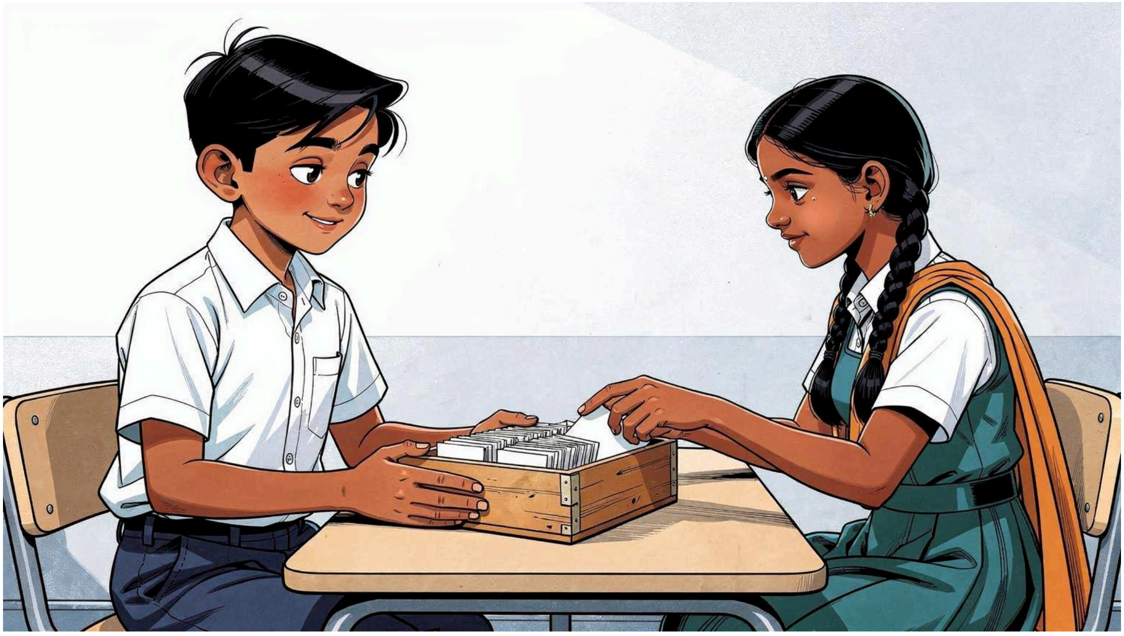
14 CHAPTER 14 Probability

WHAT THIS CHAPTER BUILDS

You will learn to count equally likely outcomes and to give the chance of an event as a fraction that always lies between 0 and 1.

STATISTICS AND PROBABILITY UNIT

IN THIS CHAPTER



Dhruv holds the box of paper chits steady while Bhavisha reaches in to draw one, every chit the same.

Getting started

Counting equally likely outcomes

A STORY

Forty chits in a box

Dhruv holds a wooden box of paper chits steady on the desk. Bhavisha reaches in to draw one.

“There are 40 chits, one for each of us,” Dhruv says. “The name that comes out reads the notice at assembly.”

“Every chit is cut the same size,” Bhavisha says. “So no name is easier to reach than any other.”

“Then what is the chance the chit has your name on it?” Dhruv asks.

“One chit out of 40,” Bhavisha says. “So $1/40$.”

“And the chance it has one of our names on it?”

“All 40 out of 40. That is 1. It cannot miss.”

“And the chance it says the headmaster’s name?” Dhruv asks, grinning.

“None of the 40,” Bhavisha says. “That is 0. No chance can be less than that, and none can be more than 1.”

Dhruv tilts the box towards her. “Go on, then. Draw.”

You will learn to count the outcomes that are equally likely, and to write every chance as a number from 0 to 1.

A coin comes up heads or tails. A die shows one of six faces. A card drawn from a deck is one of fifty-two.

Toss a coin in your head. List every result it could give. You should find two: heads and tails. Nothing about the coin favours one side over the other.

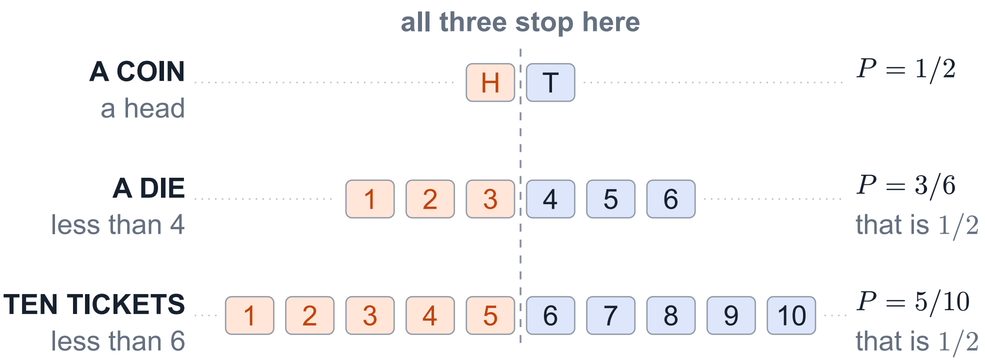
That is what makes the count useful. When every outcome is equally likely this way, the probability of an event is one count: favourable outcomes over total outcomes. That number is always between 0 and 1, inclusive. It is never negative. It is never bigger than one whole.

Two coins, two dice, and a bag of coloured marbles all reduce to the same count. We list every outcome first, then we divide. We check the list against the count each worked example gives.

RULE
List every outcome first, and check none is more likely than the rest, before dividing.

KEY
Probability is one count: favourable outcomes over all outcomes, once every outcome is equally likely.

Different counts, the same share



A probability is the count of the marked outcomes over the count of all of them. One of two, three of six and five of ten are different counts of the same share, so every marked block ends on the same line.

Probability is a share, not a count: doubling both the favourable and the total outcomes leaves it unchanged.

A SCHOLAR INDIA REMEMBERS

Many thinkers before Dignaga accepted what a trusted person said as a way of knowing. They also accepted a comparison with something similar. Dignaga did not. He kept only two ways: what you see, and what you work out from it. In probability we work the same way. We do not trust a feeling about a coin. We list the outcomes, and we count them.

Dignaga · the kingfisher

IN THE WILD

Picking one folded chit from a box holding one chit per student, to choose a class monitor, is fair because every chit is equally likely to come out. Each student's chance is then a plain count: 1 out of the number of chits in the box.

CHECK YOURSELF

1. This chapter's opening rule says the probability of an event, once every outcome is equally likely, is

- **A.** the number of favourable outcomes divided by the number of unfavourable outcomes
- **B.** the number of all possible outcomes divided by the number of favourable outcomes
- **C.** the number of favourable outcomes multiplied by the total number of outcomes
- **D.** the number of favourable outcomes divided by the number of all possible outcomes

2. Why must the outcomes being counted be equally likely before this rule can be used?

- **A.** it assumes every outcome has the same chance
- **B.** outcomes that are not equally likely cannot be counted or listed at all
- **C.** equally likely outcomes always come in an even number, like 2, 4, or 6
- **D.** the rule only works for probabilities greater than one half

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IN EVERY CHAPTER

Before you start

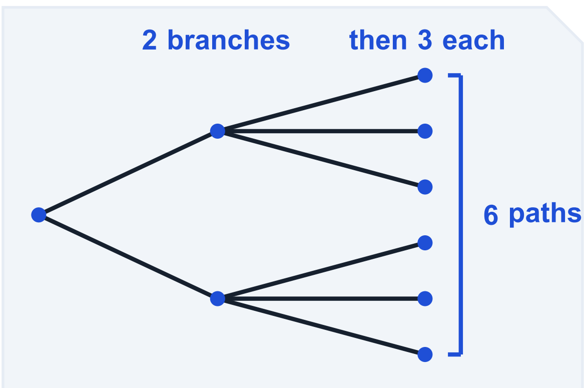
DO THIS FIRST

You already found the probability of a coin or a die. Now count outcomes from two objects. Try each check below.

- **Equally likely outcomes** (Class 9, Ganita Manjari, page 157).
  Here: every count here first checks that no outcome is favoured.
  Check: A fair coin has 2 equally likely outcomes, each with $P = 1/2$.
- **The probability formula** (Class 9, Ganita Manjari, page 161).
  Here: every $P(E)$ here divides favourable outcomes by this same total.
  Check: A die has 6 outcomes, 3 of them even, so $P = 3/6 = 1/2$.
- **Sample space** (Class 9, Ganita Manjari, page 160).
  Here: two dice give 36 ordered pairs, all listed before any counting.
  Check: Two coins give a sample space of 4 outcomes: HH, HT, TH, TT.
- **Tree diagrams** (Class 9, Ganita Manjari, page 168).
  Here: the 36 outcomes for two dice come from this same branching.
  Check: A two-step choice with 2 then 3 branches gives $2 \cdot 3 = 6$ paths.
- **Between zero and one** (Class 9, Ganita Manjari, page 173).
  Here: every probability worked out here also falls in this same range.
  Check: $P = 5/36$ lies between 0 and 1.

If any of these felt new, read the page named before going on.

Two branches, then three each



2 · 3 = 6

Each first-step branch opens into three second-step branches, so the paths multiply: 2 · 3 = 6.

A tree branches into 2 paths, then 3 more from each, giving 2 times 3 equals 6 outcomes.

Trials, experiments, and events

A random experiment is a trial with an uncertain result. Every result it could give is known in advance, even though the result itself is not.

Tossing a coin is one example. Nobody can say ahead of time whether it lands heads or tails. But only those two results are possible, and that much is known before the coin is even tossed.

An event is one or more of an experiment's possible results. Getting a head, when a coin is tossed, is one such event.

Name every possible result first. Only then does calling something an event make sense. We can try this on a die throw: we list all six results first, then check the list against what a die can show.

Your turn: a spinner has four equal sectors, coloured red, blue, green and yellow. List every result one spin can give. (Answer: four results, red, blue, green and yellow, since the spinner has exactly those four sectors and nothing favours one over another.)

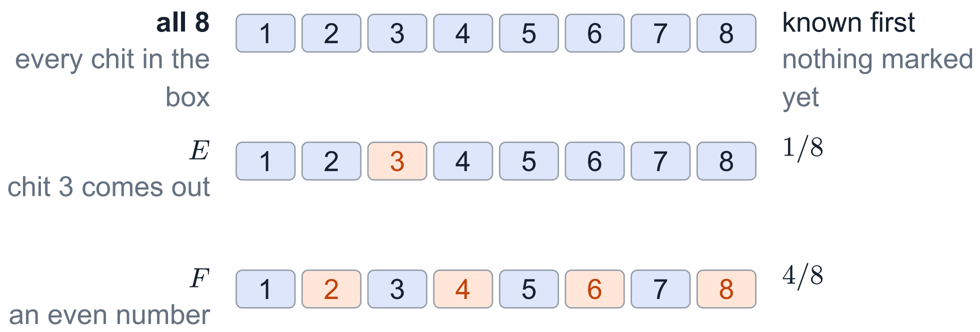
TRY
Is 'getting a head' an event?

Answer: Yes. It is one of the two possible results of tossing a coin.

BACK
Counting equally likely outcomes · p. 480

RULE
Name every possible result before calling anything an event.

The whole list first, then the event picked out of it



The box's eight chits are all known before anything is drawn, and an event is simply some of those eight picked out.

CHECK YOURSELF

3. A random experiment is best described as a trial where

- **A.** the result is completely unpredictable, and even the possible results are unknown
- **B.** the result is always the same, no matter how many times the trial is repeated
- **C.** the result depends on how carefully the trial is carried out
- **D.** the possible results are known, but which one occurs is not

4. A student says: "flipping a coin is a random experiment because we can never know anything about it in advance." What is wrong with this?

- **A.** the outcomes are known in advance — only which one occurs isn't
- **B.** nothing is wrong — a random experiment truly tells us nothing in advance
- **C.** the coin must be flipped many times before it counts as a random experiment
- **D.** a random experiment only applies to games of chance, not everyday actions

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Equally likely outcomes

Outcomes are equally likely when nothing favours one over another. A fair coin's head and tail are equally likely for this reason: nothing about a fair coin favours one side over the other.

Not every experiment keeps this property at every level. Picture a bag holding more red balls than blue ones. Drawing any one ball is equally likely for every ball in the bag. But drawing a red ball is not equally likely to drawing a blue one.

We count the balls first: more reds than blues means the colours cannot be equally likely, even though each single ball still is.

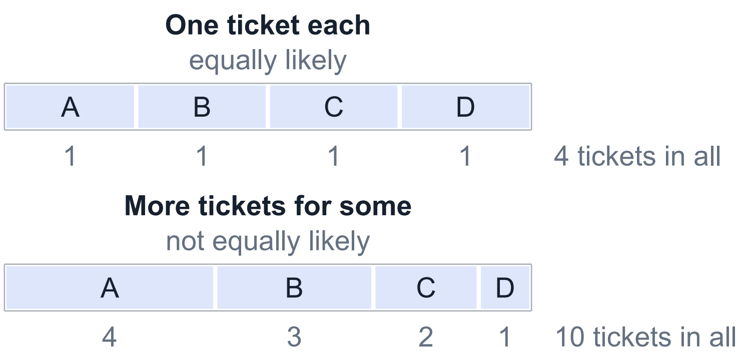
Being equally likely does not automatically pass from individual outcomes to the categories built out of them. We check which level is being counted: individual items, or groups of them.

Your turn: a bag holds 2 red balls and 5 blue balls. Is red as likely as blue? (Answer: no, since each of the 7 balls is equally likely, but 5 are blue and only 2 are red.)

CAUTION
More red balls than blue means 'red' and 'blue' are not equally likely, even though each individual ball is.

KEY
Equally likely means no reason to expect one outcome over another.

The same four names, and only one of these draws is fair



The same four names are drawn twice, and only the draw that gives every name one ticket makes the four outcomes equally likely.

A SCHOLAR INDIA REMEMBERS

What does equally likely mean, exactly? Say it without the word likely. Each outcome has the same chance as every other outcome. A fair die has 6 faces, and each face has the same chance of coming up. A die with one heavy corner does not.

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CHECK YOURSELF

5. Outcomes of a trial are called equally likely when

A. there are exactly two possible outcomes to choose between

B. no outcome is favoured over another

C. each outcome has occurred the same number of times so far

D. the outcomes are listed in a fixed, agreed order

6. A bag has 5 red balls and 1 blue ball. A student says the red and blue outcomes are equally likely, since there are only two colours. What is wrong with this?

- **A.** nothing is wrong — with two colours, each has probability 1/2
- **B.** the outcomes can never be called equally likely once there is more than one ball of any colour
- **C.** the bag needs equal numbers of every colour before probability can even be discussed
- **D.** only the six balls are equally likely, not the two colours

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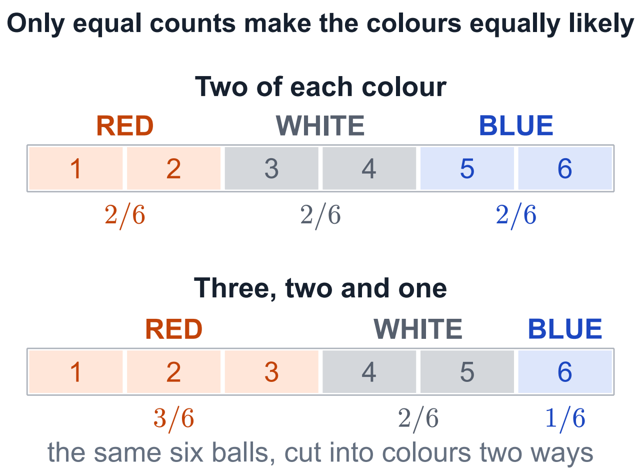
The ideas

Count outcomes, not categories



Both bags hold six equally likely balls, but the colours are equally likely only in the bag where every colour has the same count.

The classical probability formula needs equally likely things to count. Falling into equally many categories is not enough.

Picture a bag of coloured balls. It has as many colour-categories as colours. But the colours are equally likely in only one case: when every colour holds exactly the same number of balls. We count the individual balls of each colour first, before deciding anything about the colours themselves.

The individual balls are what is actually equally likely, not the colours. Each ball is as likely to be drawn as any other. Each colour has an equal share of the probability only when every colour holds exactly the same count. With 2 red balls and 1 blue ball, is red really as likely as blue?

Count the individual, equally likely items first. We group them into categories only afterwards.

Your turn: a box holds 1 black pen and 3 blue pens. Find $P(\text{blue})$. (Answer: $3/4$, not $1/2$, since 3 of the 4 pens are blue, and only the blue-against-black split counts, not the number of colours.)

CAUTION
Three colours in the bag does not mean each colour has probability $1/3$.

RULE
Count the individual, equally likely items first; group them into categories only afterwards.

CHECK YOURSELF

7. A bag holds 4 green and 2 yellow discs, and one is drawn at random. Which statement is correct?

- **A.** the six discs are equally likely; the colours are not
- **B.** the two colours are equally likely, since there are only two colours to choose from
- **C.** the six discs are not equally likely, since some are green and some are yellow
- **D.** neither the discs nor the colours are equally likely until the bag is shaken

8. Why does treating the two colours as equally likely give the wrong probability in the discs example?

- **A.** it assumes equal colour counts, which is false here
- **B.** because colours cannot be assigned a probability at all
- **C.** because there is no formula for the probability of drawing a colour
- **D.** because a disc's colour changes depending on which one is drawn

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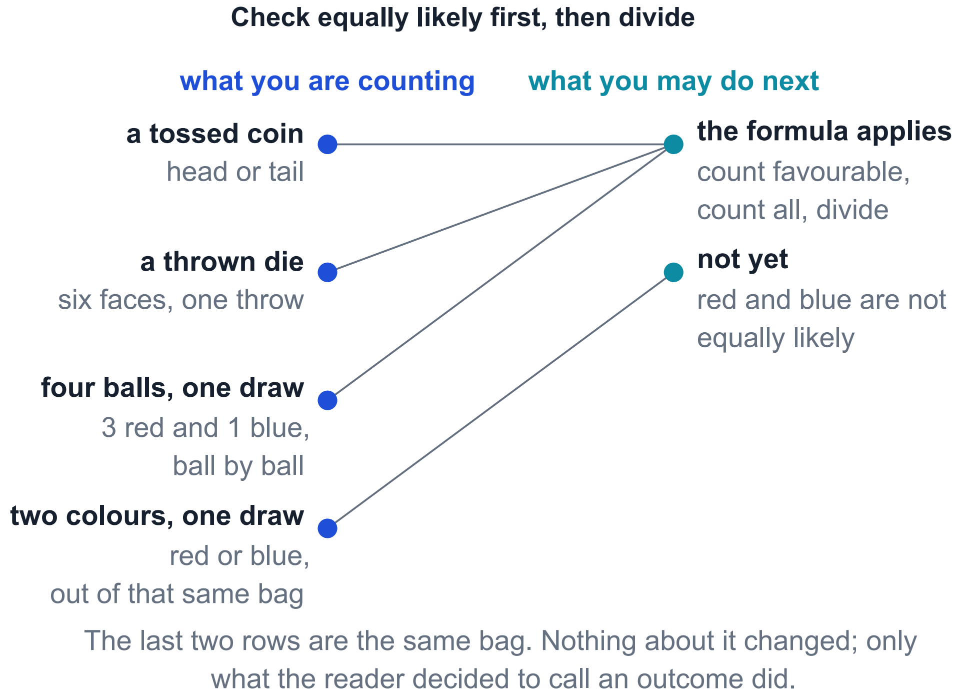
The classical probability formula



The formula applies to the first three counts, whose outcomes are equally likely, but not to the fourth, the same bag counted by colour.

The probability of an event E is written $P(E)$. It is the number of outcomes favourable to E, divided by the number of all possible outcomes.

This formula holds only when every outcome is equally likely. Without that condition, the same division counts nothing meaningful.

A fair die's six faces meet this condition. So do a fair coin's two sides. So does a well-shuffled deck's fifty-two cards. Once the outcomes are confirmed equally likely, we divide favourable by total.

The same one formula applies every time: favourable outcomes, divided by all possible outcomes, once they are confirmed equally likely.

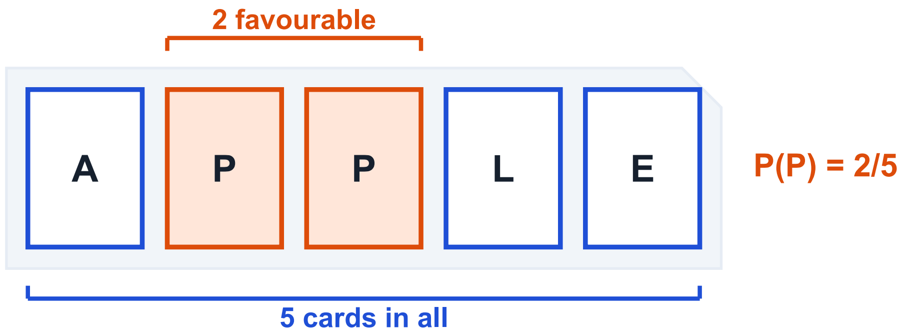
Your turn: five letter cards spell APPLE. Find $P(\text{P})$. (Answer: $2/5$, since 2 of the 5 cards, the two Ps, are favourable, out of 5 cards in all.)

BACK
Equally likely outcomes · p. 483

1795 — Pierre-Simon Laplace gave this definition of probability.

KEY
$P(E)$ = favourable outcomes over all possible outcomes, once they are equally likely.

Five cards, two of them P



Each card is equally likely to be drawn, so P(P) is the two P cards over all five: 2/5.

The word has four different letters but five cards, and the formula counts cards: two favourable over five in all.

IN THE WILD

Mendelian genetics predicts a $3:1$ ratio of dominant to recessive traits. It treats the four combinations of a monohybrid cross as equally likely outcomes, then counts how many favour each trait: the classical probability formula, applied to inherited alleles instead of a coin or a die.

CHECK YOURSELF

9. $P(E)$ is defined as

- **A.** number of all possible outcomes/(number of outcomes favourable to E)
- **B.** (number of outcomes favourable to E)/number of all possible outcomes
- **C.** (number of outcomes favourable to E)/(number of outcomes NOT favourable to E)
- **D.** (number of outcomes favourable to E) − (number of all possible outcomes)

10. This formula only gives a correct probability under one condition. What is it?

- **A.** the event E must have exactly one favourable outcome
- **B.** the total number of outcomes must be an even number
- **C.** every outcome of the experiment must be equally likely
- **D.** the experiment must be repeated a few times first

2 more in this set online.

Elementary events sum to one

An event with exactly one outcome is called an elementary event. Getting a head on a coin toss is one. So is getting a 3 on a die throw.

Every experiment breaks down into its full set of elementary events, one for each possible outcome. Take a die: it has six elementary events, one for each face.

The probabilities of all the elementary events of an experiment always add up to 1. We add the die's six probabilities together, $1/6$ six times over, and land on exactly 1.

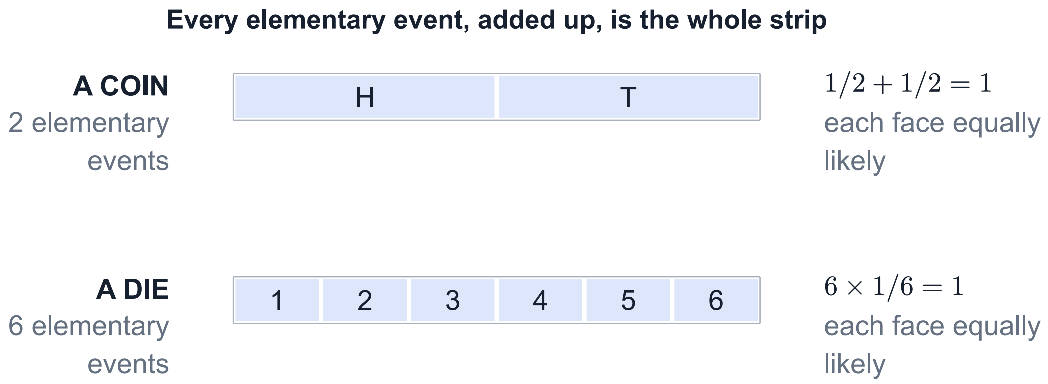
That total is a check worth running on any finished list of probabilities. If the elementary events do not add to 1, something in the counting has gone wrong.

Your turn: a spinner has three equal sectors, numbered 1, 2 and 3. Find each sector's probability, and check they add to 1. (Answer: each sector is $1/3$, and $1/3 + 1/3 + 1/3 = 1$.)

TRY
A die has six elementary events. What do their probabilities add to?

Answer: 1: six times $1/6$.

KEY
Every elementary event, added up, accounts for the whole experiment.



A coin's strip splits into two equal pieces and a die's strip into six, one piece for each equally likely outcome, together making one whole.

CHECK YOURSELF

11. An event with exactly one outcome, such as "getting a 3" on a die throw, is called

- **A.** a compound event
- **B.** an impossible event
- **C.** an elementary event
- **D.** a complementary event

12. Why must the probabilities of all the elementary events of an experiment always add up to 1?

- **A.** because there are always exactly two elementary events in any experiment
- **B.** because each elementary event must have probability exactly $1/2$
- **C.** the elementary events cover every outcome exactly once
- **D.** because probability is measured out of a total of 1 percent

1 more in this set online.

Between zero and one, always

The probability of any event E satisfies $0 \leq P(E) \leq 1$. It can never fall below zero, and it can never rise above one whole. We give both ends of that range their own name.

An event that can never happen is called an impossible event, and it carries $P(E) = 0$. An event that must always happen is called a sure event, or a certain event, and it carries $P(E) = 1$.

No genuine probability is ever negative or bigger than one whole.

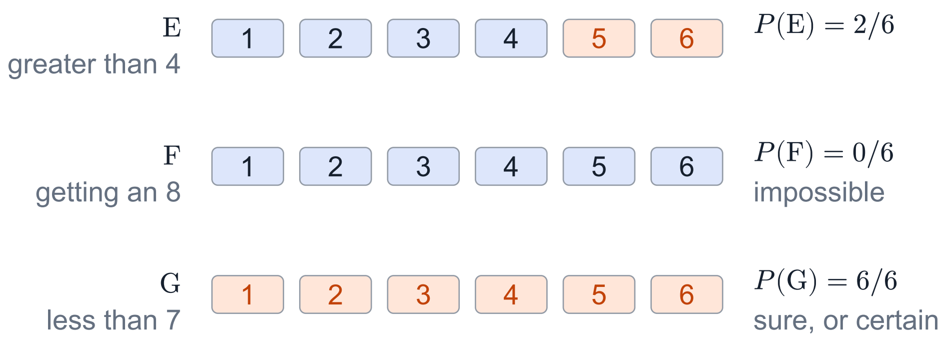
Your turn: someone claims an event has probability $5/4$. What went wrong? (Answer: more favourable outcomes were counted than there are outcomes in all, since no probability can ever be bigger than 1.)

BACK
The classical probability formula · p. 486

RULE
$P(E) = 0$ means impossible; $P(E) = 1$ means certain.

KEY
Probability never goes below 0 or above 1.

Zero, some, and all of the same six faces



Three rows shade the same six die faces for three events, from no faces shaded to all six.

A SCHOLAR INDIA REMEMBERS

A friend says the chance of rain is 1.5. Do you need to see the working? No. You can reject it at once. A probability counts favourable outcomes out of all the outcomes. The part cannot be bigger than the whole. So every probability is between 0 and 1, or equal to one of them.

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CHECK YOURSELF

13. For any event E, $P(E)$ always satisfies

A. $0 < P(E) < 1$

B. $-1 \le P(E) \le 1$

C. $0 \le P(E) \le 100$

D. $0 \le P(E) \le 1$

14. A student calculates a probability of 1.2 for an event. What does this tell you?

A. nothing is wrong — probabilities above 1 are possible for rare events

B. the event must be a sure event, since 1.2 rounds to 1

C. the event is impossible, since its probability lies outside the normal range

D. a mistake was made somewhere, since probability can never exceed 1

2 more in this set online.

Complementary events

For any event E, the event 'not E' is called the complement of E. It happens exactly when E does not, and one of the two always does.

The two probabilities always add to one. $P(E) + P(\text{not E}) = 1$, so $P(\text{not E}) = 1 - P(E)$. We find the other by one subtraction, once either probability is known.

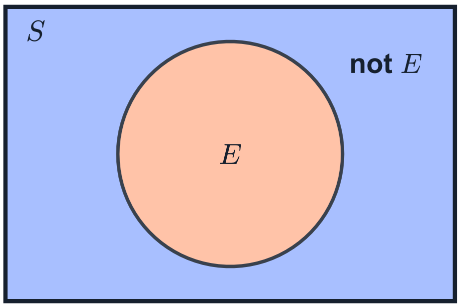
Finding $P(\text{not E})$ this way is often the shorter route, especially when 'not E' is easier to count than E itself. The subtraction only gives the true complement when 'not E' really is everything E is not: nothing more, nothing less.

Your turn: $P(E) = 3/8$. Find $P(\text{not E})$. (Answer: $5/8$, since $P(\text{not E}) = 1 - 3/8 = 5/8$.)

RULE
Know $P(E)$ already? Find $P(\text{not E})$ by subtracting from 1. Never recount directly.

KEY
Complement means 'everything the event is not', and the two probabilities always add to one.

"Not E" is everything S is, minus E



A circle marks event E inside a rectangle, and the rest of the rectangle marks its complement, together filling the whole shape.

CHECK YOURSELF

15. For any event E, the complement "not E" is the event that

- **A.** happens at the same time as E, but with a different outcome
- **B.** happens exactly when E does not happen
- **C.** always has exactly the same probability as E
- **D.** can never actually occur in a real trial

16. Why does $P(E) + P(\text{not } E)$ always equal 1?

- **A.** together, E and "not E" cover every outcome exactly once
- **B.** because E and "not E" are always equally likely
- **C.** because every experiment has exactly two possible events in total
- **D.** because probability values are always written as fractions that must add to one whole

2 more in this set online.

Outcomes from more than one object

Some experiments involve more than one object at once. Two coins tossed together is one example, and two dice thrown together is another.

The outcome of such an experiment is a pair, and which object shows what matters to the count. We list every outcome of tossing two coins before reading on.

Two coins give four equally likely outcomes: HH, HT, TH, TT. We check that list against this one: four outcomes, not three. HT and TH sound similar in words, one head and one tail either way. But they are two different outcomes: it matters which coin shows the head.

For n objects, each with k equally likely outcomes, the combined experiment has k^n equally likely outcomes. Two dice give $6 \cdot 6 = 36$ ordered pairs, since each die has six faces and there are two dice.

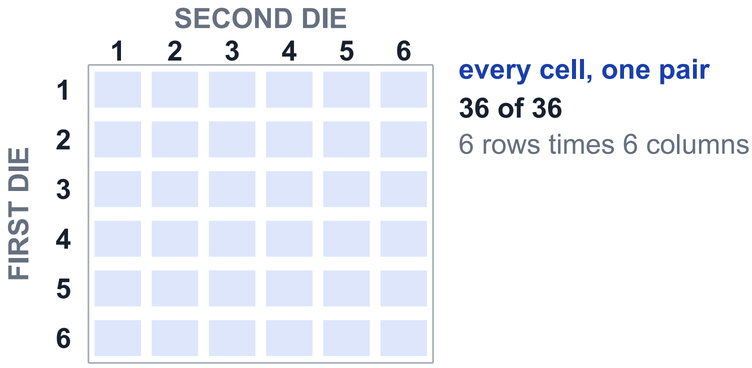
Your turn: two spinners each have three equal sectors, A, B and C. Find how many ordered outcomes one spin of both gives. (Answer: $3^2 = 9$: AA, AB, AC, BA, BB, BC, CA, CB, CC.)

CAUTION
HT and TH are two different outcomes, not one. The coin that shows the head matters.

BACK
Equally likely outcomes · p. 483

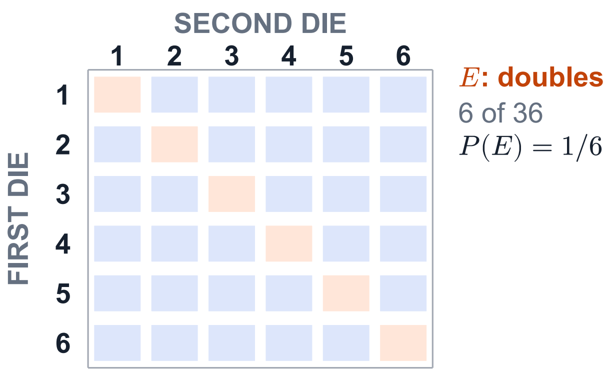
KEY
For n objects each with k equally likely outcomes, the combined outcomes are k^n.

Two dice, thirty-six ordered pairs



Rolling two dice gives ordered pairs, so the sample space is a six-by-six grid, empty before any event is marked.

Doubles run down the grid's own diagonal



On the same grid, the six doubles fall in a straight line along the diagonal, since a double is a cell where both dice agree.

CHECK YOURSELF

17. **When two coins are tossed together, the equally likely outcomes are**

- **A.** HH, TT — two outcomes, since only matching pairs count
- **B.** HH, HT, TH, TT — four outcomes in all
- **C.** HT, TH — two outcomes, since matching pairs are not really different
- **D.** HH, HT, TH, TT, HH — five outcomes, counting HH once for each coin

18. Why does tossing two coins give four outcomes rather than three?

A. because a coin can secretly land on its edge, adding a fourth case

B. because probability rules always require an even number of outcomes

C. which coin shows the head matters — HT and TH differ

D. because two coins must always be tossed one after the other, never together

2 more in this set online.

Common mistakes

Two coins give four outcomes, not three

Toss two coins in your head. List every outcome. Tossing two coins looks like it should give three outcomes: two heads, two tails, or one of each. So surely each of those three has probability $1/3$?

That count is wrong. The four outcomes HH, HT, TH and TT are all equally likely. Which coin shows the head matters, even though 'one of each' sounds like a single description.

Exactly one head has probability $2/4 = 1/2$, not $1/3$. Two of the four equally likely outcomes, HT and TH, give exactly one head, and we can count them again to check.

Listing three descriptions of an outcome is not the same as listing three equally likely outcomes. Only the full list of all four gives the correct count: HH, HT, TH, TT. Check for this trap in every sample space, and count the outcomes yourself from scratch.

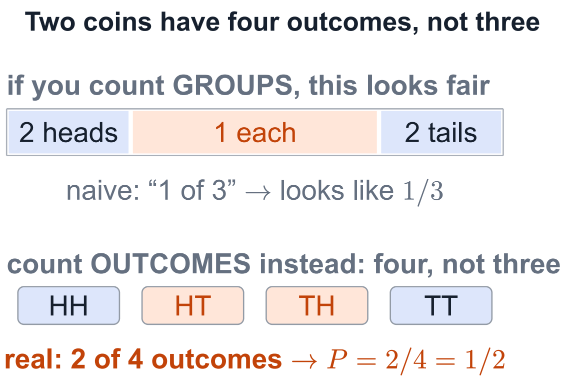
Your turn: two coins are tossed. Find $P(\text{two tails})$. (Answer: $1/4$, since only TT of the four equally likely outcomes gives two tails.)

TRY
Of HH, HT, TH, TT, how many give at least one head?

Answer: Three: all but TT.

CAUTION
Listing three descriptions of an outcome is not the same as listing three equally likely outcomes.

KEY
Four equally likely outcomes, not three unequal ones.



Grouping two coins' results into three bands looks fair, but splitting the wide middle band into its two separate outcomes gives the count of four.

THREE COINS ARE TOSSED. WHAT IS THE PROBABILITY OF EXACTLY TWO HEADS?

WEAK

Group the results by how many heads show: three heads, two heads, one head, no heads. Four descriptions, so exactly two heads looks like $1/4$.

But 'two heads' can happen with the tail on the first coin, on the second, or on the third: three different tosses. 'Three heads' can happen in only one way. The four descriptions are not equally likely, so $1/4$ cannot be right.

STRONG

List the $2 \cdot 2 \cdot 2 = 8$ equally likely outcomes: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

Exactly two heads are HHT, HTH and THH, so the probability is $3/8$. The four descriptions cover $1 + 3 + 3 + 1 = 8$ outcomes between them.

A SCHOLAR INDIA REMEMBERS

Two heads, two tails, or one of each. Are those three outcomes of one kind? No. One of each can happen in two ways: HT and TH. So there are four equally likely outcomes. The chance of one head and one tail is $2/4 = 1/2$.

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WATCH OUT

The trap: Listing two heads, two tails and one of each, the student divides evenly and calls each outcome $1/3$.

The correction: Each of the four two-coin outcomes is equally likely, at probability $1/4$, so exactly one head totals $1/2$.

CHECK YOURSELF

19. A student says: "Tossing two coins gives three outcomes — two heads, two tails, or one of each — so each has probability $1/3$." What is wrong with this?

- **A.** nothing is wrong — each of the three outcomes does have probability $1/3$
- **B.** the mistake is only in the fraction $1/3$; the three outcomes are listed correctly
- **C.** the four outcomes are equally likely, giving $1/2$
- **D.** two coins cannot be tossed together at all — they must be tossed one after another

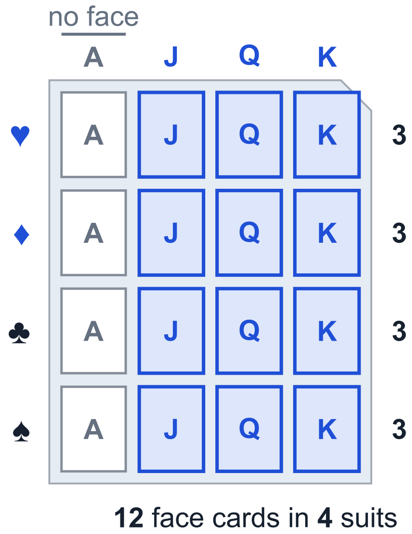
20. Two coins are tossed. A student lists the outcomes as "both heads", "both tails", and "mixed", assigning each probability $1/3$. What is $P(\text{exactly one head})$ really?

- **A.** $1/3$, matching the student's three-outcome count
- **B.** $1/2$ — "mixed" is really two outcomes, HT and TH
- **C.** $1/4$, since only HT counts as "mixed", not TH
- **D.** $2/3$, since two of the student's three listed outcomes are not "both heads"

3 more in this set online.

Twelve face cards, not sixteen

Three faces in each suit, and the ace has none



Only the jack, the queen and the king carry a face: 3 in each of the 4 suits, 12 in the deck. The ace shows one suit symbol and no face.

A grid of playing cards marks only the jack, queen and king as face cards, 12 in all, with the ace excluded.

A deck has four aces, four kings, four queens and four jacks. That looks like sixteen face cards, one card of each rank in every suit.

It is not sixteen. Only the jack, the queen and the king carry a face. Each suit holds three of them, so the deck holds 12 face cards in all. An ace shows a single suit symbol and no face at all. It is never counted among the face cards.

We can check this suit by suit: the jack, the queen and the king of hearts carry a face, and the ace of hearts does not.

Your turn: how many of the hearts are face cards? (Answer: three, the jack, the queen and the king of hearts. The ace of hearts is not one.)

BACK
The classical probability formula · p. 486

TRY
How many cards in a deck are neither a face card nor an ace?

Answer: $52 - 12 - 4 = 36$: nine cards in each of the four suits.

CAUTION
The ace shows a single pip and no face. It is never counted among the face cards.

FIRST TRY
~~A deck has 4 aces, 4 kings, 4 queens and 4 jacks. I called all 16 of them face cards and got $P(\text{not a face card}) = 1 - 16/52 = 36/52$.~~

SECOND LOOK
The ace carries a single pip, not a face. Only the king, the queen and the jack carry a face, 3 in every suit, 12 in the deck. So $P(\text{not a face card}) = 1 - 12/52 = 40/52 = 10/13$.

Count the ace with the number cards, not with the ones that carry a face.

ONE CARD IS DRAWN FROM A DECK. WHAT IS THE PROBABILITY IT IS A RED FACE CARD?

WEAK
Count an ace, a king, a queen and a jack in each red suit: 8 red face cards, so $P(\text{red face card}) = 8/52 = 2/13$.

But look at the ace of hearts. It carries a single heart and no face, so it cannot be a face card. The count of 8 is two too many.

STRONG
Only the jack, the queen and the king carry a face, three in each red suit, so there are $3 \cdot 2 = 6$ red face cards and $P(\text{red face card}) = 6/52 = 3/26$.

The other 20 red cards are the ace and 2 to 10 in each red suit: $2 \cdot 10 = 20$, and $20 + 6 = 26$.

Worked examples

WORKED EXAMPLE · A coin toss: head and tail

1. one coin, two equally likely outcomes: H, T — *We list both possible results first, before naming any event.*
2. E = 'getting a head' has 1 favourable outcome out of 2: $P(E) = 1/2$ — *This is the classical probability formula, applied to the simplest two-outcome experiment.*
3. F = 'getting a tail' has 1 favourable outcome out of 2: $P(F) = 1/2$ — *The same formula, applied to the other elementary event.*
4. $P(E) + P(F) = 1/2 + 1/2 = 1$ — CHECK ***Check that head and tail add to 1.*** *They are the coin's only two elementary events, so their probabilities must sum to a whole.*

RULE
Use this check on any experiment: do all its elementary events' probabilities add to 1?

TRY
Told only $P(E) = 1/2$ for 'head', how do you get $P(F)$ for 'tail'?

Answer: Subtract from 1: $1 - 1/2 = 1/2$.

CHECK YOURSELF

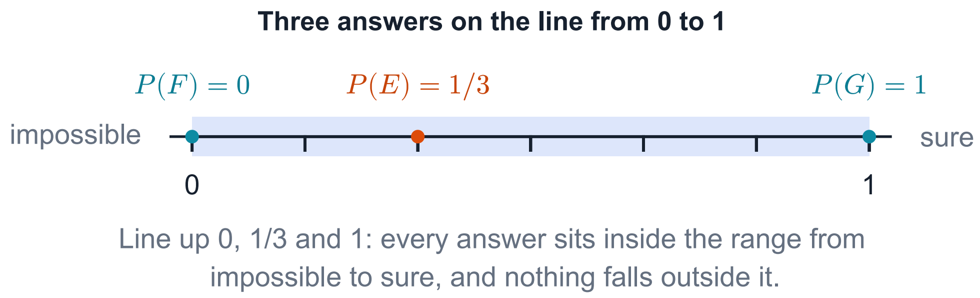
21. A coin is tossed once. Let E be "getting a head". $P(E)$ equals

- **A.** 1
- **B.** 0
- **C.** 1/2
- **D.** 1/4

22. In a single coin toss, why do $P(\text{head})$ and $P(\text{tail})$ each equal exactly 1/2?

- **A.** because a coin toss is always fair by law
- **B.** because heads and tails are opposite sides of the same coin
- **C.** because 1/2 is the only fraction less than 1
- **D.** two equally likely outcomes, one each for head and tail

1 more in this set online.



A number line from 0 to 1 marks P(F) = 0 at impossible, P(E) = 1/3, and P(G) = 1 at sure.

WORKED EXAMPLE · **A die throw: three events classified**

1. one die, six equally likely outcomes: $1, 2, 3, 4, 5, 6$ — *We list every outcome first, before deciding which ones satisfy each event.*
2. E = 'a number greater than 4' has outcomes $5, 6$: $P(E) = 2/6 = 1/3$ — *4 itself does not satisfy 'greater than 4', so only 5 and 6 count.*
3. F = 'getting 8' has no favourable outcome: $P(F) = 0/6 = 0$ — *This is an impossible event: 8 is not among the die's six faces.*
4. G = 'a number less than 7' has outcomes 1 to 6, all six: $P(G) = 6/6 = 1$ — *This is a sure event: every face of the die is less than 7.*
5. $0 \le 0 \le 1/3 \le 1 \le 1$ — **CHECK** ***Check that all three answers fall inside 0 to 1.*** *Line up 0, $1/3$ and 1: every one sits inside the range a probability must, from impossible up to sure.*

TRY
A fair die is thrown once. Let E be 'an odd number', F be 'getting 9', G be 'a number at most 6'. Find $P(E)$, $P(F)$, $P(G)$.

Answer: $P(E) = 3/6 = 1/2$, $P(F) = 0$, $P(G) = 1$: ordinary, impossible, and sure events.

CAUTION
'Greater than 4' means 5 and 6 only. 4 itself does not count, so $P(E) = 2/6$, not $3/6$.

RULE
List all six outcomes before deciding which ones satisfy the event.

FIND THE PROBABILITY OF ANY EVENT ON ONE DIE.

1. **List the six outcomes** Write out every result the die can show: 1, 2, 3, 4, 5, 6.
2. **Pick out the favourable ones** For the event, list which of those six outcomes satisfy it. 'Greater than 4' picks out 5 and 6.
3. **Divide by six** Divide the count of favourable outcomes by 6, the total number of outcomes. 'Greater than 4' gives $2/6 = 1/3$.
4. **Check the range** Every answer must sit between 0 and 1. An event with no favourable outcome gives 0, and an event every outcome satisfies gives 1.

CHECK YOURSELF

23. A fair die is thrown once. What is $P(\text{a number greater than } 4)$?

- **A.** $1/3$
- **B.** $2/3$
- **C.** $1/2$
- **D.** $4/6$

24. In the die example, $P(\text{getting } 8) = 0$. What does this illustrate about the classical formula?

- **A.** no favourable outcomes among the possible ones gives 0
- **B.** that the formula fails whenever the desired outcome is a large number
- **C.** that a six-sided die secretly has an eighth face that is never rolled
- **D.** that the total number of outcomes must be increased to include 8

2 more in this set online.

WORKED EXAMPLE · Marbles in a box: counting individuals

1.	box holds 3 blue, 2 white, 4 red marbles, 9 in all, one drawn at random	*The 9 individual marbles are equally likely, not the 3 colours.*
2.	$P(W) = 2/9$	*2 of the 9 marbles are white.*
3.	$P(B) = 3/9 = 1/3$	*3 of the 9 marbles are blue. We divide by 9, the total marbles, never by the 3 colours.*
4.	$P(R) = 4/9$	*4 of the 9 marbles are red.*
5.	$P(W) + P(B) + P(R) = 2/9 + 3/9 + 4/9 = 9/9 = 1$	CHECK ***Check that the three probabilities add to 1.*** *White, blue and red between them cover every marble in the box, so the three elementary events sum to a whole.*

BACK
Count outcomes, not categories · p. 485

CAUTION
$P(B) = 1/3$ here because 3 of 9 marbles are blue, not because there are three colours.

RULE
Divide by the total marbles, 9: never by the number of colours, 3.

Nine marbles, equally likely one at a time

BLUE			WHITE		RED			
1	2	3	4	5	6	7	8	9
$P(B) = 3/9$			$P(W) = 2/9$		$P(R) = 4/9$			

Every marble is one equally likely outcome, and the three colour bands are simply different widths because they hold different numbers of marbles.

CHECK YOURSELF

25. In the marbles example (3 blue, 2 white, 4 red, 9 in all), $P(\text{white})$ is

- **A.** 1/3
- **B.** 2/9
- **C.** 2/3
- **D.** 1/9

26. Why are the nine individual marbles equally likely here, but the three colours are not?

- **A.** each marble is equally likely; the colours are not
- **B.** because marbles are physical objects, and colours are just names
- **C.** because there are more marbles than colours
- **D.** because the bag was shaken before drawing

1 more in this set online.

WORKED EXAMPLE · A card draw: an ace, or not

1. one card drawn from a well-shuffled deck of 52 cards, all equally likely

The deck's 52 cards are the equally likely outcomes.

2. E = 'the card is an ace': 4 aces in the deck, so $P(E) = 4/52 = 1/13$

There is one ace per suit, and four suits.

3. $P(\text{not E}) = 1 - 1/13 = 12/13$

This is the complement rule: every non-ace card, found by subtracting from 1 rather than recounting all 48 of them directly.

4. $1/13 + 12/13 = 13/13 = 1$

CHECK ***Check that the ace probability and the not-ace probability add back to 1.*** *An event and its complement must always sum to a whole deck.*

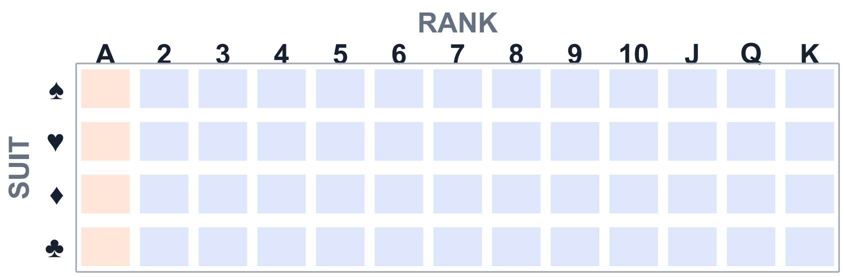
BACK
Complementary events · p. 489

TRY
How many kings are in the deck, and what is $P(\text{king})$?

Answer: 4 kings, one per suit: $P(\text{king}) = 4/52 = 1/13$.

RULE
Identify how the deck splits into 4 suits of 13 ranks, before counting favourable cards.

Four suits of thirteen ranks, one ace column



E: an ace — 4 of 52 cards, $P(E) = 1/13$

48 of 52 are not aces, $P(\text{not E}) = 12/13$

The deck laid out as four rows of thirteen columns highlights the four aces, one in each suit's row.

CHECK YOURSELF

27. One card is drawn from a well-shuffled deck of 52. $P(\text{ace})$ equals

- **A.** $1/4$
- **B.** $4/13$
- **C.** $1/52$
- **D.** $1/13$

28. Why is $4/52$ simplified to $1/13$ rather than left as $4/52$?

- **A.** because $4/52$ is not a valid probability until reduced
- **B.** $4/52$ and $1/13$ are the same value, simplified
- **C.** because the deck actually only has 13 cards once suits are ignored
- **D.** because dividing by 4 accounts for the four aces being drawn one at a time

1 more in this set online.

WORKED EXAMPLE · Not green, by the complement rule

1.	a bag of 3 green and 7 blue counters, 10 in all, one drawn at random	*We list the 10 counters first: they are the equally likely outcomes, not the two colours.*
2.	$P(\text{green}) = 3/10$	*Three of the ten counters are green.*
3.	$P(\text{not green}) = 1 - 3/10 = 7/10$	*This is the complement rule: subtract the green probability from 1, with no second count needed.*
4.	7 blue counters out of 10: $P(\text{not green}) = 7/10$	CHECK ***Check the direct count of blue counters agrees.*** *Seven of the ten counters are blue, which matches 7/10 found by subtraction.*

TRY
A box holds 5 pencils, 2 of them red. Find the probability that a pencil drawn at random is not red.

Answer: $P(\text{red}) = 2/5$, so $P(\text{not red}) = 1 - 2/5 = 3/5$.

CAUTION
Subtract the probability from 1, never the count from the total. It is $1 - 3/10$, not $10 - 3$.

RULE
Subtract from one only when you already know the first probability. Otherwise just count.

FIND THE PROBABILITY THAT AN EVENT DOES NOT HAPPEN.

1. Count the favourable outcomes Count how many outcomes make up the event itself. In the counters example, 3 of the 10 counters are green.

2. Divide to find the probability Divide that count by the total to get the event's own probability. $P(\text{green}) = 3/10$.

3. Subtract from one The probability of the event not happening is 1 minus that answer. $1 - 3/10 = 7/10$.

4. Check by counting directly Count the outcomes that make up 'not happening' on their own, and check the two answers agree. 7 of the 10 counters are blue, which matches $7/10$.

WORKED EXAMPLE · Two dice: a sum of eight

1. two dice, 36 equally likely outcomes since $6 \cdot 6 = 36$

   Each die has six faces, and both dice are thrown together.

2. E = 'the sum is 8' has favourable pairs $(2,6)$, $(3,5)$, $(4,4)$, $(5,3)$, $(6,2)$

   We list every ordered pair that sums to 8, none omitted and none repeated.

3. $P(E) = 5/36$

   Five favourable pairs out of the 36 equally likely ordered pairs.

4. $0 \leq 5/36 \leq 1$

   CHECK ***Check that the answer falls inside 0 to 1.*** *Every probability answer must fall inside this range, whenever a count is finished.*

BACK
Outcomes from more than one object · p. 490

TRY
How many ordered pairs sum to 7?

Answer: 6: $(1,6)$, $(2,5)$, $(3,4)$, $(4,3)$, $(5,2)$, $(6,1)$.

RULE
List every ordered pair before counting: $(2,6)$ and $(6,2)$ each count separately.

Which five of thirty-six sum to eight

SECOND DIE

FIRST DIE	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

E: the sum is 8
5 of 36
$P(E) = 5/36$

Fixing the total at eight picks out a slanting line of five cells, since raising one die while lowering the other keeps the sum unchanged.

Tara throws two dice and gets 4 and 5: one of the 36 throws, each as likely as any other.

FIND THE PROBABILITY OF AN EVENT ON TWO DICE.

1. **Count the total outcomes** Two dice give $6 \cdot 6 = 36$ ordered pairs, since each die has six faces.
2. **Decide which pairs the event wants** Pick the rule the event sets, such as the two faces summing to 8.
3. **List every pair in both orders** Write out every ordered pair that satisfies it, such as $(2, 6)$ and $(6, 2)$. These are two different pairs, not one.
4. **Divide by thirty-six** Divide the count of favourable pairs by 36, the total. Five pairs sum to 8, so $P(E) = 5/36$.
5. **Check by a second route** Read the first die from 2 to 6 and see that each value has exactly one partner making 8. Five values give five pairs.

CHECK YOURSELF

29. Two dice are thrown together. The probability that the two numbers add up to 8 equals

A. $1/11$

B. $4/36$

C. $5/11$

D. $5/36$

30. Why are there 36 equally likely outcomes to count from, rather than the 11 possible sums from 2 to 12?

- **A.** because 11 is simply the wrong count — there are actually 12 possible sums
- **B.** the 36 pairs are equally likely; the sums are not
- **C.** because two dice can only be thrown one at a time, doubling the outcomes
- **D.** because a sum of 8 can only be reached in one particular way

2 more in this set online.

Wrapping up

IN EVERY CHAPTER

Recap

- FORMULA: $P(E)$ = favourable outcomes over total outcomes. A die's 'greater than 4' gives $P(E) = 2/6 = 1/3$.
- BOUNDS: $0 \leq P(E) \leq 1$, always. An impossible event gives $P(E) = 0$, and a sure event gives $P(E) = 1$.
- COMPLEMENT: $P(E) + P(\text{not E}) = 1$. An ace gives $1/13 + 12/13 = 1$.
- ELEMENTARY EVENTS: every elementary event's probabilities add to 1. A die's six faces each give $1/6$, and six of those make 1.
- EQUALLY LIKELY: count individual outcomes, not categories. A box of 9 marbles gives each marble $1/9$, not each of 3 colours $1/3$.

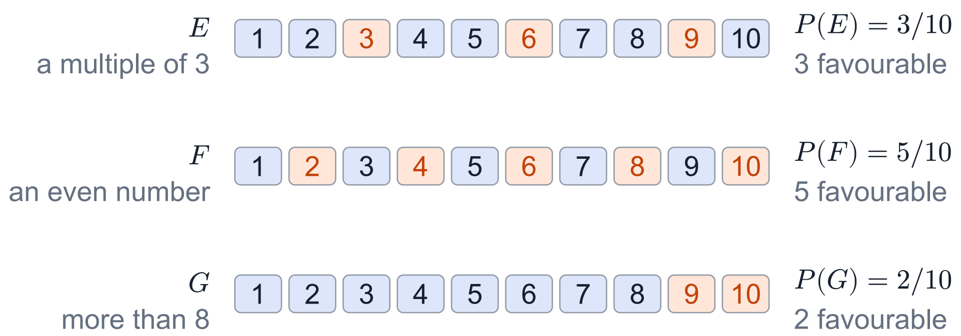
We count every probability here the same way: outcomes divided by outcomes, applied to the sample space. Try one last check yourself: recount any worked example above from scratch, and see that you land on the same answer.

KEY
One formula, applied to a set of equally likely outcomes.

BACK
Trials, experiments, and events · p. 482

RULE
Check equally likely first, count favourable and total second, divide last.

One list of ten, three different questions



The same ten tickets answer all three questions, and only the count of favourable ones changes from row to row.

CHECK YOURSELF

31. This chapter's opening claim is that every probability problem reduces to one formula. What is that formula, in words?

- **A.** favourable outcomes over total outcomes
- **B.** count the total outcomes, then subtract the favourable ones
- **C.** list every possible outcome and pick the most likely one
- **D.** average the number of favourable and unfavourable outcomes

32. Across this chapter's coin, die, card, marble, and dice-pair examples, what is the one skill that actually varies from problem to problem?

- **A.** which formula to apply, since each situation uses a different one
- **B.** whether to add or multiply the final probabilities
- **C.** finding the right counts each time
- **D.** how many decimal places to round the final probability to

1 more in this set online.

CHECK YOURSELF: THE WHOLE CHAPTER

33. This chapter's rule is for equally likely outcomes. It finds the probability of an event by

- **A.** multiplying the favourable outcome count by the total outcome count
- **B.** dividing the total outcome count by the favourable outcome count
- **C.** dividing the favourable outcome count by the total outcome count
- **D.** subtracting the favourable outcome count from the total outcome count

34. A bag contains 5 red balls and 3 blue balls, all equally likely to be drawn. What is $P(\text{red})$, the probability of drawing a red ball?

- **A.** $3/8$, the fraction of blue balls instead of red
- **B.** $8/5$, the total number of balls divided by the number of red balls
- **C.** 5, the number of red balls, without dividing by the total
- **D.** $5/8$, the number of red balls divided by the total number of balls

3 more in this set online.

IN EVERY CHAPTER

Where you will meet this

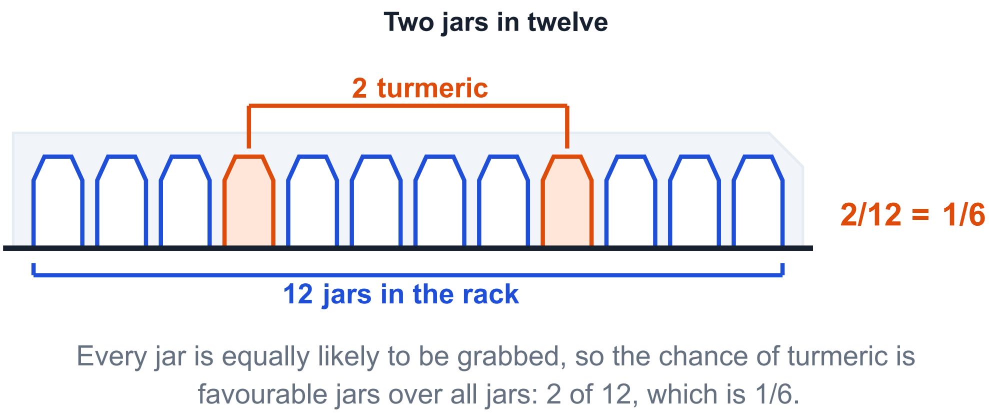
You judge a chance more often than you think, from a raffle ticket to a bus seat. Here are seven places the probability formula gives you a real number.

- **Needing a six to start in Ludo.** In Ludo, a piece leaves home only when you roll a 6 on a fair die.
  The maths: the six faces are equally likely, and just one of them is favourable.
  The chance of a 6 on one roll is $1/6$. So on average one roll in six lets a piece out.
- **Winning a shop's lucky draw.** A shop's lucky draw box holds 200 entry slips, one of them yours. 8 slips will be drawn as winners.
  The maths: each slip is equally likely to be drawn, so your chance is the 8 winning slips over

all 200.
The chance of winning is $8/200 = 1/25$.

- **Grabbing the right spice jar in the dark.** A rack holds 12 identical-looking jars. 2 of them are turmeric.
  The maths: each jar is equally likely to be grabbed, so the chance of turmeric counts favourable jars over all twelve.
  The chance of grabbing turmeric is $2/12 = 1/6$.

BACK
Count outcomes, not categories · p. 485



Two of twelve identical jars are turmeric, giving a probability of 2 over 12, which is 1/6.

- **A quiz app picking today's question.** A quiz app's question bank has 40 questions. 6 are about history.
  The maths: the chance of a history question is the number of history questions over all the questions in the bank.
  The chance is $6/40 = 3/20$.
- **Picking a good shirt from a trader's box.** A trader's box holds 100 shirts, and 12 of them have a defect. A customer picks one at random.
  The maths: getting a good shirt is the complement of getting a defective one, so the two chances add to 1.
  The chance of a defective shirt is $12/100 = 3/25$. So the chance of a good one is $1 - 3/25 = 22/25$.
- **Getting a window seat on a bus.** A bus has 40 seats, and 16 of them are window seats. Your ticket gets a seat at random.
  The maths: the chance of a window seat is the number of window seats over all the seats on the bus.
  The chance of a window seat is $16/40 = 2/5$.
- **Winning a move in a board game.** A game uses two spinners, each numbered 1 to 4. You win if the two numbers add to 5.
  The maths: two spinners give an outcome pair, and each of the equally likely pairs counts once.
  There are $4 \cdot 4 = 16$ equally likely pairs. Four pairs add to 5, so the chance of winning is $4/16 = 1/4$.

Your turn. A parking lot has 60 spaces, and 15 are reserved for staff. If a space is chosen at random, what is the chance it is a staff space? *Answer:* The chance is $15/60 = 1/4$.

Practice: Exercise 14.1

PRACTICE SET

1. PRACTICE A bag contains 4 red and 8 green counters, identical apart from colour. One counter is drawn at random. Find the probability that it is red, and the probability that it is not red. (Worked in full below — read it, then do the next two the same way.)

SOLUTION — Q1 · **Worked in full**

1.	$n(S) = 4 + 8 = 12$	*every counter is equally likely, so the total is the number of COUNTERS, not the number of colours*
2.	$n(\text{red}) = 4$, so $P(\text{red}) = 4/12 = 1/3$	*count the favourable counters, divide by the total, and reduce*
3.	$P(\text{not red}) = 1 - 1/3 = 2/3$	*the event 'not red' is the complement of 'red', and the two probabilities always add to one*
4.	the check — count the green counters directly, $8/12 = 2/3$	*the rule and the direct count must agree; if they disagree, one of the two counts is wrong*

2. PRACTICE A tray holds 8 mangoes, of which 2 are unripe. One mango is picked at random. Find the probability that it is unripe, and the probability that it is not unripe. (The same two steps as above — the total first, then subtract from 1.)
3. PRACTICE A bag contains only white and black beads. The probability of drawing a white bead at random is $3/7$. Find the probability of drawing a black bead. (Nothing to count here — the first probability is already given, so the rule does the rest.)
4. PRACTICE A fair coin is tossed once. Find the probability of getting a tail.
5. PRACTICE A die is thrown once. Find the probability of getting a number less than 3.
6. PRACTICE A card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn is a king.
10. PRACTICE A carton has 20 bulbs, of which 3 are defective. One bulb is drawn at random. Find the probability that it is not defective.
11. PRACTICE Two coins are tossed together. Find the probability of getting at least one head.
12. PRACTICE Two dice are thrown together. Find the probability that the sum of the numbers on the two dice is 10.
13. PRACTICE A bag contains only red and blue marbles. The probability of drawing a red marble is $2/5$. Find the probability of drawing a blue marble.
14. PRACTICE A bag contains only red balls. One ball is drawn at random. Find the probability that the ball drawn is blue.
15. PRACTICE A spinner is divided into 8 equal sectors numbered 1 to 8. Find the probability that the spinner stops on a number less than 9.

7. PRACTICE A card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn is not a face card.

8. PRACTICE A bag contains 5 red, 3 green and 2 yellow discs, identical apart from colour. One disc is drawn at random. Find the probability that it is green.

9. PRACTICE A box contains 6 good pens and 4 defective pens. One pen is drawn at random. Find the probability that the pen drawn is defective.

16. PRACTICE State whether the statement is true or false, and justify: 'The probability of an event is always a whole number.'

17. PRACTICE A student says: 'A die has six faces, numbered 1 to 6. Since 6 is even, the probability of an even number is 1/6.' Find the error, and give the correct probability.

18. PRACTICE A die is thrown once. Verify that the probabilities of its six elementary events add up to 1.

ANSWERS

1. $P(\text{red}) = 4/12 = 1/3$, and $P(\text{not red}) = 1 - 1/3 = 2/3$ — counting the green counters directly gives $8/12 = 2/3$ as well.
2. $P(\text{unripe}) = 2/8 = 1/4$, and $P(\text{not unripe}) = 1 - 1/4 = 3/4$.
3. $P(\text{black}) = 1 - 3/7 = 4/7$. 4. 1 favourable outcome out of 2: $P(\text{tail}) = 1/2$.
5. Numbers less than 3 are $1, 2$: $P(E) = 2/6 = 1/3$.
6. 4 kings in the deck: $P(\text{king}) = 4/52 = 1/13$.
7. Face cards number 12 (4 jacks, 4 queens, 4 kings): $P(\text{face}) = 12/52 = 3/13$, so $P(\text{not face}) = 1 - 3/13 = 10/13$.
8. $5 + 3 + 2 = 10$ discs in all: $P(\text{green}) = 3/10$.
9. $6 + 4 = 10$ pens in all: $P(\text{defective}) = 4/10 = 2/5$.
10. $P(\text{defective}) = 3/20$, so $P(\text{not defective}) = 1 - 3/20 = 17/20$.
11. Outcomes HH, HT, TH, TT; at least one head is HH, HT, TH: $P(E) = 3/4$.
12. Pairs summing to 10: $(4, 6)$, $(5, 5)$, $(6, 4)$: $P(E) = 3/36 = 1/12$.
13. $P(\text{red}) + P(\text{blue}) = 1$, so $P(\text{blue}) = 1 - 2/5 = 3/5$.
14. No blue ball is in the bag, so drawing one is an impossible event: $P(\text{blue}) = 0$.
15. Every one of the 8 numbered sectors shows a number less than 9, so this is a sure event: $P(E) = 8/8 = 1$.
16. False. A probability satisfies $0 \le P(E) \le 1$ and is a whole number only at the two extremes, 0 and 1; most probabilities, like 1/2 or 3/10, are not whole numbers.
17. The error: 6 being even is a fact about one face's label, not a count of favourable outcomes. The even-numbered faces are $2, 4, 6$, three faces out of six, so $P(\text{even}) = 3/6 = 1/2$.
18. Each face has probability 1/6: $1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 6/6 = 1$.

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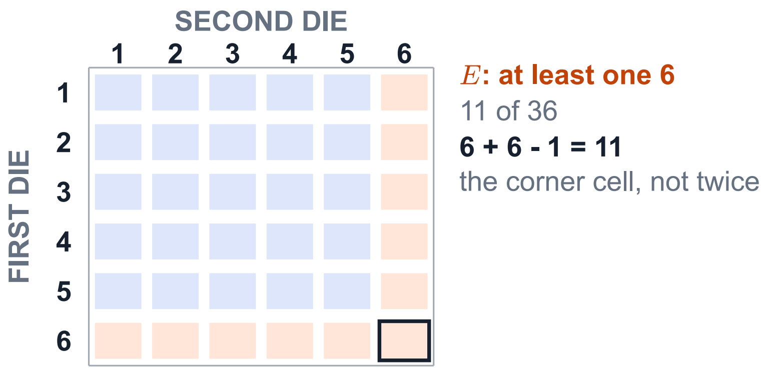
BACK
The classical probability formula · p. 486

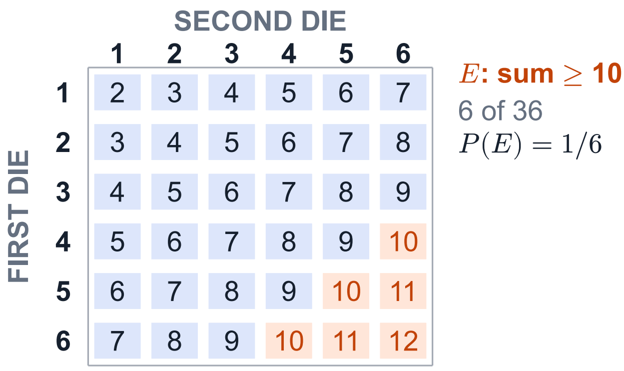
Further practice

At least one 6: a row, a column, one shared cell



A row and column of six overlap in one cell, so eleven cells mark at least one six, with the shared cell outlined.

Ten or more crowds into the far corner



Cells with a total of ten or more cluster into a solid corner of the grid, rather than fall along a single line.

PRACTICE SET

19. PRACTICE A fair die, with faces numbered 1 to 6, is thrown once. Find the probability of getting a multiple of 3.

> **SOLUTION — Q19 · Worked in full**
>
> **1.** $n(S) = 6$ — *The die has six equally likely outcomes, 1 to 6.*
>
> **2.** $n(E) = 2$ — *The multiples of 3 from 1 to 6 are 3 and 6.*
>
> **3.** $P(E) = 2/6 = 1/3$ — *Divide favourable outcomes by the total and reduce.*

20. PRACTICE One card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card is a black king.

21. PRACTICE A box contains 50 discs, numbered 1 to 50, one number per disc. One disc is drawn at random. Find the probability that the number on it is divisible by 7.

28. PRACTICE If $P(E) = 0.37$ for an event E, what is $P(\text{not E})$?

A. 0.63
B. 1.37
C. -0.37
D. 0.37

29. PRACTICE A fair die, with faces numbered 1 to 6, is thrown once. Find the probability of getting a number that is both even and prime.

22. PRACTICE A spinner is divided into 5 equal sectors, numbered 1 to 5. Find the probability that the spinner stops on an odd number.

23. PRACTICE A cricket team has 11 players, wearing jersey numbers 1 to 11. One player is chosen at random to be the captain. Find the probability that the captain's jersey number is greater than 8.

24. PRACTICE Which of the following values cannot be the probability of an event?

A. 0.2
B. -0.4
C. 3/5
D. 1

25. PRACTICE A sweet shop has a box of 30 ladoos: 12 besan, 10 motichoor and 8 kaju. One ladoo is picked at random from the box. Find the probability that it is not a motichoor ladoo.

26. PRACTICE A person is equally likely to be born on any day of the week. Find the probability that the day of birth is a day whose name starts with the letter S.

27. PRACTICE A jar holds 9 blue and 15 pink glass bangles, all the same size. One bangle is picked at random from the jar. Find the probability that it is not pink.

30. PRACTICE One card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn shows a number from 2 to 10, not an ace or a face card.

31. PRACTICE Two fair coins are tossed together. Find the probability of getting two tails.

32. PRACTICE A number is chosen at random from the integers 1 to 10. What is the probability that it is a multiple of 4?

A. 1/10
B. 1/5
C. 3/10
D. 2/5

33. PRACTICE Two fair dice, each numbered 1 to 6, are thrown together. Find the probability that the sum of the numbers on the two dice is 9.

34. PRACTICE Two fair dice, each numbered 1 to 6, are thrown together. Find the probability that the sum of the numbers on the two dice is at least 11.

35. PRACTICE Two fair dice, each numbered 1 to 6, are thrown together. Find the probability that the sum of the numbers on the two dice is a prime number.

36. PRACTICE A box contains 30 cards, numbered 1 to 30, one number per card. One card is drawn at random. Find the probability that the number on it is neither a multiple of 3 nor a multiple of 5.

ANSWERS **19.** $P(E) = 1/3$ **20.** $P(E) = 1/26$ **21.** $P(E) = 7/50$ **22.** $P(E) = 3/5$ **23.** $P(E) = 3/11$ **24.** B — -0.4. **25.** $P(E) = 2/3$ **26.** $P(E) = 2/7$ **27.** $P(E) = 3/8$ **28.** A — 0.63. **29.** $P(E) = 1/6$ **30.** $P(E) = 9/13$ **31.** $P(E) = 1/4$ **32.** B — 1/5. **33.** $P(E) = 1/9$ **34.** $P(E) = 1/12$ **35.** $P(E) = 5/12$ **36.** $P(E) = 8/15$
Full solutions: **manthika.com/learn/math10-ch14**