 = 335.5$ cm²	*add only the three surfaces on the list*
5.	the check — adding the two solids' own totals instead gives $297 + 115.5 = 412.5$ cm², which is 77 cm² too much, and $77 = 2 \cdot 38.5$	*that is exactly the two circles hidden at the join, one from each solid; if your answer is bigger than the true one by two circles, you added both totals*

2. PRACTICE The same cylinder, radius 3.5 cm and height 10 cm on its flat base, but with a cone of the same radius and slant height 12 cm fixed on top instead of the hemisphere. Find the total surface area, taking $\pi \approx 22/7$. (The same three surfaces are exposed — only the formula for the top one changes, to πrl.)
3. PRACTICE A cube of edge 14 cm has a hemisphere of radius 3.5 cm fixed on its top face. Find the total surface area, taking $\pi \approx 22/7$. (Which face loses a circle here, and what does it gain in exchange?)
4. PRACTICE A cylindrical tin of radius 7 cm and height 10 cm is topped by a hemisphere of the same radius. Find its total surface area, taking $\pi \approx 22/7$.
6. PRACTICE A cube of edge 7 cm has a hemispherical depression of radius 3.5 cm carved into its top face. Find the total surface area of the remaining solid, taking $\pi \approx 22/7$.
7. PRACTICE A tent is shaped like a cylinder of radius 2.8 m and height 2.1 m, topped by a cone of the same radius and slant height 2.8 m. Find the area of canvas needed, taking $\pi \approx 22/7$.
8. PRACTICE A wooden toy is a cone of height 4 cm and radius 3 cm fixed on a hemisphere of the same radius. Find the total surface area, taking $\pi = 3.14$.
9. PRACTICE Two identical cubes of edge 5 cm are joined face to face to form a cuboid. Find the total surface area of the resulting cuboid.

5. PRACTICE A cone of radius 3.5 cm and slant height 10 cm is fixed onto a hemisphere of the same radius, forming a toy. Find its total surface area, taking $\pi \approx 22/7$.

ANSWERS **1.** Cylinder 220 cm², hemisphere 77 cm², base 38.5 cm²; total 335.5 cm².
2. Cone $= \pi rl = (22/7) \cdot 3.5 \cdot 12 = 132$ cm², so $220 + 132 + 38.5 = 390.5$ cm².
3. $6a^2 = 6 \cdot 196 = 1176$ cm², then $1176 - 38.5 + 77 = 1214.5$ cm².
4. Curved surface area of the cylinder $= 2 \cdot \pi rh = 440$ cm². Curved surface area of the hemisphere $= 2 \cdot \pi r^2 = 308$ cm². The cylinder's own flat bottom stays exposed: $\pi r^2 =$ 154 cm². Total $= 440 + 308 + 154 = 902$ cm².
5. Curved surface area of the cone $= \pi rl = 110$ cm². Curved surface area of the hemisphere $= 2 \cdot \pi r^2 = 77$ cm². Both flat faces vanish at the join, so the total is $110 + 77 =$ 187 cm².
6. Total surface area of the cube $= 6a^2 = 294$ cm². The depression removes a flat circle of area $\pi r^2 = 38.5$ cm² and adds a curved bowl surface of $2 \cdot \pi r^2 = 77$ cm². Total $=$ $294 - 38.5 + 77 = 332.5$ cm².
7. Curved surface area of the cylinder $= 2 \cdot \pi rh = 36.96$ m². Curved surface area of the cone $= \pi rl = 24.64$ m². No floor and no hidden flat circle to account for in canvas. Total canvas $= 36.96 + 24.64 = 61.6$ m².
8. Slant height $l = \sqrt{3^2 + 4^2} = 5$ cm. Curved surface area of the cone $= \pi rl = 47.1$ cm². Curved surface area of the hemisphere $= 2 \cdot \pi r^2 = 56.52$ cm². Total $= 47.1 + 56.52 =$ 103.62 cm².
9. The joined cuboid measures 10 cm by 5 cm by 5 cm. Total surface area $= 2(lb + bh + hl) = 2(50 + 25 + 50) = 250$ cm² — equivalently, the two cubes' combined 300 cm² minus the two 25 cm² faces hidden at the join.
Full solutions: **manthika.com/learn/math10-ch12**

BACK
Surface area and volume of boxes · p. 400

Further practice

PRACTICE SET

10. PRACTICE A test tube is a hollow cylinder of radius 3.5 cm and height 10 cm, open at the top and closed at the bottom by a hemisphere of the same radius. Taking the glass as having no thickness, find its total surface area, taking $\pi \approx 22/7$.

SOLUTION — Q10 · **Worked in full**

1.	$S_1 = 2 \cdot 22/7 \cdot 3.5 \cdot 10 = 220$	*Curved surface area of the cylindrical wall, using its radius and height.*
2.	$S_2 = 2 \cdot 22/7 \cdot 3.5^2 = 77$	*Curved surface area of the hemisphere at the closed bottom, using the same radius.*

SOLUTION — Q10 · **Worked in full**

3. $S = 220 + 77 = 297$

The tube is open at the top, so no flat circle appears anywhere — the total is just the two curved surfaces added.

11. PRACTICE A cube of edge 10 cm has a smaller cube of edge 4 cm placed centrally on its top face, entirely within it. Find the total surface area of the combination.

12. PRACTICE A cuboid of length 10 cm, breadth 8 cm and height 6 cm has a hemispherical depression of radius 3 cm carved into its top face. Find the total surface area of the remaining solid, taking $\pi = 3.14$.

13. PRACTICE An overhead water tank is a cylinder of radius 7 m and height 10 m, topped by a conical roof of the same radius and height 24 m, sealed with a flat base where it rests on its supports. Find the total surface area of sheet metal needed to build it, taking $\pi \approx 22/7$.

14. PRACTICE A cylindrical block of radius 3 cm and height 10 cm has a conical depression of the same radius and depth 4 cm hollowed into one flat end. Find the total surface area of the remaining solid, taking $\pi = 3.14$.

15. PRACTICE A hemisphere of radius r is fixed onto one flat end of a cylinder of the same radius. Rohan adds the cylinder's own total surface area to the hemisphere's own total surface area to get the combination's total surface area. By how much has he over-counted?

A. πr^2
B. $2\pi r^2$
C. $3\pi r^2$
D. $4\pi r^2$

16. PRACTICE A cone is fixed onto one flat end of a cylinder of the same radius, forming a single solid. Which surfaces are left out of the combination's total surface area?

A. The cylinder's curved surface
B. The cone's curved surface
C. The cylinder's top flat circle and the cone's flat base
D. The cylinder's bottom flat circle

17. PRACTICE A toy is a hemisphere of radius 5 cm topped by a cone of the same radius. The total height of the toy is 17 cm. Find its total surface area, taking $\pi = 3.14$.

18. PRACTICE A wooden box is a cuboid measuring 20 cm by 15 cm by 10 cm, standing on its 20 cm by 15 cm face on a table. A cubical block of edge 5 cm is glued centrally on the box's top face. The box is to be painted, except for the face resting on the table. Find the area to be painted.

19. PRACTICE A decorative bead is made by joining two cones of the same radius 8 cm base to base — one of height 6 cm and the other of height 15 cm. Find its total surface area, taking $\pi = 3.14$.

ANSWERS **10.** 297 cm². **11.** 664 cm². **12.** 404.26 cm². **13.** 1144 m². **14.** 263.76 cm². **15.** B — $2\pi r^2$. **16.** C — The cylinder's top flat circle and the cone's flat base. **17.** 361.1 cm². **18.** 1100 cm². **19.** 678.24 cm².
Full solutions: **manthika.com/learn/math10-ch12**

Practice: Exercise 12.2

PRACTICE SET

1. PRACTICE A cylinder of radius 7 cm and height 6 cm is topped by a cone of the same radius and height 9 cm. Find the volume of the solid, taking $\pi \approx 22/7$. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	work out $\pi r^2 = (22/7) \cdot 7^2 = 154$ cm² once, and keep it	*both parts stand on the same circle, so this number is used twice and is exact*
2.	volume of the cylinder $= \pi r^2 h = 154 \cdot 6 = 924$ cm³	*a cylinder is its base circle carried straight up its height*
3.	volume of the cone $= (1/3)\pi r^2 h = (1/3) \cdot 154 \cdot 9 = 462$ cm³	*a cone on the same base takes a third of the height's worth*
4.	volume of the solid $= 924 + 462 = 1386$ cm³	*nothing is hollowed out here, so the two volumes simply add — a join hides faces, never volume*
5.	the check — the cone counts as a cylinder 3 cm tall, so the whole solid is a plain cylinder $6 + 3 = 9$ cm tall, and $154 \cdot 9 = 1386$ cm³	*two different routes to the same number, so a slip in either one shows up at once*

2. PRACTICE A solid cylinder of radius 7 cm and height 9 cm has a conical cavity of the same radius and depth 9 cm hollowed out from one end. Find the volume of the remaining solid, taking $\pi \approx 22/7$. (The same cone as above, 462 cm³ — this time taken away instead of added.)

3. PRACTICE A toy is a hemisphere of radius 3 cm with a cone of the same radius and height 10 cm fixed on its flat face. Find the volume of the toy, taking $\pi = 3.14$. (A hemisphere needs no height of its own — its volume is $(2/3)\pi r^3$.)

6. PRACTICE A solid cylinder of radius 6 cm and height 14 cm has a conical cavity of the same radius and depth 9 cm hollowed from one end. Find the volume of the remaining solid, taking $\pi = 3.14$.

7. PRACTICE A gulab jamun is shaped like a cylinder of radius 1.5 cm with a hemisphere of the same radius at each end, giving a total length of 6 cm. Find its volume, taking $\pi = 3.14$.

4. PRACTICE A solid is a cylinder of radius 7 cm and height 10 cm, topped by a cone of the same radius and height 6 cm. Find its volume, taking $\pi \approx 22/7$.

5. PRACTICE A toy is a hemisphere of radius 3 cm topped by a cone of the same radius and height 8 cm. Find its volume, taking $\pi = 3.14$.

8. PRACTICE A wooden pen stand is a cuboid of length 8 cm, breadth 6 cm and height 3 cm, with two conical holes of radius 0.7 cm and depth 1.5 cm drilled into it. Find the volume of wood left in the stand, taking $\pi \approx 22/7$.

9. PRACTICE A solid is formed by scooping a hemispherical cavity of radius 3 cm out of each end of a solid cylinder of radius 3 cm and height 10 cm. Find the volume of the remaining solid, taking $\pi = 3.14$.

ANSWERS **1.** Cylinder 924 cm³, cone 462 cm³; total 1386 cm³.
2. $154 \cdot 9 = 1386$ cm³ for the cylinder, less 462 cm³ for the cavity; 924 cm³ remains.
3. Hemisphere $= (2/3) \cdot 3.14 \cdot 27 = 56.52$ cm³, cone $= (1/3) \cdot 3.14 \cdot 9 \cdot 10 = 94.2$ cm³; total 150.72 cm³.
4. Volume of the cylinder $= \pi r^2 h = 1540$ cm³. Volume of the cone $= 1/3 \cdot \pi r^2 h = 308$ cm³. Both parts are joined, so they add: $1540 + 308 = 1848$ cm³.
5. Volume of the hemisphere $= 2/3 \cdot \pi r^3 = 56.52$ cm³. Volume of the cone $= 1/3 \cdot \pi r^2 h = 75.36$ cm³. Total $= 56.52 + 75.36 = 131.88$ cm³.
6. Volume of the cylinder $= \pi r^2 h = 1582.56$ cm³. Volume removed by the cavity $= 1/3 \cdot \pi r^2 h = 339.12$ cm³. Remaining $= 1582.56 - 339.12 = 1243.44$ cm³.
7. The cylinder's own length is $6 - 2 \cdot 1.5 = 3$ cm. Volume of the cylinder $= \pi r^2 h = 21.195$ cm³. Volume of each hemisphere $= 2/3 \cdot \pi r^3 = 7.065$ cm³, so both give 14.13 cm³. Total $= 21.195 + 14.13 = 35.325$ cm³.
8. Volume of the cuboid $= lbh = 144$ cm³. Volume of one conical hole $= 1/3 \cdot \pi r^2 h = 0.77$ cm³, so two holes remove 1.54 cm³. Wood left $= 144 - 1.54 = 142.46$ cm³.
9. Volume of the cylinder $= \pi r^2 h = 282.6$ cm³. Volume of each hemispherical cavity $= 2/3 \cdot \pi r^3 = 56.52$ cm³, so both remove 113.04 cm³. Remaining $= 282.6 - 113.04 = 169.56$ cm³.
Full solutions: **manthika.com/learn/math10-ch12**

Further practice

PRACTICE SET

10. PRACTICE A wooden cube of edge 8 cm has a hemispherical hole of radius 3 cm scooped out of its top face, to hold a candle. Find the volume of wood left in the cube, taking $\pi = 3.14$.

SOLUTION — Q10 · **Worked in full**

1. $V_1 = 8^3 = 512$ *Volume of the cube on its own.*

SOLUTION — Q10 · Worked in full

2.	$V_2 = 2/3 \cdot 3.14 \cdot 3^3 = 56.52$	*Volume of the hemisphere scooped out of the top.*
3.	$V = 512 - 56.52 = 455.48$	*The hemisphere is hollowed out, so its volume is subtracted from the cube's.*

11. PRACTICE A municipal water tank is a cylinder of radius 9 m and height 15 m, topped by a hemispherical dome of the same radius. Find its volume, taking $\pi = 3.14$.

12. PRACTICE A solid is formed by fixing a cone onto a cylinder of the same radius. Which statement about its volume is always true?

A. It equals the sum of the two solids' volumes.

B. It is less than the sum of the two solids' volumes, because of the hidden face.

C. It is greater than the sum of the two solids' volumes, because of the hidden face.

D. It equals the volume of the larger solid alone.

13. PRACTICE A cubical block of wood has edge 10 cm. A conical hole of radius 3 cm and depth 4 cm is drilled into one face. Find the volume of wood left, taking $\pi = 3.14$.

14. PRACTICE A solid metal ball of radius 6 cm fits exactly inside a cylindrical box of the same radius and height 12 cm. Find the volume of empty space inside the box around the ball, taking $\pi = 3.14$.

15. PRACTICE A paperweight is a cuboid of length 20 cm, breadth 15 cm and height 8 cm, with a hemisphere of radius 6 cm fixed on its top face. Find the total volume of the paperweight, taking $\pi = 3.14$.

16. PRACTICE A solid metal rod is made of two cylinders joined end to end: a base cylinder of radius 7 cm and height 40 cm, topped by a narrower cylinder of radius 3.5 cm and height 20 cm. If 1 cm³ of the metal has a mass of 8 g, find the mass of the rod in kilograms, taking $\pi \approx 22/7$.

17. PRACTICE A rectangular stone block measures 2 m by 1.5 m by 1 m. A cylindrical hole of radius 14 cm is drilled all the way through it, along its height of 1 m. Find the volume of stone left, in cubic metres, taking $\pi \approx 22/7$. Convert the radius to metres first.

18. PRACTICE A wooden bead is made by joining two cones of the same radius 5 cm base to base — one of height 6 cm and the other of height 9 cm. Find its volume, taking $\pi = 3.14$.

19. PRACTICE A solid hemisphere of radius 6 cm has a conical cavity of the same radius and depth 6 cm scooped out from its flat face. Find the volume of the remaining solid, taking $\pi = 3.14$.

ANSWERS **10.** 455.48 cm³. **11.** 5341.14 m³. **12.** A — It equals the sum of the two solids' volumes. **13.** 962.32 cm³. **14.** 452.16 cm³. **15.** 2852.16 cm³. **16.** 55.44 kg. **17.** 2.9384 m³. **18.** 392.5 cm³. **19.** 226.08 cm³.
Full solutions: **manthika.com/learn/math10-ch12**